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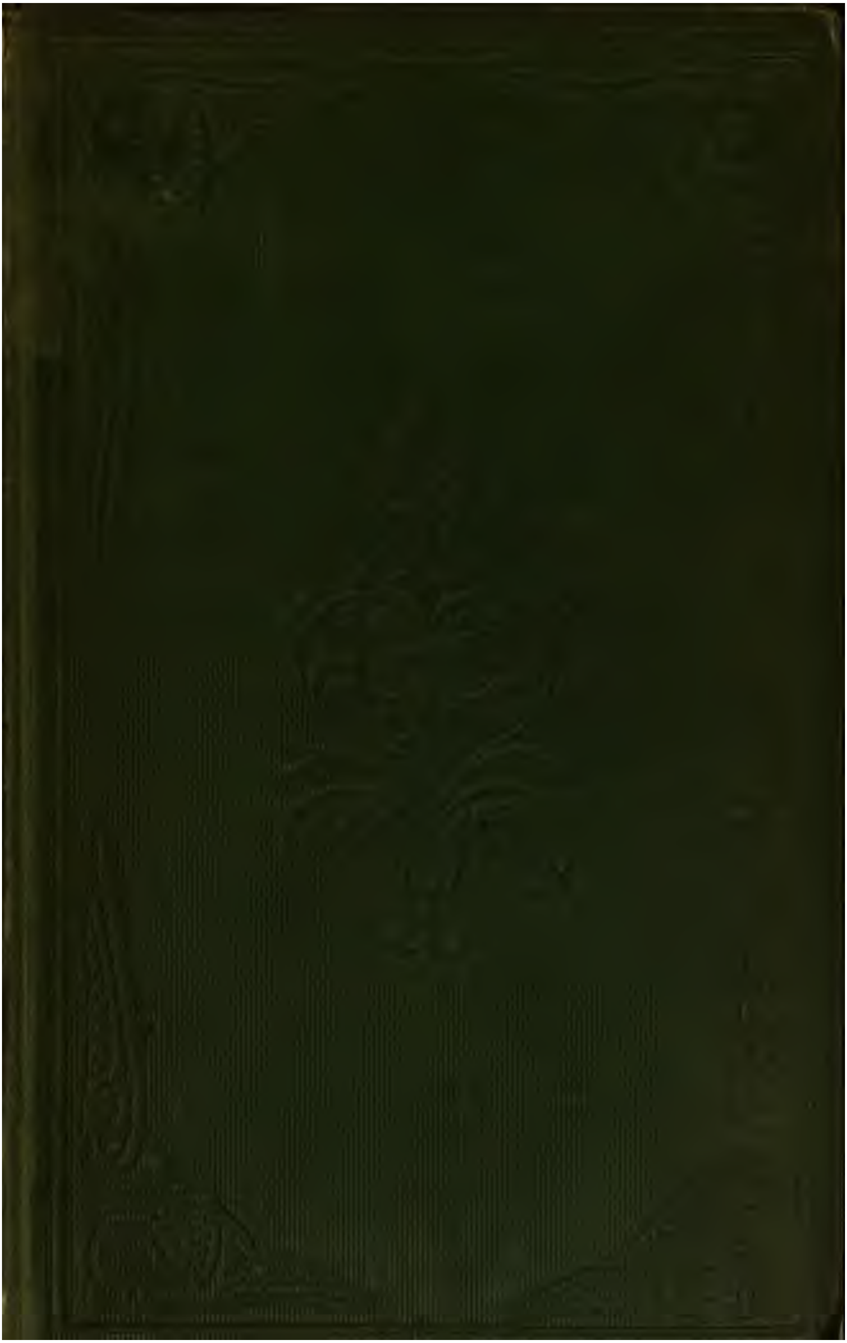

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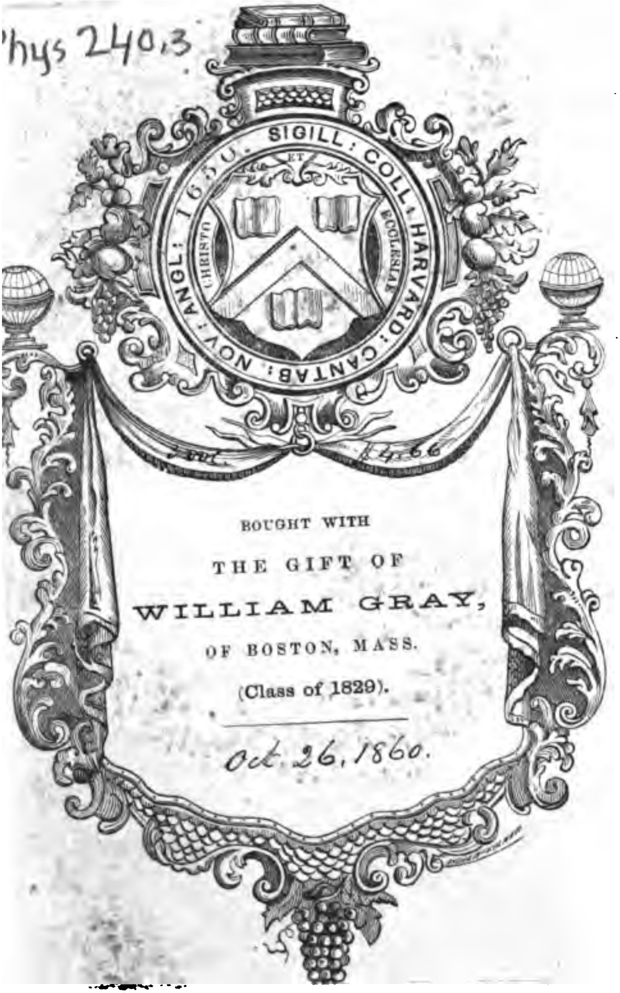

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ELEMENTS

OF

PHYSICS.

BY

Carl Friedrich

C. F. PESCHEL,

PRINCIPAL OF THE ROYAL MILITARY COLLEGE, DRESDEN,

ETC.

TRANSLATED FROM THE GERMAN,

WITH NOTES,

BY E. WEST.

Illustrated with Diagrams and Woodcuts.

PART I.

PONDERABLE BODIES.

SECOND EDITION.

LONDON:
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TRANSLATOR'S PREFACE.

THE study of the physical sciences has become of late almost universal; partly because it has been found well suited to cultivate habits of observation and reasoning, which cannot fail to be of use in the business of life, and partly because the practical application of scientific principles to the details of almost every department of manufacturing and agricultural industry, has rendered some acquaintance with natural philosophy and chemistry indispensable. Instruction in these branches of knowledge now forms part of the education given to most boys at school: to meet their requirements so many treatises have already been published, that to add to the number may seem a mark of presumption. To this objection the translator was not altogether insensible whilst preparing the present Text-book for the press; he believes, however, that on examination it will be found that the manual now offered to the public occupies ground not taken up by any existing treatise on Natural Philosophy, for the works on this science in our language are, almost without exception, either purely theoretical, or they are, in the strictest sense of the term, popular.

The present work is of a mixed character, being a systematic treatise, adapted for the use of schools, excluding difficult mathematics, but retaining the mathematical method. It contains many numerical illustrations to exercise the student in the application of the formulæ, besides copious tables, which will give it a value to the practical man. The foreign measures have, for the most part, been reduced to English standards, and a few notes have been added to the original matter.

The Berlin "Militair Litteratur Zeitung, 1843, No. 29.," and the "Repertorium der deutschen und überlandischen Litteratur, 1844, 19^e Heft," have noticed Mr. Peschel's work in commendatory terms.

In rendering this work into English, the translator has endeavoured to give a faithful version without doing violence to the idioms of our own language, and has inclosed in brackets the additions which he has made in two or three instances to the original text. From page 233. to page 243., he has substituted the methods given by Baily and Littrow for the barometric measurement of altitudes for that of the author, which was both more involved than theirs, and also too long for the space that could be afforded for it in the translation. A very few Notes have been added, either explanatory of the text, or for the sake of giving the opinions of eminent philosophers, when these have not altogether coincided with what the author advances.

AMERSHAM,
Jan. 27. 1845.

PREFACE

TO

THE SECOND EDITION.

In preparing this Second Edition for the press the whole volume has been carefully revised: our own standards of measurement have been used throughout: much new matter is introduced, space for which was obtained by cancelling what appeared to be of minor utility: many paragraphs have been re-written, errors have been corrected, and every effort has been made, without altering the essential character of the work, to adapt it thoroughly for the use of the English Student.

Amersham,
June 12. 1854.

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ELEMENTS

OF THE

PHYSICS OF PONDERABLE BODIES.

SECTION I.

I. THE PHYSICAL PROPERTIES OF BODIES.

§ 1.

The term Nature is used to signify the assemblage of all the bodies in the Universe; it includes, therefore, whatever exists and is the subject of change. Of the existence of these bodies we are rendered conscious by the impressions they produce on our senses; we are further aware that the condition of these bodies is subject to a variety of changes, whence we infer that external causes are in operation which bring about these alterations.

The object of *Physical Science* is, therefore, to investigate nature under these two aspects. That branch of it whose end is to discover what is characteristic of different bodies, in order to arrange or classify them, is called *Natural History;* the range, however, of this division must be understood as being limited to the changes which organised bodies undergo during life. That department of science which is called *Natural Philosophy*, or *Physics*, comprehends an inquiry into the causes which excite the changes that take place in the material world, and into the laws by which these changes are regulated,

Natural Philosophy, or *Physics*, in the more restricted sense of the term, teaches the general properties of *unorganised* bodies, the forces which act upon them, the laws they obey, the results thus produced, and all those *external* changes which leave the substance of the body unaffected.

Chemistry, on the contrary, treats of the individual properties of bodies, by which, as regards their material constitution, they may be distinguished one from another; it also investigates the transformations which take place in the interior of a body, transformations by which the very substance of the body is altered and remodelled ; and, lastly, its aim is to detect those laws by which chemical changes are regulated. That division of this science which has for its object the resolution of bodies, and the investigation of their elements, is called *analytical chemistry ; synthetical chemistry*, on the contrary, by varied combinations of their elements, teaches us which are similar substances ; and lastly, the doctrine of *Definite Proportionals* shows the precision with which chemical combinations are formed in certain definite proportions by weight and volume.

Natural philosophy is a science of *observation* and *experiment*, for by these two modes we deduce the varied information we have acquired about material bodies: by the former we notice any changes that transpire in the condition or relations of any body as they spontaneously arise without interference on our part ; whereas, in the performance of an experiment, we purposely alter the natural arrangement of things to bring about some particular condition that we desire. To accomplish this, we make use of appliances called *philosophical* or *chemical apparatus*, the proper use and application of which it is the office of *Experimental Physics* to teach.

> If we notice that in winter water becomes converted into ice, we are said to make an observation: if, by means of freezing mixtures or evaporation, we cause water to freeze, we are then said to perform an experiment.

These experiments are next subjected to calculation, by which are deduced what are sometimes called *the laws of nature*, or *the rules, in accordance with which, if such and such conditions be given, such and such results will necessarily follow.* To express these laws with the greatest possible brevity mathematical symbols are used. When it is not practicable to represent them with mathematical precision, we must be contented with inferences and assumptions based on analogies, or with probable explanations or *hypotheses.*

> A hypothesis gains in probability the more nearly it accords with the ordinary course of nature, the more numerous the experiments on which it is founded, and the more simple the explanation it offers of the phenomena it is intended to account for.

§ 2.

The *physical properties* of bodies are those external signs by which the existence of matter in its various forms is made evident to our minds ; the senses constitute the medium through which this knowledge is communicated.

> All our senses, however, are not equally made use of for this purpose ; we are generally guided in our decisions by

the evidence of sight and touch. Still sight alone is frequently incompetent, as there are bodies which cannot be perceived by that sense, as, for example, all colourless gases; again, some of the objects of sight are not material, as, the shadow, the image in a mirror, spectra formed by the refraction of the rays of light, &c. Touch, on the contrary, decides indubitably as to the existence of any body.

These properties may be divided into *essential*, or *principal*, and *non-essential*, or *accessory*. The former are such as we find common to all bodies, and without which we cannot conceive of their existing; the latter, though not necessary to bodies, yet becoming known to us as the result of closer investigations, as well as on account of their applications, are worthy of distinct notice.

§ 3.

The essential properties of every body are *Extension* and *Impenetrability*.

Extension is that property, in consequence of which every body occupies a certain limited space. It is the condition of the mathematical idea of a body; by it the *volume* or size of the contained space, as well as its boundary, or *figure*, is determined.

It is well known that the extension of bodies is expressed by the three dimensions, length, breadth, and thickness. The computation from these data follows geometrical rules.—Regular and irregular figures; crystals, globes.

Impenetrability is evinced in the fact that a second body cannot enter into the space occupied by another body without previously thrusting out the former.

Matter, then, is whatever occupies space, and possesses extension and impenetrability; all bodies are matter with fixed boundaries.

One might be led perhaps to imagine that this property belonged only to solids, since we see them penetrating both air and water; but on a closer observation aided by experiments it is demonstrated that this essential property of matter is common to the above-named and to all similar bodies.

First Exp. Fill with water a hollow cylinder into which a piston fits accurately;—the piston cannot now be thrust into the water, thus showing it to be impenetrable.

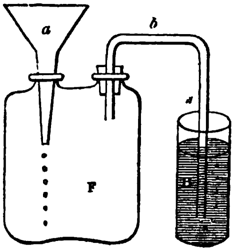
Second Exp. Invert a glass in water or any other liquid; the air not being able to escape prevents the entrance of the fluid, and is thus proved to be an impenetrable body.

One ingenious application of this property is the diving-bell.

The difficulty of pouring liquor into a vessel having only one small hole arises from the impenetrability of the air, as the liquid can run into the vessel only as the air makes its escape. The following experiment will illustrate this fact.—

Third Exp. In one mouth (*a*) of a two-necked bottle insert a funnel, and in the other (*b*) a siphon, the longer leg of which is immersed in a glass of water. Now let water be poured into the funnel *a*, and it will be seen that, just as it descends into the vessel F, the air makes its escape through the tube *b*, as is proved by the ascent of the bubbles in the water in B.

Fig. 1.



§ 4.

The non-essential properties of bodies are: *compressibility, expansibility, porosity, divisibility, elasticity,* and *gravity.*

Compressibility is that property of bodies by virtue of which they may be made to occupy a smaller space; and *expansibility* is that in consequence of which they may be made to fill a larger, without in either case altering the quantity of matter they contain.

Both these changes are produced in all bodies, as we shall see presently, by the action of heat; many bodies may also be reduced in bulk by other external causes, as pressure, percussion, &c.—Rolling, hammering, and stamping of metals.

§ 5.

Since all bodies admit of compression and expansion, it follows of necessity that there must be interstices between their minutest particles.

That property of matter, in consequence of which it does not completely fill the space bodies occupy, but leaves holes between their particles, is called *porosity;* these *pores* are themselves filled with other matter, commonly with atmospheric air. In many cases the pores are perceptible by the naked eye; in others, by the aid of the magnifying glass, and even when by this means they cannot be rendered visible, their existence may be inferred from experiments.

Sponge, cork, wood, bread, &c., are bodies whose pores may be noticed by the naked eye. The human skin appears full of them when viewed through a magnifying glass: the porosity of water is shown by the ascent of air-bubbles when its temperature is raised.

Exp. Pour some quicksilver into a piece of leather, press it, and there will be a fine metallic shower, proving the porosity of this animal substance.

The *mass* and *density* of bodies are estimated according to their various degrees of porosity. The mass represents the number of material particles in a body: the greater the mass for the same volume, the less porous the substance will be. Density expresses the relation of the masses when the volumes are equal: *i. e.* "*of two bodies, that is the more dense which with equal bulk contains the greater mass.*"

1) Let one body A have a density D and a mass M, and another body B of equal volume the density d, and the mass m; then $D : d = M : m$.

2) Two bodies of equal masses, A and B, whose volumes are V and v; then $D : d = v : V$.

3) Two bodies A and B, whose masses M and m, and whose volumes V and v, are unequal; in this case

$$D : d = Mv : mV$$
$$= \frac{M}{V} : \frac{m}{v}.$$

§ 6.

The *divisibility* of bodies is that property in consequence of which, by various mechanical means, such as beating, pounding, grinding, &c., we can reduce them to particles homogeneous to each other, and to the entire mass; and these again to smaller, and so on.

The mathematical divisibility of matter may be carried on *ad infinitum;* though its division, by mechanical means, is subject to limitation. Many examples, however, prove that it may be carried on to an incredible extent: we are furnished with numerous instances among natural objects, whose existence we can detect only by means of the most acute senses, or even by supplying these with artificial aids; the size of these objects we can calculate only approximately.

---

§ 6. It has been sometimes said that matter is capable of *infinite* divisibility; extension may certainly be divided *ad infinitum* in a geometrical sense, supposing that the instruments of division are mere mathematical points or lines; although no limit has yet been set to the continual subdivision of matter, it seems probable that a point may be reached at which it will cease, and that the process may be carried to a different extent in different bodies. The existence of those planes called *planes of cleavage*, and the regularity of form observable in crystalline bodies, seem to indicate that the ultimate atoms of some bodies at least are possessed of a fixed and determinate figure.

1) *Mechanical subdivisions* for purposes connected with the arts, are *ex. gr.* the grinding of corn, the pulverising of sulphur, charcoal and saltpetre, for the manufacture of gunpowder, &c.— Homœopathy affords an instance of a remarkably extended application of this property in bodies.

2) *Divisibility of metals.* — Some metals, particularly gold and silver, are susceptible of a very great divisibility. In the common gold lace, the silver thread of which it is composed is covered with gold so attenuated, that the quantity contained in a foot of the thread weighs less than $\frac{1}{8000}$ of a grain. An inch of such thread will therefore contain $\frac{1}{72000}$ of a grain of gold; and if the inch be divided into 100 equal parts, each of which would be distinctly visible to the eye, the quantity of the precious metal in each of such pieces would be $\frac{1}{7,200,000}$ of a grain. One of these particles examined through a microscope of 500 times magnifying power, will appear 500 times longer, and the gold covering it will be visible, having been divided into 3,600,000,000 parts, each of which exhibits all the characteristics of this metal, its colour, density, &c.

3) *Divisibility of colouring matter.* — Dyes are likewise susceptible of an incredible divisibility. With 1 grain of blue carmine 10 lbs. of water may be tinged blue. These 10 lbs. of water contain about 617,000 drops. Supposing now, that 100 particles of carmine are required in each drop to produce a uniform tint, it follows that this one grain of carmine has been subdivided 62 millions of times.

4) *Minuteness of organised bodies and parts of bodies.* — According to Biot, the thread by which a spider lets herself down is composed of more than 6000 single threads. The single threads of the silkworm are also of an extreme fineness.

Our blood, which appears like a uniform red mass, consists of small red globules swimming in a transparent fluid called serum. The diameter of one of these globules does not exceed the $\frac{1}{4000}$ part of an inch; whence it follows that one drop of blood, such as would hang from the point of a needle, contains at least one million of these globules.

But more surprising than all is the microcosm of organised nature in the Infusoria, for a more exact acquaintance with which we are indebted to the unwearied researches of Ehrenberg. Of these creatures, which for the most part we can see only by the aid of the microscope, there exist many species so small that millions piled on each other would not equal a single grain of sand, and thousands might swim at once through the eye of the finest needle. The coats-of-mail and shells of these animalcules exist in such prodigious quantities on our earth that, according to Ehrenberg's investigations, pretty extensive strata of rocks, as, for instance, the smooth slate near Bilin, in Bohemia, consist almost entirely of them. By microscopic measurements 1 cubic line of this slate contains about 23 millions, and 1 cubic inch about 41,000 millions of these animals. As a cubic inch of this slate weighs 220 grains, 187 millions of these shells must go to a grain, each of which would consequently weigh about the $\frac{1}{187,000,000}$ part of a grain. Conceive further that each of these animalcules, as microscopic investigations have proved, has his limbs, entrails, &c., the possibility vanishes of our forming the most remote conception of the dimensions of these organic forms.

5) *Divisibility of odorous matter.* — In cases where our finest instruments are unable to render us the least aid in estimating the minuteness of bodies, or the degree of subdivision attained; in other words, when bodies evade the perception of our sight and touch, our olfactory nerves frequently detect the presence of matter in the atmosphere, of which no chemical analysis could afford us the slightest intimation.

Thus, for instance, a single grain of musk diffuses in a large and airy room a powerful scent that frequently lasts for years; and papers laid near musk will make a voyage to the East Indies and back without losing the smell. Imagine, now, how many particles of musk must radiate from such a body every second, in order to render the scent perceptible in all directions, and you will be astonished at their number and minuteness.

In like manner a single drop of oil of lavender evaporated in a spoon over a spirit-lamp, fills a large room with its fragrance for a length of time.

§ 7.

Elasticity is the name given to that property in bodies by virtue of which, when their form is in any manner altered, they resume, of themselves, their original dimensions as soon as the external force which acted upon them is removed. Many bodies return to their original form, even after it has been changed by the action of some considerable force, whilst others recover it only partially. The degree of elasticity, measured by the force against which a body shows itself perfectly elastic, is therefore various in different bodies; so also is the limit to a body's elasticity, *i. e.* the minimum force which is required to produce a permanent change in the position of its component particles, or even a separation of them. If a force is applied exceeding the limit of a body's elasticity, either it is torn, or it does not again resume its original form and volume. Perfectly elastic bodies, on the contrary, are such as resist the operation of any force, however great and however long-continued its action.

The following are a few out of a large number of highly elastic bodies; viz. tempered steel, caoutchouc, ivory, whalebone, &c. Some gases possess absolute elasticity, *i. e.* the limit of their elasticity is unknown to us.

First Exp. Let an ivory ball fall on a marble slab smeared with some colouring matter. The point struck by the ball shows a round speck which will have imprinted itself on the surface of the ivory without its spherical form being at all impaired.

Fluids under peculiar circumstances exhibit considerable elasticity; this is particularly the case with molten metals, more evidently sometimes than in their solid state. The following pretty experiment illustrates this fact with regard to antimony and bismuth.

Second Exp. Place a little antimony and bismuth on a piece of charcoal, so that the mass when melted shall be about the size of a peppercorn; raise it by means of a blowpipe to a white heat, and then turn the ball on a sheet of paper so folded as to have a raised edge all round. As soon as the liquid metal falls, it divides itself into many minute globules, which hop about upon the paper and continue visible for some time, as they cool but slowly; the points at which they strike the paper, and their course upon it, will be marked by black dots and lines.

The recoil of cannon-balls is owing to the elasticity of the iron and that of the bodies struck by them.

§ 8.

And lastly, *Gravity* is evidently a property common to all terrestrial bodies, since they constantly exhibit a tendency to approach the earth and its centre. In consequence of this tendency, all bodies, unless supported, fall to the surface of the earth, and if prevented by any other bodies from doing so, they exert a pressure on these latter.

This is one of the most important properties of terrestrial bodies, and the cause of many phenomena, of which a fuller explanation will be given presently.

II. PHYSICAL FORCES IN GENERAL.

§ 9.

Whatever we observe as causing a body to exist under certain conditions, or whatever changes any of its relations, is called *Force*. Hence, whatever causes or tends to cause motion in a body, or whatever changes motion, is called Force. Hence, also, Force either produces pressure or motion. When Force produces pressure, the consideration of it belongs to the science of Statics, or the doctrine of Equilibrium; when motion is the result of Force, then we have the science of Dynamics, or the doctrine of Motion.

The nature of these unseen causes we are ignorant of; we know of their existence only by the effects they produce, and with these we become acquainted solely through the medium of our senses. Hence, whilst their operations are going on they appear to us always in connexion with something material, which, in one way or other, affects our senses.

§ 10.

We shall find, though not always upon a superficial inspection, that the approaching and receding of bodies, or their component parts, is the basis of all those results produced by the various physical forces that come under our notice. In other words, that the universally operating principle is that of *Attraction* and *Repulsion*.

§ 11.

Experience proves that these universal forces are at work in two essentially different modes. They are in operation either in the interior of a body, amidst the particles which compose it, or they extend their influence through a wide range, and act upon the masses of bodies in space: the former distinguished as *Molecular attraction*, and the latter as the *Attraction of Gravitation*.

§ 12.

There is yet one other modification of this general principle, a kind of transition from the effects of gravitation to those of molecular attraction, viz. the attraction and repulsion exhibited by bodies on their surfaces, when they mutually touch. This force is spoken of as the attraction of *Adhesion*.

III. OF MOLECULAR ATTRACTION IN GENERAL.

§ 13.

The *Molecular attraction* of matter chiefly affects the form of bodies by the immediate operation of their particles on each other.

According to this explanation, all bodies are merely a collection or *aggregate* of minute particles, called atoms, and are formed by the attractive and repulsive forces acting on them at immeasurably small distances.

Two very different hypotheses have been formed to explain the nature of matter, or the mode of its formation, the one known as the *atomic* theory, the other the *dynamic:* the founder of the former and earlier was Leucippus. He considered the basis of all bodies to be extremely fine particles, differing in form and nature, which he supposed to be dispersed throughout space, and to which his follower Epicurus first gave the name of atoms. To these atoms he attributed a rectilinear motion, in consequence of which such as are homogeneous united, whilst the lighter were dispersed through space. The author of the second hypothesis was the famous Kant. He imagined all matter existed or was originated by two antagonist and mutually counteracting principles, which he called attraction and repulsion, all the predicates of which he referred to motion. — Most modern philosophers, and foremost amongst these Ampère and Poisson, have adopted a hypothesis combining the features of both the preceding. They regard the atoms as data, deriving their origin from the Deity as the First Cause, and consider their innate attractive and repulsive force as a necessary condition to their combination in bodies. The main features of this hypothesis are borrowed from Aristotle, inasmuch as he supposed the basis of all bodies to be the four elements known to the ancients, the particles of which, endued with certain powers, constituted bodies. — According to Ampère, all bodies consist of equal particles, and these again of molecules that up to a certain distance attract each other. Their distance from each other he supposed to be regulated by the intensity of the attractive and repulsive forces of their matter, partly by the repulsion arising from the undulations of an ether within them, and partly by the attraction which is directly as the mass and inversely as the square of the distance. And lastly, these molecules consist of atoms held together by their attractive and repulsive forces, the latter of which preponderates. To whatever body the molecules belong, they are invariably hard and polyhedrons in figure. Mechanical force separates only the particles; the force generated by the undulations of the atoms may resolve the compound molecules into more simple ones; but chemical agents alone can still farther resolve these into atoms. According to Poisson, all ponderable bodies consist of extremely small molecules, combined by attraction with heat, electrical and magnetic matter. Very little heat exists in the interstices between the molecules, the greater quantity is therefore attached to them, for which reason the effects of the caloric seem to proceed immediately from them. The molecules attract each other and attract heat, whilst this last principle is repulsive to itself, and both forces diminish so rapidly as the distance of the molecules increases, that they become quite inappreciable at any distance that we can observe. The molecules, however, are so small, that the distance at which a diminution in the forces takes place, is always a multiple of its diameter, and consequently an incalculable number of such molecules is conflicting at the same time with each one of these forces. The two forces do not follow the same laws with regard to their decrease by distance: there is, however, a certain point at which a stable equilibrium obtains between them.

§ 14.

Consequently, according to this assumption, the molecular forces are the effective causes which hold together the particles of bodies. Through them the atoms (molecules) approach to a certain distance, where they gain a fixed and unalterable position with respect to each other.

The power with which the particles adhere in this stable condition is called the *force of cohesion*. It is measured by the resistance it offers to the mechanical separation of bodies.

On the degree of this force the three states depend in which matter comes under our notice, called its *aggregate forms*, and distinguished as *solid*, *liquid*, and *gaseous*. These different states of matter result from certain definite relations under which the molecular attraction and repulsion establish their equilibrium; there are three cases, viz. two extremes and one mean. The first extreme is that in which attraction predominates among the atoms; this produces the *solid* state. In the other repulsion prevails, and the *gaseous* form is the consequence. The mean obtains when the attraction is only slightly in excess, and then matter presents itself under the liquid form.

These three forms or conditions of matter may for the most part be readily distinguished by certain external peculiarities; there are, however, especially between solids and liquids, so many imperceptible degrees of approximation, that it is sometimes difficult to decide where the one form ends and the other begins. It is further an ascertained fact that many bodies (perhaps all), as for instance water, are capable of assuming all the three forms of aggregation.

The peculiar properties belonging to each of these states will be explained presently, when solid, liquid, and aëriform bodies come severally under our notice.

§ 15.

The molecular forces may so act upon the atoms of dissimilar bodies as to cause a perfect combination or union of their atoms; they may also produce a separation between the combined atoms or molecules in such a manner as entirely to change the individual properties of the matter thus put into a state of active transposition. This effect of the molecular forces is called *chemical attraction* or *affinity.*

IV. OF GRAVITATION IN GENERAL.

§ 16.

The *principal external forces,* viz. those of *gravitation,* are the result of the mutual action of the masses of bodies through intervening space.

A distinction is made between the gravitation of the celestial bodies, or *universal gravitation, i. e.* the phenomena arising from the mutual action of those large bodies on each other, and the gravitation of our earth, or *terrestrial gravity, i. e.* the phenomena which must be attributed to the mutual operation of terrestrial objects; in other words, the effects which the earth produces on all bodies forming with itself one whole.

Both these species of gravitation are displayed in the approach and receding of bodies in space; or, expressing otherwise the same idea, they both generate *Motion.*

SECTION II.

I. OF MOTION IN GENERAL.

§ 17.

The *Motion* of any body is its change of place. *Rest,* on the contrary, is the state when a body remains in the same place.— Motion reigns throughout Nature, and probably absolute rest is nowhere to be found in the universe.

Apparent Motion and Rest.—To a person on board a swift-sailing vessel, objects on shore seem to be moving in a direction contrary to that in which the ship is going; a similar appearance is presented by the trees on a road-side to a person travelling rapidly along the road.—The apparent motion of the sun and the apparent rest of the earth.

§ 18.

A cause must always exist for motion, as being an alteration in the state of a body (§ 9.); during the motion a certain time must necessarily elapse; and lastly, the moving body must pass over a certain distance in a definite direction, which will depend on the causes of its motion. The chief points to be regarded will therefore be:—

1) The *cause* of motion, or the *motive power*.
2) The *path* and *direction* of the moving body.
3) The *distance* passed over.
4) The *velocity* during the continuance of its motion.

§ 19.

Motive Powers are either *primary* or *secondary*.

To the former class belong such as act constantly on matter; they are gravity both universal and terrestrial.

Secondary motive powers, on the contrary, are only transitory in their effects, as, *ex. gr.*, bodies set in motion by impact, by wind and water-mills, by elasticity, by steel-springs, by the force of expansion,—as the compressed air in a pair of bellows, in the syringe of a fire-engine, &c.; to this class too may be added the effects of steam, and gunpowder; and those produced by animal power, whether of men or beasts.

Forces, with regard to the duration of their action, are divided into impulsive and continuous. The former are such as act on the body for a very short space of time; as, for instance, the mutual pressure of two bodies, which impinge; the force of the string which impels the arrow shot from a bow: the latter, on the contrary, continue in operation for a definite portion of time, as the force of gravity on a falling body.

Forces which act according to the same laws are called *similar*, whilst such as follow different laws are called *dissimilar*.

All forces may be considered as magnitudes which can be measured by the effects they produce; their intensity is expressed either by numbers, or by lines proportionate to such numbers.

§ 20.

The *path* of a body in motion is the line, straight or curved, which it traces out.

The *direction*, if the body move in a right line, is represented by that line; but if the body move in a curved line, the direction at any point of its path is the line touching the curve at that point.

§ 21.

The *Distance* passed over by a moving body is the length of the path. Every motion requires a certain time; and by comparing the distance passed over with the time occupied, we obtain the *Velocity* of a body.

The path of any moving body is estimated by some measure of length; as lines, inches, feet, miles, semi-diameters of the earth, &c. Time, in like manner, is reckoned in seconds, minutes, hours, days, months, or years.

Of two bodies, that has the greater velocity which passes over the greater distance in the same time. The unit of time, according to which velocities are usually expressed in works on Natural Philosophy, is the second.

§ 22.

The velocity may either continue the same during the whole time a body is in motion, or it may vary. In the former case equal spaces are described in equal times; in the latter unequal spaces are passed over in equal times.

Uniformly accelerated motion is that in which the velocity increases equally in each successive equal portion of time; *uniformly retarded motion* when the velocity diminishes by the same law. In both these varieties of motion a distinction is observed between a body's *initial* and *final velocity*. The former is that which a body has when it begins to move, the latter that which it has acquired after the lapse of a certain time. Both are measured by the space which the body would have passed over in one second with the uniform initial or final velocity, as the case may be.

Let V represent the velocity, D the distance, and T the time; then in the case when equal distances are passed over in equal times,

$$\left.\begin{array}{r} 1)\ V=\frac{D}{T} \\ \text{then } 2)\ D=V\,.\,T \\ 3)\ T=\frac{D}{V} \end{array}\right\}$$

In comparing different velocities and distances we obtain

$$4)\ V : v = \frac{D}{T} : \frac{d}{t}$$

$$5)\ D : d = VT : vt$$

The only uniform motion on the earth is its revolution on its axis; the movements of machinery are only approximations to uniformity. Illustrations of uniformly accelerated and retarded motion are numerous, as will be seen shortly.

§ 23.

The following variations may exist in the motion of a body: —

1) The whole body may change its place, all the parts of which it consists advancing with equal rapidity, and not returning to the place originally occupied: this is called *direct motion.*

A stone thrown, or a cannon-ball shot from a gun, are instances of this kind of motion.

2) The body may change its place, as in the former case, but return again in a direction opposite to that in which it first moved; this is a *swinging*, *oscillating*, or *vibratory motion*

The motion of a pendulum, or that of a string of any musical instrument strained tight by the finger and then let go, belongs to this class.

3) The different points of a body may at the same time move in different directions; this is called a rotatory motion. *Rotation on an axis* implies, in addition to this rotatory motion, that all the parts of the body shall revolve around an imaginary right line in a state of rest, called the *axis.*

The wheels of any piece of machinery have a rotatory motion round their axes; the diurnal and annual motions of the earth, a rolling ball, the wheels of a waggon as it travels, are illustrations both of direct and rotatory motion.

§ 24.

When a body moves in a straight line, with a velocity either uniform or varying in any manner by the action of a force in that line, its motion is called *simple*; but if several causes of motion operate together, the motion produced is called *compound.*

The motions to be compounded may be arranged in three ways, as regards their direction; viz.: —

1) The motions may be in the same direction.
2) They may be in exactly opposite directions.
3) They may be inclined at some angle.

The motion produced by the composition of several others is called the *resultant motion.*

In the first of the above cases the motion produced is the sum of the component motions, and the direction remains unaltered. In the second, it is their difference in the direction of the greater motion.

Lastly, when they make some angle with each other, the position of the body at any time may be found as follows: suppose that one cause of motion would carry the body from a to b in the same time in which from another cause it would have passed over *ac* (*figs.* 2. and 3.), then the position of the body at the end of that time will be the opposite corner, *d*, of the parallelogram found by drawing through *b* and *c* lines, *bd* and *cd*, parallel respectively to *ac* and *ab*.

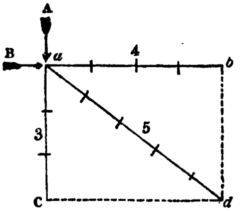
If each of the component motions be uniform, which would occur if they arose from impulsive forces at *a*, the body will trace out the diagonal with a uniform velocity.

> Motions which obey this law are, *ex. gr.*, the sailing of a vessel up a stream, ferrying across a river, the fall of a body from the mast-head of a ship when she sails; the movements which take place in vessels or carriages, or on the earth, are unaffected by their uniform motion.

Also, if *ab* and *ad* represent the magnitudes of two forces acting at a point and tending to produce motion, the amount of this tendency, that is the *resultant pressure*, is represented by the diagonal *ad* of the parallelogram *abdc*.

The magnitude and direction of this resultant may be found by the aid of trigonometry.

Fig. 2.



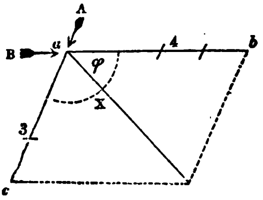
Let the two component forces form a right angle (*fig.* 2.), then

$ad = \sqrt{ab^2 + ac^2}$, Eucl. I. 47.

or $R = \sqrt{A^2 + B^2}$

But if they form any other angle, x (*fig.* 3.), then

Fig. 3.



$$ad = \sqrt{ab^2 + ac^2 - 2ab.\ ac.\ \cos.\ (180 - x)}$$

$$= \sqrt{ab^2 + ac^2 + 2ab.\ ac.\ \cos.\ x}$$

$$\text{or } R = \sqrt{A^2 + B^2 + 2AB.\ \cos.\ x}$$

Substituting ϕ for the angle *bad* formed by the component B with the resultant, then

$$\tan.\ \tfrac{1}{2}\,(2\,\phi - x) = \frac{A - B}{A + B}\ \tan.\ \tfrac{1}{2}\,x$$

Example. Let A = 3, B = 4, and $x = 120°$,

$$R = \sqrt{3^2 + 4^2 + (2 \times 3 \times 4), \text{ cos. } 120°}$$
$$= \sqrt{25 - 24 + (\tfrac{1}{2})}$$
$$= \sqrt{13}$$
$$= 3{\cdot}605 \text{ nearly.}$$

Again let forces 15 and 8 act at a right angle.

$$\text{then } R = \sqrt{15^2 + 8^2}$$
$$= \sqrt{225 + 64}$$
$$= \sqrt{289}$$
$$= 17.$$

§ 25.

The Parallelogram of Forces serves —

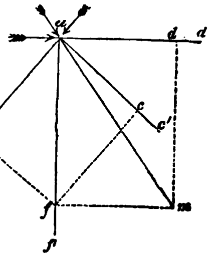
1) For the *Composition of Forces, i. e.* instead of several forces acting in different directions on one point, to find a resultant. — The single force to be substituted for the different forces is found, according to the preceding §, by obtaining the resultant for any two of these forces, from this and the third force compound a second resultant, and so proceed, till no more forces remain. The last diagonal, to which this construction leads, will give the direction and intensity of the resultant of all the components.

Thus in *fig.* 4. the forces *ab* and *ac* give the resultant *af*; this and *ad* give the resultant *am*, which is therefore the resultant of the three forces *ab*, *ac*, and *ad*.

2) The *Resolution of Forces, i. e.* the finding of several forces, their directions, and intensities, which together would be equivalent to the given single force.

Let *am* (*fig.* 4.) represent the direction and intensity of a given force acting on the point *a*; if this be resolved in the directions *ad′* and *af′*, the intensities *ad*, *af*, of the two components may be obtained by drawing *md*, *mf*, parallel to *ad′*, *af′*. In like manner, resolve *af* into *ac* and *ab*, *am* has therefore been resolved in three directions into the components *ab*, *ac*, *ad*.

Fig. 4.



Two forces acting at any angle whatever cannot cancel each other, but three or more may. Thus, *ex. gr.*, the effects of the two forces, *ab*, *ac*, *fig.* 4., may be neutralised by a third force, *fa*, acting in the direction *fa*, and the three forces, *ab*, *ac*, *ad* may be cancelled by a fourth *am*, acting in the direction *ma*.

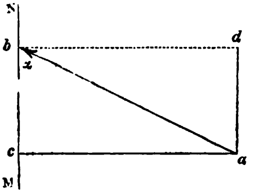
§ 26.

The full effect of a force acting on a body is produced only in the case when its direction is *perpendicular* to the surface of the body ; at any other angle a part only of the force is expended on the body.

By the resolution of any force acting obliquely into two others, one of which is supposed to be perpendicular, and the other parallel to the side of the body acted on, we obtain in the perpendicular component a measure of the actual effect produced on the body, and hence deduce the loss on the original force.

Let *ab* be the original force acting under the angle against the surface MN. Resolve *ab* into the perpendicular *ac*, and the line *ad* parallel with the plane. The length of *ac* will be the absolute effect produced by the force *ab* against MN, and *ab*—*ac* will give the loss. By trigonometry we obtain $ac = ab \,.\, \sin.\, \alpha$, and $ad = ab \,.\, \cos.\, \alpha$.

Fig. 5.



As illustrations we may adduce the ascent of a paper kite in the air, and the motion of the vanes of a windmill.

§ 27.

The cause of motion is not to be found in matter itself, it is merely the medium through which any *motive power* whatever becomes evident by its effects. On the contrary, *every body displays a tendency to remain in its present condition, whether that be one of rest or of motion, until some external cause act upon it to produce a change in its condition.* However, before a body can assume this opposite condition, a certain time must elapse sufficient for the condition of every atom in the mass to become altered. This property of matter is called its *inertia.*

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§ 27. Simple as the idea of the absolute *inertia* of matter may appear to our minds, this property of bodies was unknown to the ancients: we are indebted to Galileo for its discovery. The conclusions of this and the preceding articles with respect to motion, were enunciated by Newton, in his famous Three Laws of Motion. 1st. Every body perseveres in a state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed thereon. 2nd. The change of motion is ever proportional to the motive force impressed, and is made in the direction of the right line in which that force is impressed. 3rd. To every action there is always opposed an equal reaction ; or the mutual actions of two bodies upon each other are always equal, and directed towards contrary parts.

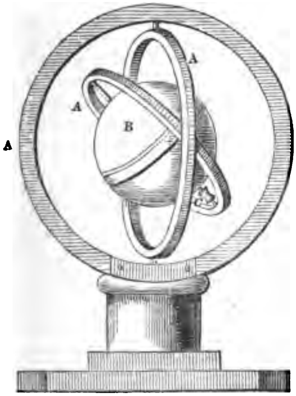
A mill-wheel at rest gradually acquires its highest velocity by the continued fall of water, and revolves long after the cause of motion has ceased.—The horses are obliged to exert a greater force in order to move a loaded waggon at starting than when it has once been set in motion.—Application of fly-wheels in machinery—The shock communicated to persons, and even to inanimate objects, by the sudden stopping or starting of the conveyance in which they are placed, is owing to their inertia.

First Exp. The apparently retrograde movement of a ball at rest on a plane, when the latter is suddenly put in motion.

Second Exp. The driving on the head of a hammer, or any similar tool upon its handle, by striking the opposite end.

Third Exp. Bohnenberger's machine for illustrating the position maintained by the axis of our earth in its revolution. This apparatus consists of one fixed and two moveable rings A, *fig.* 6., at right angles to each other, and of a massive metal ball B, moveable about its axis, fixed within the smallest ring. To this axis is attached a small roller *c.* Now give to the innermost ring, and consequently to the axis of the ball, a slanting position, fasten to a peg in the roller a fine cord, provided at the end with a slip-knot, and wind it up on the roller: next give a smart pull to the string so as completely to unwind it, and the ball will be set in rapid motion; the apparatus may now be placed in any position whatever, and the axis of the revolving globe will constantly retain the same place and direction. The ring in which the axis of the ball terminates, otherwise so easy to move, will then withstand a considerable pressure.

Fig. 6.



§ 28.

Any body in motion becomes itself a motive power, when it tends, in consequence of its inertia, to set in motion some second body. The intensity of such a moving force, or the momentum of the body, depends on the mass of ponderous atoms it contains, and on their common velocity, since every atom must contribute in an equal degree towards imparting motion to this second body. Consequently, the momenta of two bodies in motion are to each other as the products of their masses into their velocities.

Let M and m represent the masses of two bodies in motion, V and v their velocities, and P and p their momenta, then the preceding law may be expressed in the following proportions:

$$1)\ P : p = MV : mv$$

whence:

$$2)\ V : v = \frac{P}{M} : \frac{p}{m}$$

$$\left.\begin{array}{l} \text{if } M = m \\ 3)\ V : v = P : p \\ \text{if } P = p \\ 4)\ V : v = m : M \end{array}\right\}$$

From the above law it is easily seen, why great masses, such as loaded ships, icebergs, &c., even though they move but slowly, exert such prodigious force upon objects with which they come in contact; and also, on the other hand, how cannon and musket balls, in consequence of their great velocities, are so destructive.

Action and *reaction* are always equal to each other, *i. e.* the effect which any force exerts, or, what is the same, its momentum is equal to the resistance which the body it sets in motion offers to it. *The intensity of the moving power is therefore estimated by the mass and velocity of the body set in motion.* To represent these forces as comparable magnitudes, they are usually expressed thus: the mass is stated which can be raised in a certain unit of time (a second or minute) through a certain space (1 foot or 1 yard) also considered as a unit.

II. PHENOMENA PRODUCED BY GRAVITY.

§ 29.

Gravitation, or the all-prevading attraction of bodies throughout space, is the unseen bond that holds together the material universe, regulating by the same laws the unvarying course of the heavenly bodies, and the descent of the smallest particle of dust. These laws, as laid down by Newton, and adopted throughout the whole scientific world, are as follows: —

The gravitating forces of bodies are to each other:

1) *directly as their masses*, and

2) *inversely as the squares of their distances; i. e.* if the mass of one body be 2, 3, 4, or 5 times greater than that of another, its gravitating force is also 2, 3, 4, or 5 times greater, and with a distance 2, 3, 4, or 5 times greater, the gravitating force will be 4, 9, 16, or 25 times less. Put M and m for the masses, D and d for the distances, G and g for the gravitating forces of two bodies, and g', for that of m at the distance D; then

from 1) we obtain

$$G : g' = M : m$$

$$\text{or } G = \frac{g' M}{m}$$

from 2)

$$g' : g = d^2 : D^2$$

$$\text{or } g' = g . \frac{d^2}{D^2}$$

from 1) and 2) it follows that

$$G : g = M d^2 : m D^2$$

$$\text{or } G = g . \frac{M d^2}{m D^2}$$

whence, if m and d are assumed = 1

$$G = g . \frac{M}{D^2},$$

which is the general expression for the above laws, M representing the attracting mass, and D the distance at which it acts upon some other body, as compared with the attracting force g of this second body, whose mass = 1 and whose distance = 1 are taken as the units of measurement.

As the mass of the sun is several hundred times greater than that of all the planets together, it is able powerfully to attract even the most remote (Neptune, whose distance from that luminary is 2800 millions of miles). In a similar manner the planets act upon their satellites. Our moon which is 60 of the earth's semi-diameters distant from its centre, is attracted, according to the above law, with a force 60 x 60 = 3600 times less than bodies on the surface of the earth.

§ 30.

The earth itself attracts all terrestrial objects according to the same laws. The cause of this operation is called *terrestrial gravitation* or *the force of gravity.*

Gravity is a combined operation of all parts of the globe, and the result may be represented as if the attraction acted only from its centre. As the earth attracts all terrestrial objects towards itself, so it is attracted towards them in return; but as their masses are extremely small when compared with that of the earth, its attraction only is apparent to us. Bouguer and De la Contamine have, however, discovered and proved that high mountains exert a perceptible influence. Gravity acts equally on all bodies at equal distances from the earth's centre, its force diminishes as the distance increases, and increases as the distance diminishes agreeably to the second law, § 29. Hence it follows that the gravity at any point of the earth's surface, which is nearer to the centre, is greater—and at places above the surface, which are more remote, it is less—in the proportion of the square of the distance.

Of Weight, both absolute and specific.

§ 31.

The simplest effect of gravity, and that which is immediately evident to the senses, is *weight*, or the pressure which every body exerts on whatever supports it. The amount of a body's weight is proportional to its mass. The measure of its weight, estimated by that of some other body, which serves as a unit, is termed the *absolute weight.* Gravity and weight stand to each other in the relation of action and re-action (1).

The absolute weight of similar bodies is proportional to their

§ 30. " The French academicians, in measuring their South American degree, were the first who found sufficient local attraction in a mountain adjacent to their observations (Chimboraço) to give any hope of making that phenomenon useful in future inquiry. Maskelyne, in 1772, suggested the employment of astronomical observation in the neighbourhood of a mountain, for the determination of the earth's mean density. Schehallien was chosen for the purpose by a committee of the Royal Society, and the result of this celebrated experiment was announced in 1775. The description of this purely statical experiment is as follows: the position of a plumb-line, in a state of deviation from the vertical of a place, on account of the attraction of a mountain is first to be accurately determined by making that plumb-line the regulator of an instrument for measuring zenith distances, and comparing the zenith distances thus obtained with those determined in other places of known differences of latitude. The weight of the plumb-line is then acted on by three forces of known direction, one of them being of known magnitude. The remaining forces (one of which is the attraction of the mountain) can then be determined; so that if the distance of the mountain be known, its mass can be compared with that of the whole earth. The next step, and the most difficult one, is to compare the mean density of the mountain with that of water, which requires an accurate knowledge of its material and size. Maskelyne's rough computation, made with such knowledge as he had of the composition of the mountain, gave the mean density of the earth nine-fifths of that of the mountain, or from four to five times that of water. Hutton's subsequent and laborious investigation, made four and a half for the ratio; but Playfair's examination of the strata of the mountain led to the inference that the result of the experiment could only be considered as placing the same ratio between four and six-tenths and four and nine-tenths. It is worth noting that Newton (*Princ.* book iii. prop. 10.) had ventured a conjecture which, as happens so frequently with him, turns out to be true. He thinks that the mean density of the earth is *between five and six times that of water.*" — *Royal Astron. Society's Monthly Notices,* vol. v. No. 28.

volumes (2); dissimilar bodies, on the contrary, have almost invariably a different absolute weight with equal volumes (3).

The relative weight of equal volumes expresses the *density* of matter; the more any body weighs with equal volume the denser it is. This density of bodies is usually expressed in numbers, that of pure water being assumed as unity, and the particular weight of each substance is called its *specific gravity* (4). Again, as (§ 4.) all matter admits of compression and expansion, it follows that the specific gravity of the same substance varies with circumstances.

1) To express the absolute weight of bodies, various units of weight are made use of in different countries—hundred-weight, pound, grain, &c.

2) The absolute weights of similar homogeneous bodies are to each other as the cubes of their similar sides; of spheres as the cubes of their diameters.

3) Equal globes of lead, iron, stone, &c., have unequal weights.

4) Let a certain volume of water weigh 1 lb.; then a sphere of equal magnitude of cast-iron will weigh 7·2 lb.; one of lead 11·3 lb.; and one of marble 2·8 lb. Whence it appears that the densities or specific gravities will be as follow: iron, 7·2; lead, 11·3; and marble, 2·8.

Of the Descent of Heavy Bodies.

§ 32.

The direction in which gravity acts on bodies at all parts of the earth is towards its centre. Every cord to which a body is freely suspended assumes this position when in a state of rest, and thus becomes a semi-diameter of the earth produced. Such a cord is called a *plumb-line;* the line which it marks out is a perpendicular; and a line or plane at right angles to it is said to be *horizontal.* At small distances, such even as several hundred feet, the lines of gravity may be considered as parallel to each other, on account of the magnitude of the earth's semi-diameter; but at greater distances where miles intervene, the convergence of the perpendiculars must be estimated according to the curvature of the earth's surface.

The convergence in one geog. mile is 1 min., and in 60 geog. miles it amounts to 1°.

§ 33.

Every body not supported, or deprived of what supports it, falls towards the surface of the earth in the direction of its line of gravity.

As gravity is a constant force, the descent of bodies, which is purely the result of this force, is necessarily a *uniformly accelerated motion.*

Again, as gravity operates with equal intensity on all bodies, *i. e.* as every individual atom of a body is attracted equally towards the earth, *all bodies must fall with equal velocity.*

The velocity acquired by a body in falling for one second serves as the measure of the force of gravity. This velocity is 32·2 feet, and the space fallen through in the first second is half of this, or 16·1 feet.

The laws that regulate the descent of heavy bodies, at a knowledge of which we have arrived by the aid of mathematical science, are the following: —

1) *The final velocities* (§ 21.) *increase as the times, or they follow in the order of the natural numbers* 1, 2, 3, *&c.*

i. e. the velocity of a falling body at the end of the 2nd second is twice, at the end of the 3d second three times as great as at the end of the first.

final V. of 1st second = 1 × 32·2 = 32·2 ft.
2nd — = 2 × 32·2 = 64·4
3rd — = 3 × 32·2 = 96·6
t th — = t × 32·2

2) *The spaces fallen through in equal successive portions of time are as the odd numbers* 1, 3, 5, 7, *&c.*

i. e. a falling body descends in the 2nd second of its fall through 3 times, in the 3rd second through 5 times the space passed over in the 1st second. Thus the space of the

1st sec. = 1 × 16·1 = 16·1 ft.
2nd — = 3 × 16·1 = 48·3 ft.
3rd — = 5 × 16·1 = 80·5 ft.
t th — = $(2t-1)$ × 16·1

3) *The spaces, counted from the beginning of the fall, or the whole spaces fallen through, increase as the squares of the times, thus* 1^2, 2^2, 3^2, 4^2, *&c.*, t^2 = 1, 4, 9, 16, *&c.*, t^2.

i. e. a body in 2 seconds falls through 4 times, in 3 seconds through 9 times, in 4 seconds through 16 times the space it descends in 1 second: thus it falls in

1 sec. 1 × 16·1 = 16·1 ft.
2 — 4 × 16·1 = 64·4
3 — 9 × 16·1 = 144·9
t — t^2 × 16·1

4) *If a body were to continue in motion with the final velocity acquired after falling a given time, and the operation of gravity were to be then suspended, the body would descend in a time equal to that*

during which it had already fallen, with a uniform motion through twice the distance of the first fall.

The space fallen through in 3 seconds is (3) 144·9 ft., the final velocity of 3 seconds (1) 96·6. Now a body continuing to fall for 3 seconds with a uniform velocity would pass through a space of $3 \times 96{\cdot}6 = 289{\cdot}8 = 2 \times 144{\cdot}9$. The above laws were first published by Galileo in 1602.

The subjoined table expresses in one view the first three laws:—

Times - - -	1,	2,	3,	4,	5,	6,	*etc.*	$-t$
The final velocities are as	2,	4,	6,	8,	10,	12,	*etc.*	$-2t$
The space for each time is as	1,	3,	5,	7,	9,	11,	*etc.*	—
The whole spaces are as	1,	4,	9,	16,	25,	36,	*etc.*	$-t^2$

Exp. Illustration of these laws by Attwood's machine. Its application and principle. — If it be required in experiments on the descent of bodies that the descending weight should fall only 1 inch in the 1st second, instead of 16·1 ft. = 193 in. nearly, then the additional weight to be placed on it must be $\frac{1}{193}$ of the entire mass that is to be set in motion; or thus, let M represent the weight of the wheel, cord, and weights attached, w the additional weight to be added on one side, which being acted on by the force of gravity, sets the whole in motion, g the velocity a body would acquire by falling freely in one second, and g' that which the weight of the machine would acquire in the same time; then we have

$$wg = (M + w)\, g'$$
$$w\,(g - g') = Mg'$$
$$w = \frac{Mg'}{g - g'}$$

Supposing $g' = 2$ inches, then $g = 386$ in., whence $w = \frac{M}{190}$ nearly.

1) In every uniformly accelerated motion, the body in a state of rest on beginning to move, has a velocity $= 0$; from that moment, however, it increases in arithmetical progression. Now let the velocity which a body has acquired by falling during 1 sec. $= g$, then its mean velocity in the 1st sec. will be $\frac{0 + g}{2} = \frac{1}{2}\, g$, and the body moving uniformly with this mean velocity will traverse the same distance in 1 sec. which it would have fallen freely in 1 sec.; *i. e.* the final velocity acquired by a body falling freely 1 sec. is double the space through which it has fallen. Taking 16 ft. as the space nearly, its velocity will be 32 ft.

In the 2d sec. the body continues its motion with the acquired velocity $g=32$ ft. in consequence of its inertia, (§ 27.), besides which gravity acts upon it as in the 1st sec., accelerating its motion, so at the end of the 2nd. sec. it will have acquired a velocity of $2g=64$ ft. The space which it will have descended in the 2nd sec. is equal to that which it would have fallen with the mean velocity between its initial and final velocities, that is, with the velocity $\frac{g+2g}{2}=\frac{3g}{2}$; and $\frac{3g}{2}$ is therefore $=3\times16=48$ ft.

It is evident that after the lapse of 3 sec. the final velocity of the body will be $3g=96$ ft., and that in this sec. it will have descended with the mean velocity of $\frac{2g+3g}{2}=\frac{5g}{2}$ through a space of $\frac{5g}{2}=5\times16=80$.

From the above it follows; —

a) that the final velocities increase as the successive seconds of time, for: —

the final velocity after the 1st sec. $=1.g=32$ ft.
– – 2d — $=2.g=64$ ft.
– – 3d — $=3.g=96$ ft.
– – t — $=t.g=32t$ ft.

b) That the spaces fallen through in each successive second are in the order of the uneven numbers, for —

the space fallen in the 1st sec. $=1.\frac{g}{2}=16$ ft.
– – 2d — $=3.\frac{g}{2}=48$ ft.
– – 3d — $=5.\frac{g}{2}=80$ ft.

Hence it follows that —

c) The spaces are as the squares of the times, for

in 1 second, the space $= 1.\frac{g}{2}=1\times16=16$
— 2 seconds —— $= 1.\frac{g}{2}+3.\frac{g}{2}=2^2.\frac{g}{2}=4\times16=64$
— 3 seconds —— $=1.\frac{g}{2}+3.\frac{g}{2}+5.\frac{g}{2}=3^2.\frac{g}{2}=9\times16=144$
— t seconds —— $=t^2.\frac{g}{2}$

Since a falling body after t seconds has acquired the velocity tg, it will, moving uniformly with this velocity, in the same time, t sec. fall through the space $t.\,tg=t^2g$, which is exactly twice the distance fallen in t sec., thus proving the 4th law.

Let h = the space, t = the time, v = the final velocity, and g = the velocity acquired in the 1st sec., then we have the following equations deducible from the preceding laws: —

$$1)\ v=gt$$

$$2)\ h=\frac{1}{2}gt^2$$

whence we obtain —

$$3)\ t=\frac{v}{g}=\sqrt{\frac{2h}{g}}$$

$$4)\ v=\sqrt{2gh}$$

$$5)\ h=\frac{v^2}{2g}$$

The following are a few examples to be solved by the application of the above formulæ: —

1) The highest known mountain in the world, the Dhawalagiri in India, rises about 25,780 feet above the level of the sea; in how many seconds would a stone fall through that height? — By the 3rd formula, $t=40$ sec.

2) With what final velocity will the stone in the case supposed above reach the ground? — By the 1st form., $v=1288$ ft.

3) How long has that body been falling which comes to the ground with a final velocity of 322 ft.? — By the 3rd form., $t=10$ sec.

4) How high is the gallery of a tower from which a stone falls in 5 seconds of time? — By the 2nd form., $h=402\frac{1}{2}$ ft.

5) From what height must a body have fallen that reaches the ground with a final velocity of 644 ft.? — By the 5th form., $h=6440$ ft.

6) With what final velocity will a bomb-shell come to the ground if it be let fall from a height of 1040 ft.? — By the 4th form., $v=258$ ft.

These laws, which we have deduced above, can, however, be exactly fulfilled, only if the moving body meet with no impediment. One obstacle constantly presenting itself, the effect of which has not yet been considered, is the resistance of the air. This is the occasion of all bodies coming to the ground somewhat later than they would if falling *in vacuo* with a similar velocity. (1.) Why the resistance offered by the air to falling bodies varies, will be more fully treated of hereafter.

1) That all bodies *in vacuo* fall with exactly equal velocity, is demonstrated by the contemporaneous descent of the coin and feather in what is generally called the guinea-and-feather experiment, as performed with the aid of the air-pump.

§ 34.

Since our earth is not a perfect sphere, having its polar diameter less than its equatorial, it follows that all bodies upon it (irrespective of inequalities of the earth's surface) have not the same distance from its centre, and consequently, by the 2d law (§ 29.), as well as by what was shown in § 30. with regard to the operations of gravity in general, the attraction of the earth must vary in different places; it will be weakest at the equator and strongest at the poles. The space through which a body falls in the 1st second of time at the poles is 16·1246 ft. whilst at the equator it is only 16·016 ft. See below the §§ on *The Pendulum.*

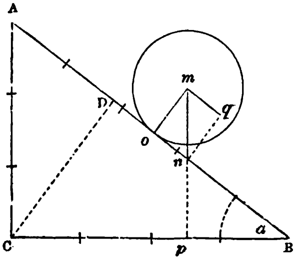
Of the Descent of Bodies on an inclined Plane.

§ 35.

A body placed upon a *plane inclined* towards the horizon glides or rolls down to the lower extremity of the plane. The attraction of the earth is the cause of this motion also, which must, therefore, without noticing any impediment, be a uniformly accelerated one. But as the body, on account of the plane, cannot take the direction of gravity, its velocity will be less than that of a body falling freely. The impressed force is resolved into two forces, one of which is expended in pressure upon the plane, whilst the other produces the accelerated motion.

The amount of the pressure on the inclined plane is such a proportion of the weight of the body, as the base of the plane is of its length; *i. e.* (see *fig.* 7.) the pressure would $= \frac{BC}{AB} \times$ weight of body, or, *the pressure on the inclined plane is to the absolute weight of the body as its base is to its length,* i. e. *as* BC : AB.

Fig. 7.



Required the pressure exerted by a body, whose weight = 100 lb., on an inclined plane whose base is 6 ft. and length 8 ft. —— Ans. 75 lb.

When the body slides down the plane, the velocity in the first second of time is diminished in a descent of this kind in the ratio of the height to the length of the plane, or (*fig.* 7.) as A C : A B; *i. e. the accelerated motion of a body descending an inclined plane, or its relative force of gravity, is to that of a body falling freely as the height to the length.*

How many feet will a body slide in 1 sec. down an inclined plane, whose length A B, *fig.* 7., = 160 ft., and whose height A C = 16 ft.? —— Ans. 1·6 ft.

The correctness of the two laws given above may be deduced from the following: — Let mp represent the vertical direction, and $mn = w$, the weight (or absolute gravity) of the body. which exerts on the plane a pressure $mo = w'$, and a downward force (its relative gravity) $mq = on = w''$ (§ 26.), and the $\angle\ a$ the inclination of the plane to the horizon, then as the $\triangle$ s mno, B A C, are similar, we obtain the following proportions:

$$\begin{aligned} 1)\ w : w' &= mn : mo \\ &= AB : BC \\ &= 1 : \cos.\ a \\ 2)\ w : w'' &= mn : mq \\ &= AB : AC \\ &= 1 : \sin.\ a \end{aligned}$$

Since the force producing motion down the plane = sin. a × weight of body,

$$\therefore g' = g \sin.\ a,$$

g' being the velocity acquired in descending the plane for one second. Consequently in t seconds the body would descend upon the plane $\frac{g}{2} \sin.\ a\ t^2$, and would acquire a velocity of $g \sin.\ a\ t$. Hence the formula for this motion may be deduced from those for bodies falling freely by substituting g sin. a for g.

Galileo first experimentally illustrated the laws of falling bodies by means of an inclined plane.

§ 36.

A body descending on an inclined plane from A to B (*fig.* 7.) acquires a final velocity equal to that which it would have had supposing it to have fallen freely from the height A C of the plane.

1st. *ex. gr.* Let the base B C = 21·444 ft., the height A C = 16·1 ft., and the length A B = 26·522 ft. The ball, in descending the plane, will acquire a final velocity of 32·2 ft., which is the same that it would have gained had it fallen perpendicularly 16·1 ft.

The final velocity for the distance A C (*fig.* 7.) will be

$$v = \sqrt{2g \,.\, \mathrm{AC}}$$
$$= \sqrt{2g \,.\, \mathrm{AB}\sin.\, a}\,;$$

for the slant side A B the final velocity will be

$$v' = \sqrt{2g \sin.\, a \;\;.\; \mathrm{AB}},$$

or thus, still referring to the same formulæ,

$$g' : g = h : h'$$
$$\therefore gh = g'h'$$

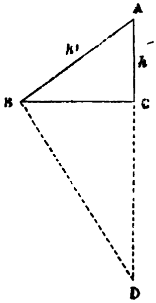
and $$\sqrt{2gh} = \sqrt{2g'h'}, \text{ or } v = v'.$$

In like manner the space passed through by a body falling freely is to that gone over on the inclined plane in equal times, as the length of the plane to its height; *i. e.* it is such a part of the space the body would have descended freely as the plane's height is of its length.

Assuming the same dimensions as before for the three sides of the inclined plane, whilst the body would fall freely from A to C through a space of 16·1 ft., it would descend on the inclined plane from A to D a distance $= \frac{\mathrm{AC}}{\mathrm{AB}} \times \mathrm{AC} = \frac{16{\cdot}1}{26{\cdot}522} \times 16{\cdot}1 = 9$ ft.

The space through which a body would fall by the action of gravity in the time that it would descend the whole length A B (*fig.* 8.) of the inclined plane, is found by letting fall a perpendicular at the point B to meet the height A C produced in D. A D will be the distance required :

Fig. 8.

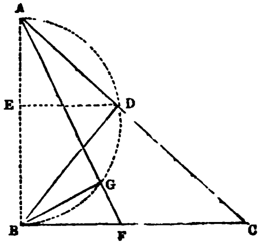


$$\therefore \mathrm{AD} : \mathrm{AB} = \mathrm{AB} : \mathrm{AC}$$

If it be required to determine geometrically what part of the length of an inclined plane a body will pass over whilst another body falls freely the height of the plane, it is merely necessary, as in *fig.* 9., to describe a semicircle on the height. The part A D, cut off by the circumference, is the distance sought; then join D B, and draw D E parallel to B C. Let there be a second inclined plane, A B F, of the same height A B, the body, would traverse the space

A F in the same time that it would fall freely from A to B. Suppose now, that there were several such planes, all of the same height, A B, the chords B D, B G, being conceived to be parts of their inclined sides, we obtain the following law, first laid down by Galileo, *that all chords terminating in either extremity of the diameter of a circle, being considered as inclined planes, will be traversed by a body in the same time that it would have fallen by the action of gravity alone, through a space equal to the diameter; i. e.* in the time that a body would have fallen from A to B, it might descend from A to D, from A to G, from D to B, or from G to B.

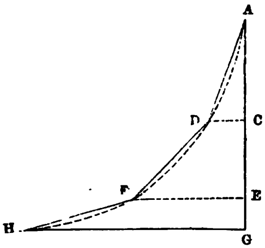
Fig. 9.



§ 37.

If several planes be joined together, as is represented by the crooked line A D F H (*fig.* 10.), a body that has traversed A D will have acquired the same velocity as it would have gained by falling perpendicularly through A C; and *if the body be supposed to begin the descent of each plane with the velocity which it had acquired in the preceding,* it is clear that the velocity at the end of the crooked line will be the same as that acquired by falling freely to G through the the same height. Now, supposing the parts A D, &c., be taken very small and very numerous, all the points A D, &c., being on the curve A D F H, the change of direction from each small plane to the next will diminish indefinitely, and then the above supposition holds; therefore the velocity acquired in falling through the curve is the same as that acquired in falling through its height, and this does not depend on the form of the curve.

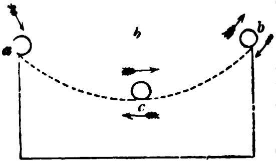
Fig. 10.



Similarly, if a body be thrown up a curve, the height to which it will ascend will be the same as that in descending which it would have acquired the given velocity. This remark will be better understood when the chapter on Projectiles has been studied.

So that a body descending from *a* in a curve, such as that in *fig.* 11., would rise to an equal height at *b*; and in returning would acquire by its fall a velocity which would carry it to *a*, and so on. This is also true of rolling bodies. Such a contrivance would furnish us with an extremely simple perpetual motion, were it not for the impediments to motion, which have not yet come under review, by which the velocity of the body would be gradually annihilated.

Fig. 11.



Of Projectiles.

§ 38.

There are two forces in operation on every body, by whatever means projected; viz. the projectile force, which is momentary, and gravity, whose action is permanent. The motion thus obtained must consequently be a compound one, the forces acting either in the same vertical, in exactly opposite directions, or obliquely. Hence the following circumstances must be observed.

§ 39.

When the projectile force acts in the direction of gravity, the path of a body is the same as it would have described by falling freely, its accelerated motion being increased in each successive portion of time in proportion to the projectile force added.

Thus, suppose a body to be projected *downwards* with the velocity v, it will fall in t seconds a space $= vt + t^2.\frac{g}{2}$.

§ 40.

If a body be thrown *vertically upwards*, its motion, on account of the counteracting influence of the force of gravity, will be *uniformly retarded*; for, if the projectile force only acted upon it, it would have a uniform motion, passing through equal spaces in equal times. The effect of the projectile force is, however, lessened every moment by the attraction of the earth acting on the ascending body, and that, too, in exactly the same manner as it accelerates the descent of falling bodies. The laws regulating the fall of bodies in these cases obtain an inverse application. A body will therefore entirely lose its velocity after a certain time, and by virtue of the attraction of the earth it will return to its surface in just the same time that was occupied in its ascent.

A body thrown perpendicularly upwards will be attracted towards the earth in the first second of time through a space of 16·1 ft., in the second sec. 3 × 16·1, in the third 5 × 16·1, &c.; or in two seconds the force of gravity will diminish the space through which the body would otherwise have ascended by 4 × 16·1, in three sec. 9 × 16·1, &c. This will be more clearly illustrated in the following example:—Suppose a body to begin its ascent with an initial velocity of 160 = 10 × 16 ft.; neglecting fractional quantities, we shall find that it moves,

in the 1st sec., $(10 \times 16)-(1 \times 16)=9 \times 16=144$ ft.
2d, $(10 \times 16)-(3 \times 16)=7 \times 16=112$—
3d, $(10 \times 16)-(5 \times 16)=5 \times 16=\ 80$—
4th, $(10 \times 16)-(7 \times 16)=3 \times 16=\ 48$—
5th, $(10 \times 16)-(9 \times 16)=1 \times 16=\ 16$—

At the beginning of the sixth second it commences its descent, and reaches the ground in the same number of seconds that were employed in accomplishing its ascent. Its final velocity is equal to its initial velocity, and indeed at any point of the body's course the velocity with which it descends is equal to that with which it ascended.

Or thus, expressing it in general terms:—

Let a body ascend vertically with a velocity v; then, but for the action of gravity, it would pass in t seconds over the space $t\,v$; this force would, however, in that time diminish the space by $\frac{1}{2}g\,t^2$, so that it would accomplish only $vt-\frac{1}{2}gt^2$; let this $=h'$. After t sec. its velocity will $=v-gt$; at the moment, however, when the body is at rest on beginning its descent, $v-gt=0$, or $v=g\,t$, $\therefore t=\frac{v}{g}$.

To determine the height a body rises, we need only substitute for t its equivalent $\frac{v}{g}$ in the equation

$$h=vt-\frac{1}{2}gt^2=\left(v-\frac{1}{2}gt\right)t,$$

and we shall have

$$h'=\left(v-\frac{g}{2}\cdot\frac{v}{g}\right)\frac{v}{g}=\frac{v^2}{2g}.$$

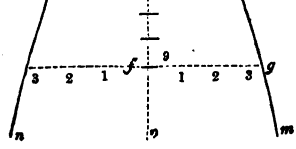
The velocity which the body acquires in falling this height is $=gt=g\,.\,\frac{v}{g}=v$; *i. e. the body at the end of its fall acquires again the same velocity with which it was projected vertically upwards.* If the time the body is in motion be t' sec., then it has reached a height $=g\,.\,\frac{t'^2}{8}$.

Experiments in which cannon-balls are shot vertically.

§ 41.

If the projectile force forms any angle whatever with that of gravity, a body set in motion by these forces will not follow the direction of either of them, but will adopt a middle course. The resulting path, which is determined by the parallelogram of motions (§ 24.), varies in direction every moment on account of the continuous attraction of the earth; the course of the body will therefore be a curved line, named a *parabola*. *Fig.* 12.

Fig. 12.

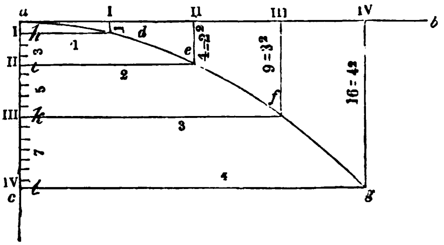


A parabola is a curve consisting of two similar sides *a m*, *a n*, which may be produced indefinitely, by being drawn in such a manner with regard to the axis *a p* passing through the vertex *a*, that the abscissæ *a b*, *a c*, *a f*, are to each other as the squares of the ordinates *b d*, *c e*, *f g*. Thus, if the former be 1, 4, 9, 16, &c. the latter will be 1, 2, 3, 4, &c.

§ 42.

If a body be projected in a horizontal direction, its path will be the *half* of a *parabola*, whose vertex *a* lies in the point of projection, *fig.* 13.

Fig. 13.



Let *a b* represent the direction of the projectile force, and *a c* that of gravity; a body *a* would be carried by the former force alone in successive seconds of time through the equal spaces, *a*I, I. II, II. III, III. IV, whilst by gravity alone it would fall in successive seconds through the spaces 1, 3, 5, 7. (§ 33.) If both forces act simultaneously on the body, it must, by the law of the parallelogram of forces (§ 24.), arrive at the end of the first second at *d*, at that of the second at *e*, of the third at *f*, and of the fourth at *g*. Again, as gravity is not a force producing its effects by impact, but is permanent in its operations, the path *a d e f g* of the body cannot be a broken line, but will be such a curve, that the abscissæ shall be as the squares of the ordinates, that is, it will be a parabola.

For in this curve the abscissæ *a h*, *a i*, *a k*, *a l*, &c., are *a h* × 1, *a h* × 4, *a h* × 9, &c., and the ordinates are *h d* × 1, *h d* × 2, *hd* × 3, &c., therefore the ratio of each abscissa to the square of the ordinate is the same.

From this figure we obtain also the following:—

a) A body projected horizontally moves in the direction of the projectile force, through a space equal to that which it would have described had it been acted on by that force alone; but arrives at a point as much below that from which it started as it would have reached had it fallen freely in the same time.

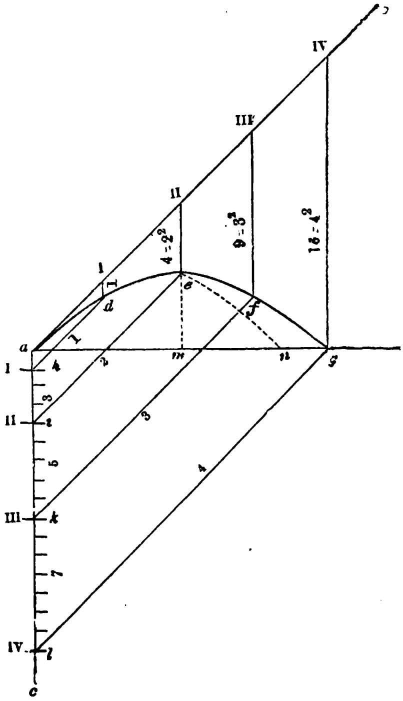
b) A body descends towards the earth through the same space that it would have fallen had it obeyed the force of gravity only during the time of its motion, inclining each moment from the original perpendicular as far in the direction of the projectile force as this would have carried it in the same time.

c) A body projected horizontally reaches the earth in exactly the same time that it would have fallen freely from the same height.

§ 43.

If a body be thrown *obliquely upwards* its path will be a *complete parabola, a d e f g, fig.* 14., whose vertex *e* will be the highest point reached by the body, which ascends one of the legs *a d e*, and descends the other *e f g*.

Fig. 14.



§ 43. If the parabolic theorem, that the greatest range is obtained at an angle of 45°, and that the ranges will be equal at equal angles above and below the

This path is obtained in the following manner: suppose the body *a* were impelled in the direction *a b*, and that the projectile force alone acted on it, it would then arrive in the first second at I, in the second at II, in the third at III, and so on. During the first second, however, gravity has drawn it through a space $a\,h = 16$ ft.; at the end of the second second, $a\,i = 4 \times 16 = 64$ ft; at the end of the third second, $a\,k = 9 \times 16 = 144$ ft.

That is, after the first second the body will have arrived at *d*, after the second at *e*, after the third at *f*, after the fourth at *g*, and so on; in other words, it will move through the points *d*, *e*, *f*, *g*, making the curve *d e f g*, which, for similar reasons to those that were advanced in the case of a body projected horizontally, is a parabola.

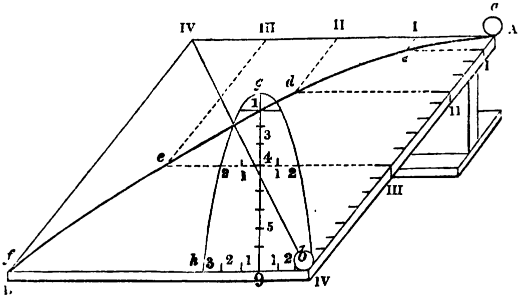
In this curve the abscissæ at the ends of successive seconds are $a\,h \times 1$, $a\,h \times 4$, $a\,h \times 9$, &c., and the corresponding ordinates are $h\,d \times 1$, $h\,d \times 2$, $h\,d \times 3$, &c., therefore the abscissæ are as the squares of the ordinates.

In consequence of the resistance of the air, the path of a body deviates from a true parabola into the figure called the *ballistic curve*, the descending side of which (*e g*, *fig.* 14.) approaches somewhat nearer to the point of projection, as *e n*.

All missiles, as shot or balls impelled by means of gunpowder, describe curves of this nature, the elements of which are calculated on mathematical principles.

Exp. A graphic illustration of the curve described by a body is afforded by means of a projectile-table. — A table of a dark colour, best of all of slate, A B, *fig.* 15., is placed at

Fig. 15.



maximum range, be experimentally verified by means of such a projectile table as is described above, it is evident that both the force with which the ball is projected, and the angle of its projection must be susceptible of measurement. This is accomplished by using a spiral steel spring, which may be any angle to the horizon ; *a* and *b* is a marble whitened with chalk kept in its place by a small groove. — If the ball *a* be struck so as to move horizontally, it will describe the semi-parabola *a c d e f*; but if the ball *b* be projected upwards at any angle whatever, it will describe the parabola *b g h*.

Of Motion round a Centre.

§ 44.

When the motion of a body arises from two causes, so that, the *centripetal force*, being one of these, constantly attracts the body towards a certain point, called the *centre of force ;* whilst the other, viz. the *tangential force*, or rather, the inertia of the body, drives it in another direction, there will result a curvilinear motion round this point.

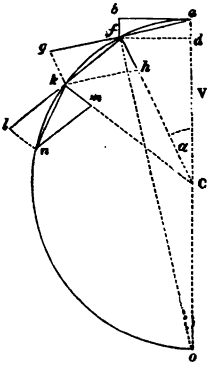
That the course of a body acted on by two forces, in the manner described above, must be curvilinear, may be deduced as follows: The attractive force acts without intermission on the body, so that the change it produces is continuous, but nevertheless it may be represented by supposing it to effect infinitely small changes of direction and velocity at infinitely small intervals of time. In *fig.* 16. let C be the centre of the attractive force, then suppose there to be a body at rest at *a*, unless supported it will fall towards C. Now, let a force act on *a* in such a manner as to drive it from *a* to *b* in the same time that the centripetal force would have drawn it from *a* to *d*, it must in this time travel the distance *a f*. In consequence of the body's inertia (§ 27.), in the next equal portion of time it would pass over $fg = af$, but the centripetal force meanwhlile would attract it to *h ;* it will therefore arrive at *k*. In the third space of time it would be carried for one reason from *k* to *l*, where $kl = fk$, and for the other from *k* to *m ;* con-

strained according to various degrees of tension ; the spring must be contained in a tube moveable about a pivot, so as to be set at the required angle by a graduated scale. In order that these experiments should coincide with the theory, care must be taken that the lower edge of the table is perfectly horizontal.

§ 43. The following extract from the late Professor Leslie's "Elements of Natural Philosophy," vol. i. p. 117., will enable the reader to form some idea of the resistance offered by the air to bodies moving with high velocities. "In the practice of gunnery, however, unless in the case of very small velocities, the parabolic theory is of little avail, the resistance of the air being so prodigious as completely to derange all the effects. Thus, a 24-pound shot, discharged at the elevation of 45°, and with a velocity of 2000 feet each second, would, *in vacuo*, reach the horizontal distance of 125,000 feet, but has its range through the atmosphere confined to 7300 feet."

sequently it must pass over the distance *k n*, and so on. Now, when the attractive force acts without intermission, and the intervals of time at which its action may be considered to be renewed, are infinitely small, consequently, as in projectile motion, the crooked line *a f k n*, which represents the path of the body, becomes a curve.

Fig. 16.



Such is the motion of the planets around the sun, and the satellites around their respective primaries. Universal gravitation acts throughout the creation as the centripetal force, and a tangential force, or initial velocity, must be assumed to have acted on these bodies imparted to them by their Maker.

As the result of the tangential force every body moving round a centre acquires a tendency to fly off from that centre; this tendency is called the *Centrifugal force.* It is the antagonist of the Centripetal force, to which it sets limits.

Fig. 16. shows that after the first period the body, if under the influence of the tangential force only, would arrive at *b*, and that it would thus have receded the distance *b f* from the centre of motion C; *b f* is therefore half the measure of the centrifugal force for that time. And since the centripetal force is opposed to the centrifugal, it compels the body to deviate from *a* to *f*, instead of proceeding from *a* to *b*.

§ 45.

The relative intensities of the centripetal and tangential forces determine the nature of the curve in which a body thus acted on will move.

If the centrifugal force be at all points equal to the centripetal, it follows that the body's path will be a circle, this being the only motion around a centre which can be described with uniform velocity.

If, on the contrary, the centrifugal force at every point of the body's path be inversely as the square of its distance from the centre, it may be proved mathematically that the curve is an ellipse, in which case the velocity at one time increases, and at another decreases. The motions of all the planetary bodies are performed in elliptical orbits.

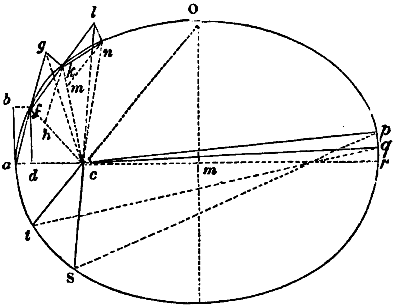
1) Referring to *fig.* 16., if we assume *a b* to be a very small

quantity, ab may be considered $= df$, whence it follows, that *when a body moves in a circular orbit, the centripetal or centrifugal force is to the tangential, as the vers. sin. to the sin. of the arc moved through.* If, then, we put $ad = P$, the centripetal, ab or $df = C$ the tangential force, and α, the angle measured at the centre of the arc af, then we have

$ad : df =$ vers. sin. α : sin. α
or $P : C =$ vers. sin. α : sin. α.

2) In an elliptical orbit the centre of the forces will be found in one of the foci c of the ellipse, *fig.* 17. All right lines drawn from this point to the circumference, as ca, cf, ck, cn, &c., are called *radii vectores.* Every body moving in an ellipse arrives at one point a of its orbit which is its nearest distance ca from c, the centre of the forces, called the *Perihelion* of the planets, and at some other point r which is its greatest distance cr, or the *Aphelion* of the planets, whence co is the *mean distance.*

Fig. 17.



3) If, as in *fig.* 17., in equal times $af = fg$,

$$\text{then } \triangle\, afc = \triangle\, fgc, \text{ Eucl. I. 38.}$$
$$\text{and } \therefore gk \text{ is parallel to } cf$$
$$\triangle\, fck = \triangle\, fgc$$
$$\therefore \triangle\, afc = \triangle\, fck$$

In like manner it may be proved that the triangle $fck =$ triangle kcn, &c.

Whence, *in an elliptical orbit equal areas are described by a radius vector in equal times.*

In n time we shall have n area, and in m time the m area; wherefore these areas are proportional to the times.

4) If the body a continue to be at an equal distance from c, as in a circular orbit, then the lines ca, cf, ck, &c.; and the

chords *af*, *fk*, *kn*, &c. are equal, *i.e.* the velocity of the moving body is the same at every point of its orbit, or in other words, the body has a uniform motion; in an elliptical orbit, on the other hand, the velocities (*ts*, *pq*, *fig.* 17.) vary; and planets move most rapidly at their perihelion and most slowly at their aphelion. — Mean velocity.

5) Suppose a body *a*, *fig.* 16., to describe the arc *af* in one second of time, then its velocity $V = af$, and if we assume *af* to be very small, the chord *af* will be so too. Then, since the triangles *adf*, *afo* in a circle are similar,

$$ad : af = af : ao$$

$$\text{or } ad = \frac{af^2}{ao}$$

Here *ad* represents the motion caused in a given time by the centripetal force, or half the measure of that force, as well as half the measure of the centrifugal force. Again, substituting for it $\frac{1}{2}G$, for the velocity *af*, V, and for the diameter $ao = 2R$, it follows: —

$$G = \frac{V^2}{R}$$

Giving to *g*, *v*, and *x* corresponding values for any other point *v*,

$$g = \frac{v^2}{r}$$

i. e. the centripetal and centrifugal forces are as the squares of the velocities, divided by the diameter of the circles described;

$$\text{or } G : g = \frac{V^2}{R} : \frac{v^2}{r}$$

if $R = r$, $G : g = V^2 : v^2$

$V = v$, $G : g = r : R$

$V : v = R : r$, *i. e.* when different circles are described in equal times:

$$G : g = R : r$$

6) According to the general laws of gravitation (§ 29.) the attractions G and *g* of all bodies are inversely as the squares of the distances D and *d*, or R and *r* from their centre; thence

$G : g = r^2 : R^2$; but by the preceding, $G : g = \frac{V^2}{R} : \frac{v^2}{r}$, thus also

$$\frac{V^2}{R} : \frac{v^2}{r} = r^2 : R^2,$$

$$\text{or } V^2 : v^2 = r : R,$$

i. e. the squares of the velocities are inversely as the distances.

7) If the times employed by *a* and *v*, *fig.* 16., to complete their circles be called T and *t*, it follows:

$$V \,.\, T = 2\,R\,\pi,$$
$$\text{and } v \,.\, t = 2\,r\,\pi,$$
$$\therefore V^2T^2 : v^2t^2 = R^2 : r^2.$$

Dividing these proportionals by the preceding in 6) we obtain

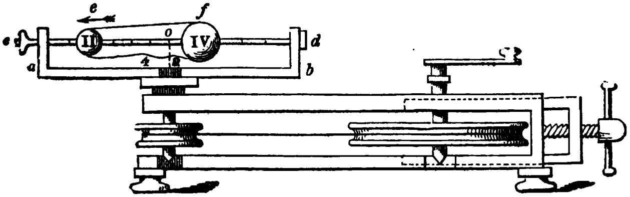
$$\frac{V^2T^2}{V^2} : \frac{v^2t^2}{v^2} = \frac{R^2}{r} : \frac{r^2}{R},$$
$$\text{or } T^2 : t^2 = R^3 : r^3;$$

i. e. the squares of the times are to each other as the cubes of the distances.

Keppler (1571—1630) deduced the laws under 3, 6, and 7 from observations made by himself on the motions of our earth and the other planets around the sun; their accuracy was established on mathematical principles by Newton, according to his laws of gravitation.

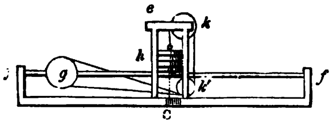
All the laws of centrifugal force may be illustrated by the whirling-table. This apparatus consists of one or two horizontal sheaves, to which a rapid motion is imparted by turning a handle fixed to the axis of one of the sheaves, the other being connected with it by means of an endless cord. Now, by turning, more or less rapidly, the wheel to which the handle is fixed, we can give various velocities to the other wheel, every point of which moreover has a velocity, increasing in proportion to its distance from the centre. By knowing how many revolutions the second wheel performs whilst the other is turned once round by the hand, the actual velocity of every point in it can be readily estimated. This machine is variously constructed; the diagram below gives one of its most simple forms.

Fig. 18.



First, to show that the centrifugal force of a body revolving in a circle at any distance from the centre is inversely as the radius, the time of revolution being supposed the same. Screw on the frame *ab*, in which a fine polished wire is fixed; on this wire the balls *e* and *f* slide, having a whole drilled through their centres, and being attached to each other by a string. The wire is graduated both ways from the point which marks the centre of the sheave, so that the balls may be placed at certain distances from that centre. If the balls *e* and *f* be placed at equal distances from *o*, they will by revolving acquire an equal velocity and tendency to fly from the centre. If their masses are equal, the centrifugal forces with which they endeavour to fly in opposite directions will be equal also. The thread, however, prevents the balls from singly following this tendency, and in the case we have supposed they will retain their respective positions unchanged. But if one of the balls be greater than the other, they may still revolve in equilibrium if their distances from the centre are inversely as their masses, or, in other words, the centrifugal force varies as the product of the masses and distances from the centre. But it is clear that, if the distances from the centre were the same, the centrifugal force would vary as the mass; therefore, the masses being the same, the centrifugal force varies as the distance.

Fig. 19.



Next, on the middle of the fly-wheel is fastened the bar *ab*; in the centre of which stands the framework *ec*, *fig.* 19., consisting of four metal wires, by which is supported the axis of the pulley *k*, which is made to move with but little friction, and below the axis of two small parallel pulleys, *k'*. At the two extremities of the bar are the supports *ff*, in which the polished metal rod *ff* is secured, on which the wooden ball *g* may slide freely, a hole being drilled through it. The ends of a fine silk cord are fastened to small staples on opposite sides of the ball *g*, which cord runs singly over the small wheels *k'*, but double over *k*. A hook carrying the weight *h* is attached to this cord. This weight *h* consists of several parallel circular plates of metal of equal weights, that can be placed one on the other as may be required, and is intended to represent the force of attraction.

If, now, this apparatus be set in motion by the fly-wheel, *fig.* 18., the ball *g* will fly from the centre, tighten the cord, and raise the weight *h*, as soon as the centrifugal force of the ball has acquired a sufficient intensity. With a little practice so uniform a motion may be imparted to the wheel that the weight *h* shall be kept suspended above the rod *ff*. For instance, if the distance of the ball from the centre of rotation be 15 inches, and the weight $h = \frac{4}{9}$ lb., by performing a certain number of revolutions, about 40 in a minute, we shall be able to balance the weight. Repeat the experiment, the distance being 10 inches, and $h = \frac{9}{9}$ lb., 73 or 74 revolutions per minute will be required to counterpoise the weight *h*.

Now, since the velocities are proportional to the radii, and inversely as the times of revolution

$$V : v = \frac{15}{74} : \frac{10}{40} = 4 : 5 \text{ nearly}$$

$$\therefore V^2 : v^2 = 16 : 20 \text{ nearly}$$

$$= r : R \text{ (see 6)).}$$

$$\text{and } \therefore \frac{V^2}{R} : \frac{v^2}{r} = \frac{16}{15} : \frac{24}{10} = 4 : 9$$

or in the proportion of the weights (see 5)).

Also,

$$T^2 : t^2 = \frac{1}{40^2} : \frac{1}{74^2} = 74^2 : 40^2 = 15^3 : 10^3$$

very nearly (see 7)).

The ball *g* has therefore on a small scale fulfilled the same laws as to the times of its revolution as the planets at their vast distances from the sun.

§ 46.

If a solid body of a spherical form revolve about an axis (§ 23.), all parts of its surface and mass will perform the revolution in the same time. Their velocities will therefore be as their perpendicular distances from the axis, *i. e.* as their radii, and consequently the centrifugal forces of the points will be directly as their radii. (Obs. 6. § 45.) The farther, therefore, the points are situated from the axis of rotation, the greater will be their tendency to fly off from it.

These particles, then, are in a state of *central motion*, such that the centrifugal force is counteracted by the *cohesive*.

This is the condition of our earth in its revolution on its axis; consequently

$$V : v = R : r$$

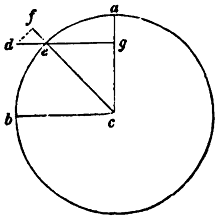
$$= \cos. L : \cos. l.$$

L and *l* being the latitudes of the places; whence

$$G : g = \cos. L : \cos. l.$$

i. e. the centrifugal force is greatest under the equator, diminishing towards the poles, where it$=0$. As gravity is the antagonist power to the centrifugal force, it follows that the former must increase as the latter diminishes from the equator towards the poles; this, however, is not exactly the ratio because it is only at the equator that these two forces act in directly opposite directions; at every other point they act obliquely, and, as a consequence, but partially. In order to obtain the vertical component of the centrifugal force, let de, *fig.* 20., be the centrifugal force g, acting at the point e; then by the parallelogram of forces (§ 24.), if df be drawn at right angles to fc, ef will represent that portion g' of the centrifugal force which tends to counteract the gravitating force, thus :—

Fig. 20.



$$g : g' = 1 = \cos. fed : 1 : \cos. l \therefore g' = g. \cos. l,$$

and $G' = G \cos. L$,

and $G' : g' = G \,.\, \cos. L : g \,.\, \cos. l;$

whence, substituting the values of G and g we obtain

$$\text{II}, \; G' : g' = \cos.^2 L : \cos.^2 l.$$

If, however, the centrifugal force be so great as to overcome the mutual cohesion subsisting between the particles of the rotating body, these will then fly off from the mass, in which case they will be projected in tangents to their orbit with a velocity proportional to the tangential force.

At the equator the centrifugal force $= \frac{1}{289}$th of that of gravity. If the velocity with which our planet revolves on its axis were to be increased 17 times, the centrifugal force would also be increased 17^2 times, or 289 times (§§ 45. 46.), and would then equal the force of gravitation.

To this cause is to be attributed the dispersion of spray by grindstones; the violent flying asunder of the portions of millstones or wheels which happen to break whilst in motion; the effects of a sling, and of the ballistæ and other engines of the ancients for throwing missiles.

§ 47.

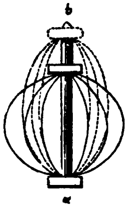
If the particles of which a body is composed are easily displaced, *i. e.* if the body be soft, a change may be effected by the action of this force in the figure of the body.

Such a body of a spherical form revolving about one of its diameters, acquires a flattened shape in the direction of this diameter or axis, because the parts that lie in the plane of the greatest circumference which can be drawn perpendicular to the axis, viz. in the plane of the body's equator, have the greatest centrifugal force, this force diminishing as the parts approach the poles or extremities of the axis.

Hence the inferences drawn as to the flattened figure of our earth.

Exp. On the vertical axis *a b*, *fig.* 21., is an armillary sphere, composed of elastic wires, fitting round the axis by means of a ring, which holds them all together. By this contrivance it is possible for the elastic rings to assume an elliptical figure, having a shorter vertical diameter. Screw this apparatus into the middle of the fly-wheel of the whirling-table, *fig.* 18., and give to the whole a rotatory motion; the wires, instead of their original circular form represented by the dotted lines, will assume, in consequence of the centrifugal force, an elliptical figure, as shown in the dark lines, *fig.* 21.

Fig. 21.



§ 48.

When a body rotates about an axis, the centrifugal force of each particle gives rise to a pressure on the axis. But if its mass be equally arranged around its axis, the centrifugal forces of its particles mutually counteract each other, and destroy the pressure on the axis. Such an axis is called a *Free Axis.* So long as the rotation of the body continues, this axis can be displaced only by the exertion of a considerable force, though it would readily assume another position on applying but a small power if the body were in a state of rest, because the centrifugal force of every particle, by virtue of its inertia, has a tendency to continue in that plane in which its motion first began.

The axis of our earth is such a free and stable axis. Its unchanging position during the earth's revolution round the sun is owing to the cause assigned above, and the various phenomena of the light and heat imparted to its surface by the sun are consequent upon its fixed position. Experiment 3, § 27. may illustrate the earth's motion.

III. THE DIFFERENT PHENOMENA PRODUCED BY ADHESION.

§ 49.

That force by means of which bodies, whether similar or dissimilar, adhere together, when their surfaces come in contact, is called *Adhesion.* (§ 11.)

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§ 48. In regular figures there is an infinite number of such axes, for example a sphere will revolve about any one of its diameters as an axis, without exerting any pressure upon it. All bodies have three free axes. The equilibrium of the centrifugal forces is *unstable* when the axis of rotation of the body once displaced recedes from the free axis; it is *stable* when the tendency of

It has been remarked above, that this force does not act with equal intensity on all matter; that its operations are influenced not merely by the mass of bodies, but also by the number of points of contact, or the extent of their touching surfaces; for which reason it is sometimes called *Superficial Attraction;* and that its effects obtain equally *in vacuo*, so that it must be independent of the atmospheric pressure.

§ 50.

The adhesion between solids is best exhibited when the touching surfaces are as smooth as possible. A certain force is requisite in such cases to separate the sides that are in contact.

First Exp. Two polished plates of glass or metal, called *adhesion-plates*, will be found to stick together, if laid one on the other and slightly pressed.

Second Exp. Two plates of common window-glass laid in like manner on each other do not adhere. *Query.* What is the cause of this?

The intensity of the adhesive force may be measured by suspending one of the plates to the arm of a balance and attaching weights to the other arm until the plates are separated. Dividing now the weight thus found by the number of square inches in one plate, the quotient will give the adhesive force for one square inch.

Among many instances of the mutual adhesion of solids we may adduce, writing with chalk, charcoal, plumbago, and the like, as also the sticking of dust to the walls of buildings and articles of dress, the coating of gun-barrels with the powder, and the polishing of gunpowder by means of black-lead.

§ 51.

Adhesion obtains further betwixt solids and liquids. Two opposite conditions demand notice: either the adhesion is stronger than the cohesive force of the liquid (§ 13.), and the solid becomes moistened, or else the latter force exceeds the former, and in that case the solid is not wetted.

First Exp. Many organised and unorganised bodies become wet on being brought into contact with water, spirit of wine, and many other liquids—the human body, clothes, &c. if exposed to rain.

Second Exp. A drop of any of these fluids placed on a wooden table, or on a plate of glass, loses its globular form, and spreads itself over the surface of the solid.

This does not happen with a drop of mercury.

the centrifugal forces is to restore the position of the body after displacement. If the body is in equilibrium about its new axis, its former position was one of *neutral* equilibrium. Hence, as disturbing causes are never absent, the only permanent revolution is about a stable axis.

Third Exp. Immerse a finger in water, it becomes wet; in quicksilver it remains dry.

Fourth Exp. A tallow candle, or a feather of any species of water-fowl, continues dry, though dipped in water.

Fifth Exp. Gold, silver, tin, lead, &c. become moist on being immersed in quicksilver, but iron and platinum do not.

Sixth Exp. Quicksilver poured into gauze will not run through, water will. Let the gauze containing the quicksilver touch the surface of the water in a glass, and the metal will immediately flow through.

Seventh Exp. *Vera's Cord-pump*—an endless cord, running round two wheels, descends below the surface of the water, which it raises, by means of the adhesion subsisting between the liquid and the rope, to which a rapid motion is imparted by the upper wheel.

On this effect of adhesion depend writing with ink, Indian-ink and colour drawing, the smearing of metals, especially iron, with oil or grease, to protect them from rust.

The measure of the absolute force of adhesion between solids and liquids is found, in the same manner as that of solids only, by the help of a balance, the solid being suspended, so that its surface just touches the surface of the fluid.

Even those fluids which do not moisten the solids immersed in them, have yet a feeble degree of adhesion to them.

Eighth Exp. Small globules of mercury continue hanging to glass, paper, and other bodies, which they do not wet, even though held in such a position that they would naturally fall.

In many of these cases, namely, those in which the solid on coming in contact with a liquid, itself passes into the latter state, we have the transition of the adhesive force into chemical solution.

This occurs with regard to gold, silver, tin, lead, &c., when these metals are immersed in quicksilver.

§ 52.

Solids which become wet on immersion in certain liquids lose this property if covered with any matter not similarly affected by that particular fluid.

First Exp. A glass bottle, or one made of an oily wood, or well coated with dust, or with *semen lycopodii*[*], imbibes no moisture on being plunged in water—spherical drops of water roll off its surface.

Second Exp. Put a ring or coin into a cup of water, cover the surface of the fluid with *semen lycopodii*, and the object can be picked out of the cup without wetting your fingers.

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[*] Club Fern.

On the circumstance of the solid's attracting moisture from the liquid, or not, as well as on the various degrees of the adhesive force, several other facts depend, as may be shown by the following experiments :—

Third Exp. Place a drop of water at the upper end of a glass tube held in a slanting direction, its adhesion will cause it to stick to the glass, whilst it descends to the lower extremity of the tube in consequence of the action of gravity; on arriving at this point it assumes a globular form, and drops off as soon as the attraction of the earth has overcome the adhesion. To this property we are to attribute the fact of fluids running down the sides of the vessel containing them, when poured out of one vessel into another.—Convenient form for drains, &c.

Fourth Exp. Smear part of the rim of a glass containing water with some hog's-lard, and then cover it with *semen lycopodii.* If the liquid be poured out over this part of the vessel it will run straight down, and not over the sides of the glass.—*Query.* Why so?

Fifth Exp. Quicksilver poured out of a glass vessel with perpendicular sides, never runs down them, but the contrary happens if a tin vessel be used.—*Query.* Why so?

Sixth Exp. A small globule of mercury may be lifted off a piece of paper by means of a glass rod, which, however, it will immediately quit if brought into contact with another globule of the same metal.—This experiment proves that a certain degree of adhesion subsists between the quicksilver and the glass, but that this force is less powerful than the mutual cohesion of the drops of mercury.

Seventh Exp. Let a drop of water fall on a plate of glass, it will be displaced by a drop of oil of turpentine, which in its turn will give way to a drop of æther or spirits of wine.—It follows from this experiment that oil of turpentine has a stronger adhesion to glass than water has, and that of æther has an adhesion stronger even than the oil.

§ 53.

The adhesive force between solids is materially increased by introducing a fluid between the surfaces in contact. The fluid fills up the existing inequalities, thus increasing the number of points of contact, for the adhesion is increased in a degree equal to the adhesion of the fluid body to the two surfaces of the solids.

First Exp. Two plates of common window glass on being wetted exhibit this property in such a manner as to stick tightly together

Second Exp. Metal or glass adhesion-plates adhere together with far greater tenacity, if breathed on previously to their being brought in contact.

The Adhesion becomes Cohesion when the liquid thus introduced changes into a solid. Hence the application for various purposes connected with the arts, of glue, cement, and solder, the process of plating, that of silvering mirrors, and the use of mortar in building.

Third Exp. A thin flake of melted tallow introduced between the plates increases their adhesion: if allowed to become cold, it requires considerable force to sever them.

§ 54.

A remarkable instance of the effects of adhesion between solids and liquids is afforded in *Capillary Attraction* and *Repulsion.*

This peculiarity consists in the fluid's either ascending the solid body that touches it, or else descending to a lower level at the point of contact with the solid, being depressed by it. The attraction takes place when the solid is of a nature to be wetted by the fluid: repulsion occurs when this is not the case.

These phenomena are most observable in the interior of glass tubes of small bore, hence called *Capillary tubes;* and the narrower the bore the greater the effect produced. They contribute much to our acquaintance with the causes of this attraction and repulsion, inasmuch as the results are more easily noticed by us in these tubes than in bodies of any other form.

First Exp. Plunge the ends of several capillary tubes of various bores in water, the fluid will rise in the interior of the tubes to a height exceeding that of the surrounding water, the respective heights reached varying as the bores. The surface of the fluid inside the tubes is invariably concave.—Now take the tubes out of the water, and a column of the liquid will be seen in each, the length of the column depending on the fineness of the bore.

These circumstances may be thus explained. The capillary attraction between the solid and fluid excites a tendency in the particles of the fluid towards those parts of the interior of the tube with which they are in contact, and thus the fluid assumes a concave form. As soon as the fluid within the tube has become concave, its elevated boundary flows together, in consequence of the attractive force exerted within the narrow limits of the capillary tube, the next row of particles in their turn acts upon the elevated surface of the fluid, when a second ascent of the boundary line takes place, and a second confluence of the elevated particles, whilst the lower ones are lifted also by their cohesion with the upper. The action of these attractive forces continues till an equilibrium is established between the weight of the column and its cohesion and the capillary attraction of the tube.

Second Exp. Now insert the same tubes in quicksilver, and the surface of the fluid within will be depressed below that of the surrounding fluid; its figure will be convex, and its depth will increase as the tube is of narrower bore.

The depression of the mercury in this case may be accounted for on grounds similar to those taken in the explanation of the preceding experiment. That is, the fluid sinks as much below its original level as will bring the depressed column of liquid into equilibrium with the capillary repulsion.

In tubes of the same material, the fluid also being the same, the height to which it ascends, or the depth that it sinks, will be inversely as the diameter of the bores. Let the inner diameters of two tubes be d and D, then the sum of the attractions of the first tube is to that of the second as the diameters or circumferences of the tubes, or as $d : D$. The attractive forces, however, are as the weights of fluid which they support, that is, of columns of the heights h and H respectively; consequently,

$$\begin{aligned} d &: D = d^2h : D^2H \\ \text{or } 1 &: 1 = dh : DH \\ \text{or } D &: d = h : H \end{aligned}$$

It is evident that the height to which the fluid is raised or the depth to which it is depressed in the same tube must depend on its specific gravity as well as on the tube, and that it will be affected by the temperature of the fluid. The heights therefore vary directly as the attractions, and inversely as the specific gravity; whence it follows that $h \times$ spec. grav. = attract. force.

The following phenomena likewise depend on the above-named principles:—

Third Exp. Dip a glass rod in any fluid that will adhere to it; water, for instance; in consequence of its attraction to the rod it will ascend it for some distance.

§ 54. "Mr. Martin says, the power by which the fluid is raised will keep it in the tube for any time without exhaling or evaporation; as he tried by hanging several capillary tubes thus charged with their fluids for months together in the summer sun, whose heat did not appear to diminish the fluids in the least sensible degree." — *Gregory's Mechanics*, vol. i. p. 414.

Fourth Exp. Smear the glass rod with fat, or sprinkle it with *semen lycopodii*, and immerse it in water; in consequence of the repulsion around the rod, the water about it will descend to a lower level.

Fifth Exp. Plunge a clean glass rod in quicksilver; in consequence of the repulsion the latter will sink about the rod.

Sixth Exp. Dip a piece of polished tin in quicksilver; by virtue of its attraction the metal will ascend the tin.

Seventh Exp. Pour any adhesive liquid, as water, into a glass, and it will assume a concave form of surface by ascending the sides of the vessel.

Eighth Exp. Draw a ring of fat within the glass, and sprinkle it with *semen lycopodii*; the water will rise to the height of this circle, and, if still more be poured in, it will rise in a convex shape, that portion around the glass being depressed.

Ninth Exp. Quicksilver in a glass vessel assumes a convex form, and in one of tin a concave.

Tenth Exp. If two plates of glass be placed with one end of each touching the other, their sides being at a small distance asunder, and be thus immersed in any adhesive fluid, it will rise between them, and the highest point will be where the plates are nearest together.

Many natural phenomena are attributable to capillary attraction, as the dampness of the ground, sand, sugar, and other similar bodies below which any moisture exists; the ascent of oil, melted tallow, or wax, as well as that of spirits of wine in the wicks of candles and lamps, the absorption of fluids by sponge, blotting-paper, &c., the swelling of wood with change of weather, and the filtering of liquids through any porous vessels that may contain them. The ascent of the sap in plants appears in part to be owing to the attraction of the capillary vessels of the plants, though some vital agency is also in operation.

To the same cause, perhaps, is to be attributed the passage of liquids and gases through the porous laminæ and capillary apertures of bodies, generally called *Endosmosis* and *Diffusion.*

Eleventh Exp. Fill a small glass vessel to the brim with spirit of wine, tie over it a piece of bladder previously softened, and put the whole apparatus under water; in the course of a few hours a portion of the water will penetrate

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Exp. 10. If the two plates be so placed at an angle with each other, that their vertical sides shall coincide, on being immersed in water or any other adhesive fluid, it will ascend between them, and form a *hyperbola.*—See *Playfair's Outlines of Nat. Phil.* vol. i. p. 187.

to the spirit of wine, whilst a still smaller quantity of the latter will make its escape. By this means the covering of the glass will become so strained, that on its being punctured with a needle, a jet will be thrown several feet high. — The passage of the one fluid is caused partly by capillary attraction, and partly by the chemical affinity of the two fluids, whilst the greater penetration of the other is due to the less powerful attraction exercised by the capillary tubes of the bladder on the water. The spirit of wine being more strongly attracted by these tubes, will flow out less readily.

The prodigious effects the application of this force on a large scale is calculated to produce, may be seen in splitting large blocks of stone by the insertion of dry wedges of porous wood and then well saturating them with water.

§ 55.

The adhesive force between solids and liquids excites visible motion in the former, provided they are light enough to be easily moved. Attraction takes place between solid bodies, whether the fluid on which they float adheres to them or not; repulsion, on the contrary, happens only when the fluid adheres to one of the solids and not to the other.

First Exp. Lay a small hollow glass sphere on the surface of some water contained in a glass vessel. — The globule, as being the movable solid, will gradually approach the side of the vessel, to which it will at length be drawn. — A glass rod immersed in the water will produce a similar effect.

Second Exp. Two glass globules floating on the water mutually attract each other, if the water adhere to the surfaces of both or of neither of them. In order that the globules may not be wetted by the water, smear them with fat, and sprinkle them with *semen lycopodii.*

Third Exp. The repulsion of a ball floating on the surface of the water and the side of the cup is seen when the water adheres to the ball and not to the cup, and conversely.

Fourth Exp. Repulsion is exhibited between two glass globules when one has water adhering to it, and the other has not.

§ 56.

Adhesion obtains likewise between liquids, when one of two conditions will result; either the adhesion of the fluids will exceed their cohesion or be less than it.

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§ 55. The word attraction refers merely to the reaction which takes place; the phenomena arise from changes in the attractions or repulsions of the contiguous fluid.

In the former case a perfect union will take place between the fluids. If the forces be equal, when contact is established, adhesion will immediately be converted into cohesion, and a homogeneous mass will be obtained. When the forces are unequal, it is frequently difficult to determine whether the union which results is a chemical combination or merely a mixture.

In the second case it is merely the surfaces of the fluids which touch without uniting; that fluid whose cohesion is cancelled by adhesion remains on the surface of the other.

First Exp. Two drops of the same fluid unite in one as soon as a contact is established between them. This is most clearly shown in the case of two globules of mercury.

Second Exp. Water, wine, spirit, milk, and several other liquids, mix when poured together. The effects of their mutual attractions are therefore stronger than the cohesion of their respective particles.

Third Exp. Water and quicksilver, or water and oil, if poured together, will not mix. — The cohesion of the quicksilver or of the oil is consequently stronger than its adhesion to the water.

Fourth Exp. Let a drop of water fall upon some quicksilver and it will spread itself over the entire surface of the metal. A similar result would follow if a drop of sweet oil be put on water, which will be overspread with a thin film reflecting the prismatic colours.

Fifth Exp. Place a drop of liver of sulphur (tersulphuret of potassium) on the surface of some water previously covered with a film of oil, and it will take the place of the oil.

Here is presented to us the phenomenon of one fluid taking the place of another, even after the adhesive force of the one is stronger or weaker than that of the other. (§ *52.*)

§ 57.

Lastly, adhesive attraction is displayed by aëriform bodies towards both solids and fluids, and is then distinguished by the name of *Absorption.* In these cases, Absorption seems frequently to be the first evidence of chemical attraction; for when gases and liquids unite it is often difficult to determine whether chemical combination has or has not taken place.

First Exp. Plunge a piece of cork quickly under water; numerous little bubbles of air will adhere to it.

Second Exp. Place a glass of water in the sunshine, or on a warm stove; small bubbles of air will arrange themselves about the sides of the vessel: this air must therefore have been contained in the water.

IV. THE PRINCIPAL PHENOMENA DEPENDING ON AFFINITY.

§ 58.

Chemical Attraction or *Affinity* (§ 15.) is the exercise of the molecular force on the atoms of dissimilar bodies.

It effects both the *combination* and *decomposition* of the atoms composing different bodies; in both cases the substances, whether combined or separated, are found to possess *individual properties* distinct from those which they had in their former state. This alteration either becomes immediately appreciable by our senses, as a change of colour, taste, &c., or it may be observed in the effects produced on other bodies.

§ 59.

In chemical language all substances are divided into *elements* or *simple* substances and such as are *compound.*

Simple substances are those which are not chemically divisible, *i. e.* such as have not yet been resolved into different substances, as sulphur, carbon, iron, &c.

Compound substances, on the contrary, are such as may be demonstrated chemically, *i. e.* by chemical decomposition, to consist of different bodies. When the elements of which bodies consist are united mechanically, or by the force of cohesion, the substances are said to be mixed, as the quartz, felspar, and mica in granite, or the saltpetre, sulphur, and charcoal in gunpowder. When a homogeneous mass is formed, differing from the substances of which it is composed, such union is termed *Chemical Combination.* The substances composing this body are called its *Elements,* and may be either simple substances, or such as are susceptible of still further decomposition; in the latter case they are called *proximate* elements.

Examples:

Compound.		*Elements.*
CINNABAR	=	MERCURY. SULPHUR.

Compound.		*Proximate Elements.*		*Ultimate Elements.*
SALTPETRE	=	POTASH	=	POTASSIUM. OXYGEN.
		NITRIC ACID	=	NITROGEN. OXYGEN.

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§ 58. Numerous experiments illustrating the changes produced in the temperature, form, colour, &c. of bodies when chemically combined, may be found in *Parkes's Chemical Catechism*, 13th ed. pp. 567—593.

§ 60.

The number of substances at present reckoned by chemists as elementary is 59. Their names, the numbers which express the proportions in which they combine by weight, and the symbols used to represent them, will be found in Table II. in the Appendix; what is further requisite to be known will be explained as we proceed.

> The student is by no means to infer that these 59 substances are absolutely elementary, and that they will at no future time be decomposed, or that the number of simple substances is unalterably determined.

The general characteristics of these bodies are for the most part the following:

With regard to their aggregate form, most of the simple substances are solid bodies at ordinary temperatures: two are liquids — Bromine and Mercury; and four gases — Oxygen, Nitrogen, Hydrogen, and Chlorine.

Taken collectively, they are usually divided into two principal classes, viz. *metals* and *non-metallic substances*; 46 of these elementary bodies belong to the former class, and 13 to the latter.

The characteristic external tests of nearly all the metals are their peculiar *metallic lustre*, and, generally speaking, their great specific gravities. The non-metallic substances, on the contrary, are for the most part *destitute* of any metallic lustre, and are usually of *low specific gravities.*

Again, a remarkable difference obtains between metallic and non-metallic bodies as regards their power of conveying heat and electricity, the former being good conductors, whilst the latter are either bad conductors or altogether non-conductors.

> The reader must not infer from the above remarks that a strongly defined line can be drawn between the metallic and non-metallic bodies, as several of these properties are found in both classes of substances, though existing in various degrees; many substances undergo an alteration in their properties with a change in their form of aggregation. Thus it appears that here, as in other departments, the most opposite classes and orders are united by intermediate members of every conceivable gradation; consequently, it is not always easy to decide whether a body belongs to the metals or to the non-metallic substances.

These elementary bodies are seldom obtained naturally in a *pure* or *isolated* state. Some of them, in combination with other substances, are widely diffused; as Oxygen, Nitrogen, Hydrogen, Carbon, Silicon, Aluminium, Potassium, and Sodium, which are the chief constituents of the crust of our earth, of its atmosphere, and of the organised forms upon it; some are found more sparingly distributed, and others in but very small quantities. Some, on account of the abundance in which they are met with, their applications in the useful arts, or from their having been discovered and used from the most remote antiquity, as, for instance, Sulphur, Iron, Copper, Tin, Lead, &c., are universally known; others are less generally so, because of their hitherto limited utility, or because they have been discovered within so recent a date.

§ 61.

The quantity of these elements and their relative proportions are extremely unequal in the three kingdoms of nature.

The unorganised kingdom is the storehouse of all the elementary bodies. As has been already remarked, we rarely find them in a state of purity, but generally combined by the force of affinity, either two and two in *pairs*, or the pairs themselves forming *double pairs*, and in some few cases even the double pairs united.

Four elements, Oxygen, Nitrogen, Hydrogen, and Carbon, constitute the chief part of the organic kingdom. The last two exist, with few exceptions, in all combinations of this class; Oxygen in most, and Nitrogen in but a few. Nearly all vegetable substances are destitute of nitrogen, whilst it is found in all animals; however, the presence or absence of nitrogen is not a decisive evidence as to the vegetable or animal origin of any particular body. The organic combinations of the above elements are distinguished from their unorganised compounds chiefly in this, that they are not *binary*, but are generally *ternary*, or *quaternary*, *i. e.* the four elements combine immediately with three or four, without first forming pairs, as in organised combinations. They follow the immediate operations of that vital principle which is beyond our comprehension; a principle to which chemical affinity plays but a second part.

§ 62.

It is an incontrovertible fact, established by modern researches in chemistry, that *simple substances, and also their compounds, invariably combine only in certain fixed proportions by weight.*

All water, whether obtained naturally or by artificial means, is found to consist of 11·09 parts of Hydrogen and

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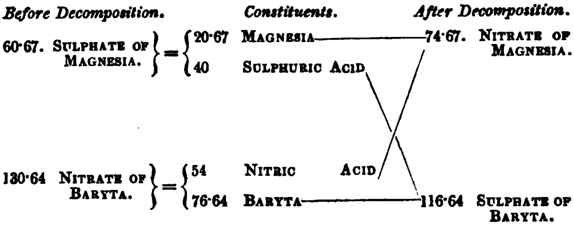
§ 62. The doctrine of Definite Proportionals was first propounded by Mr. Dalton, who built on it his atomic theory of the chemical constitution of bodies. Dr. Thomson was the first who publicly explained these views, which are greatly indebted for their promulgation in this country to Dr. Wollaston, the inventor of the scale known by his name. The scale much resembles a sliding rule, and greatly economises the time of the chemist, by presenting him mechanically with results that would otherwise be obtained only by tedious calculation.

88·91 Oxygen. A larger proportion of one or both of these elements will not enter into chemical combination.

A second law is, that *when two compound bodies mutually decompose one another, their constituents are in the exact proportions requisite for the resulting compounds.*

Thus if Sulphate of Magnesia be mixed with Nitrate of Baryta the two bases exchange acids, and two new salts result, viz. Sulphate of Baryta and Nitrate of Magnesia, and there is no excess of either acid or base.



On the above facts are founded these three universal laws:—

1) *That the parts by weight in which bodies mutually combine stand in a fixed numerical relation to each other.*

2) *That in every chemical combination the parts by weight of one element may be replaced by certain weights of other substances.*

Suppose that *a* combines with *b* in the proportion of 10 : 7, *b* with *c* in that of 7 : 6, it follows that *c* will unite chemically with *a* in that of 6 : 10. For instance, if we know that 8 parts of Oxygen combine with 39 of Potassium, and these again with 16 of Sulphur, we are sure at once that the proportion between Oxygen and Sulphur will be 8 : 16, and in a combination of 39 parts of Potassium with 8 of Oxygen, the latter may be replaced by 16 of Sulphur.

3) *That the weights which mutually represent each other, at once furnish the proportion in which these equivalent substances combine.*

If the bodies *a*, *b*, *c*, *d* combine chemically as 10 : 7 : 6 : 3, then *b*, *c*, *d* will combine as 7 : 6 : 3.

If the proportion has been obtained in which any one constituent combines with all the rest, we are furnished in the numbers thus known with the ratio in which these elements are all combined together.

As oxygen enters into combination with all the simple substances (Fluorine?), and its compounds are therefore most numerous, it has been adopted as the unit in comparing these proportionate weights: again, as hydrogen has the smallest combining proportion, and combines with many other substances, it has been employed by some chemists as the unit, and is generally used in England. The particular numbers that thus represent the proportions in which bodies combine are called *Atomic weights*, or *equivalents*, and that part of chemistry which treats of the mode in which these numbers are determined and applied, is called the *doctrine of chemical Proportions.*

In Table II., Appendix, the equivalents are given both for Oxygen = 100, and for Hydrogen = 1, opposite the names of the elements.

§ 63.

Many elements combine not only in that particular proportion expressed by the numbers of the equivalents contained in the table, but even unite with other elements in 2, 3, 4, or 5 different proportions.

By comparing these various proportions, it has been discovered that the proportion of that element which admits of increase is always an exact multiple of the simple equivalent, and that the compounds obey the two following laws, since either

A single proportion of one element combines with 1, 2, 3, 4, 5 *proportions of the other*, or

Two proportions of the one combine with 3, 5, 7, 9 *proportions of the other.*

Not only simple substances, but also their compounds, combine only in definite proportions. This may be proved either by direct experiment, or by adding the atomic weights of their constituent parts. If the equivalents which represent the elements constituting any body be added together, we shall have the proportion in which this body will combine with others.

Ex. gr. Potash consists of 1 equivalent of *Potassium* and 1 of *Oxygen*,
whence the equivalent for *Potash* is $39 + 8 = 47$.
Carbonic Acid, consists of 1 equivalent of *Carbon* and 2 of *Oxygen*,
whence the equivalent for *Carbonic Acid* is $6 + 16 = 22$.
Water consists of 1 equivalent of *Hydrogen* and 1 of *Oxygen*,
whence the equivalent for *Water* is $1 + 8 = 9$.

§ 64.

In exhibiting the composition of chemical combinations in a simple and intelligible manner, as *Chemical Formulæ*, the simple substances are represented by the initial letters of their Latin names, and if two or more elements begin with the same letter, a second is added.

One equivalent of a simple substance is expressed by writing merely the chemical symbol thus, 1 equivalent of *Potassium* = K. — Several equivalents are represented by the addition of a small figure above or below the symbol, thus 5 equivalents of *Oxygen* $= O_5$ or O^5.

As oxygen is so frequently combined with the other simple substances, an abridged mode of representing it is frequently adopted, and for every equivalent of oxygen a *dot* is placed over the symbol of that body with which it is united; for 2, 3, 4, *etc.* equivalents, a corresponding number of *dots* is employed.

Ex. gr., Potash $= K\,O$ or $\dot{K}$
Carbonic Acid $= C\,O_2$ or $\ddot{C}$
Water $= H\,O$ or $\dot{H}$
Sulphuric Acid $= S\,O_3$ or $\dddot{S}$

Several equivalents of similar combinations of the first order are represented by a number prefixed to the formula, thus

3 equivalents of *Carbonic Acid* $= 3\,C\,O_2$ or $3\,\ddot{C}$

The chemical formulæ for the combinations of the second order, the double pairs, are constructed by writing together the formulæ of the pairs composing them; thus

Nitrate of Potassa = 1 *Potash* + 1 *Nitric Acid*
$= K\,O,\ N\,O_5$ or $\dot{K},\ \overset{\cdot\cdot\cdot\cdot\cdot}{N}$

Combinations of the third order, those of double pairs with each other, as also mixtures, are represented by writing the formulæ joining them together by the sign + thus

Sulphate of Alumina and Potash (*Alum*) = 1 *Sulph. Pot.* +
1 *Sulph. Alum.* +
24 *Water*
$= K\,O,\ S\,O_3 + Al_2O_3,\ 3\,S\,O_3 + 24\,H\,O$

Gunpowder, a mixture of
1 Nitrate of Potash
1 Sulphur
3 Carbon
$= K\,O,\ N\,O_5 + S + 3\,C.$

§ 65.

The binary compounds, or chemical combinations of the first order, are commonly named and classified according to the non-metallic substance they contain. Hence, they are distinguished as compounds of oxygen, or *Oxides*; of chlorine, or *Chlorides*; of sulphur, or *Sulphurets*; of phosphorus, or *Phosphurets*; of carbon, or *Carburets*, &c.

The nomenclature of compounds of this class follows this rule, that the name of the class precedes that of the simple substance.

Ex. gr., Oxide of Iron, Oxide of Copper, Oxide of Lead, &c.;
Chloride of Iron, Sulphuret of Iron, Carburet of Iron, &c.

Combinations of the metals with each other are generally called *Alloys*, those of quicksilver with any other metal are named *Amalgams*, as amalgam of gold, amalgam of silver, amalgam of tin, &c.

Bronze is an alloy of copper and tin;
Brass of copper and zinc.

Binary combinations, considered generally, differ widely as regards their chemical affinity; they are either *Acid*, *Basic*, or *Neutral*.

§ 66.

The *Acids* are found in every form of aggregation, and most of those which are solid admit of being dissolved in water. They may be generally known by their peculiar sour taste; they have also the property of reddening vegetable blues, and exhibit a chemical affinity of the bases.

The last-named property is their most characteristic mark, as several acids are found wanting the first two peculiarities, especially such as are insoluble in water, *ex. gr.*, *Silicic acid.*

Most acids consist of combinations of non-metallic elements; some, however, consist of one such body and a metal.

Oxygen acids, derived from the inorganic kingdom, have usually *a simple radical;* they are chiefly binary compounds of oxygen with a non-metallic substance; a few only have a metallic base, and are hence called *metallic-acids.*

The following are given as instances of oxygen acids, viz.

Sulphuric Acid, compounded of 1 equivalent of Sulphur and 3 of Oxygen = SO_3 or $\dddot{S}$.

Carbonic Acid, comp. of 1 Carbon and 2 Oxygen = CO_2 or $\ddot{C}$.

The following are *Metallic Acids*, viz.

Arsenious Acid, comp. of 1 Arsenic and 3 Oxygen = $As.\ O_3$ or $\overset{\cdots}{As}$.

Antimonic Acid, comp. of 1 Antimony and 5 Oxygen = $Sb.\ O_5$ or $\overset{\cdot\,\cdot\,\cdot\,\cdot\,\cdot}{Sb}$.

Acids having a compound radical are nearly all of organic origin. They are either *Vegetable* acids, which are free from Nitrogen, or *Animal* acids, into whose composition this element enters. The base or radical of the former is composed of Carbon and Hydrogen, that of the latter invariably contains Nitrogen.

Two instances are given below of vegetable and one of animal acids, viz.

Acetic Acid, comp. of 4 Carbon, 3 Hydrogen, and 3 Oxygen $= C_4 H_3 O_3$.

Gallic Acid, comp. of 7 Carbon, 1 Hydrogen, and 3 Oxygen $= C_7 HO_3$.

Cyanic Acid, comp. of 2 Carbon, 1 Nitrogen, and 1 Oxygen $= C_2 NO$.

In the class of acids which has just been noticed the characteristic element was Oxygen; there is another class which, as having Hydrogen for their base, are called *Hydrogen Acids*. In the inorganic kingdom binary compounds of this element are to be found with certain non-metallic substances (Sulphur, Chlorine, Bromine, Iodine, Selenium, and Fluorine), and with the metals arsenic, antimony, and tellurium; in the animal and vegetable kingdoms it forms ternary combinations with Nitrogen and Carbon. In chemical nomenclature, Hydrogen acids are distinguished by the prefix *hydro*.

From among many the following instances may suffice, viz.

Hydrochloric Acid, comp. of 1 Hydrogen and 1 Chlorine $= H\ Cl$.

Hydrosulphuric Acid, comp. of 1 Hydrogen and 1 Sulphur $= H\ S$.

Hydrocyanic Acid, comp. of 2 Carbon, 1 Nitrogen, and 1 Hydrogen $= C_2NH$.

§ 67.

The characteristic distinction of all *Bases* is their strong chemical affinity for the acids. Those which can be dissolved in water have an acrid taste, and possess the property of changing vegetable blues green, and vegetable yellows brown.

The mineral alkalies consist chiefly of binary combinations of Oxygen with the metals; the animal and vegeto-alkalies, on the contrary, are for the most part composed of the four constituents of organic nature, namely, Oxygen, Hydrogen, Nitrogen, and Carbon.

A few of the principal Alkalies and Alkaline earths are,

Oxide of Potassium, or *Potash*, comp. of 1 Potassium and 1 Oxygen $= KO$ or $\dot{K}$.

Oxide of Sodium or *Soda*, comp. of 1 Sodium and 1 Oxygen $= Na.\ O$ or $\dot{Na}$.

Oxide of Calcium or *Lime*, comp. of 1 Calcium and 1 Oxygen $= Ca.\ O$ or $\dot{Ca}$.

Oxide of Magnesium or *Magnesia*, comp. of 1 Magnesium and 1 Oxygen $= Mg.\ O$ or $\dot{Mg}$.

§ 68.

The most important and most generally diffused acids and bases are the *oxygen acids* and the *oxides*. Both, however, may contain different quantities of oxygen united with the same radical, *i. e.*, the oxygen may—agreeably to the law laid down in § 63—unite itself in various proportions with the other element of the compound; the degree of the quantity of oxygen present is shown by a slight change in the appellation.

If two acids are thus formed, that which contains the larger portion of oxygen is distinguished by the termination *ic*, whilst the name of the other ends in *ous*, as,

Sulphuric Acid $= SO_3$ or $\dddot{S}$, and

Sulphurous Acid $= SO_2$ or $\ddot{S}$.

In a similar manner the different quantities of Oxygen contained in a base are marked by the terms *Oxide*, *Binoxide*, *Teroxide*, &c., that containing the largest proportion of Oxygen is called the *Peroxide*.

Oxide of Lead $= Pb.O$ or $\dot{Pb}$.

Binoxide (Peroxide) of Lead $= Pb.O_2$ or $\ddot{Pb}$.

§ 69.

The *indifferent* or *neutral combinations* of this class may be known by their possessing the properties neither of acids nor of alkalies, and especially by their exhibiting no marked chemical affinity.

The term *Suboxide* is used to denote certain inferior oxides which do not unite with acids, and which contain less oxygen than the basic protoxide of the same element.

When 2 equivalents of a metal combine with 3 of oxygen the resulting compound is termed a *Sesquioxide*.

Sesquioxide of Aluminium (Alumina) $= Al_2.\ O_3$.

Sesquioxide of Iron $= Fe._2\ O_3$.

§ 70.

Many compounds of the first order, in addition to the elements of which they are chemically composed, contain also a portion of water, without which indeed some of them cannot exist. In this combination they are known as *Hydrated Oxides*, or *Acids*,

Hydrated Sulphuric Acid (Oil of Vitriol) $= SO_3 + HO$

Hydrate of Lime $= Ca.O + HO$.

§ 71.

The Metals are usually arranged in classes according to their affinity for Oxygen and the nature of their oxides.

1°. *Metals whose oxides are Alkalies :* Potassium, Sodium, and Lithium. These are remarkable for their affinity for oxygen, in consequence of which they oxidize rapidly if exposed to air or water; they are of low specific gravity and fusible below the heat of boiling water.

2°. *Metals whose oxides are alkaline Earths :* Barium, Strontium, Calcium, and Magnesium; their oxides being Baryta, Strontia, Lime, and Magnesia.

3°. *Metals whose oxides are Earths :* the most important being Aluminium, whose Sesquioxide, Alumina, is the basis of all clays.

4°. *Metals whose oxides are powerful Bases :* Manganese, Iron, Chromium, Nickel, Cobalt, Copper, Zinc, Bismuth, Lead, Cadmium, and Uranium.

5°. *Metals whose oxides form weak Bases, or Acids :* Arsenic, Antimony, Tin, &c.

6°. *Metals whose oxides are reduced to the metallic state by heat alone :* Gold, Silver, Platinum, Mercury, Rhodium, &c. These metals are not easily oxidized, and consequently retain their metallic lustre unimpaired; this circumstance, and the fact that they are not oxidized by exposure to a red heat, led the ancients to regard them as the perfect or noble metals.

§ 72

The combinations of the second order are produced by the union of acids with their bases, and are called *Salts*. Thus we obtain double pairs, and in every pair there is commonly found a similar element, which in most cases is oxygen.

The names of the salts are composed of the acid and base, as *Nitrate of Potash*, $KO + NO_5$.

The same acid may combine in several proportions with a base. That particular combination in which the acid and the base are present in the single equivalent is called a *Neutral Salt ;* when two equivalents of the acid are united to one of the base a *Bisalt* results, as Bicarbonate of Soda, $Na.O + 2CO_2$; when one equivalent of acid is united to two of base a *Disalt* is found, so Dichromate of Lead, $2Pb.O + Ch.O_3$. The terms *sub* and *super*, which were formerly employed to denote the alkaline or acid reaction of salts, without reference to their composition, are not now used in scientific works.

A remarkable relation always exists between the equivalent of oxygen in the base, and the equivalents of acid in the salts of that base, they being always equal in number; thus, Neutral Salts of a protoxide contain one equivalent of Acid, as Sulphate of Potash $= K.O + SO_3$; those of a sesquioxide, three equivalents of Acid, as sulphate of the Sesquioxide of Iron $= Fe_2O_3 + 3SO_3$.

§ 73.

The *Haloid Salts* form a distinct class of salts. They are especially compounds of the first order, inasmuch as they are binary combinations of a small group of non-metallic substances, both elementary and compound, with metals. To this class belong the following substances, viz. *Chlorine*, *Bromine*, *Iodine*, *Fluorine* and *Cyanogen*, and in consequence of this property which they possess of forming saline compounds with the metals they are called *Haloids*.

The haloid salts are variously named. Their appellation is formed from their binary composition, as, *Chloride of Potassium*, *Chloride of Sodium* (culinary salt). When dissolved they may be regarded as double pairs, the Hydrogen of the water with the haloid furnishing a Hydrogen-acid, and the Oxygen of the water with the radical giving an Oxide: thus, $Na.Cl. + HO = HCl. + Na.O$, hence, when dissolved, Chloride of Sodium may be regarded as Hydrochlorate of Soda.

§ 74.

As regards their aggregate form, most of the salts are solids at ordinary temperatures; many of them are soluble in water, and then are susceptible of *crystallisation*, *i. e.* when separated from the water in which they are dissolved, they return to the solid state and assume regular forms. In their crystallised state salts usually contain a portion of water, called their *Water of Crystallisation*.

In addition to these properties common to almost all bodies of this class, some possess peculiarities from which they derive distinctive appellations.

Deliquescent Salts are such as, when crystallised, attract so much moisture from the atmosphere, that they either return again to the liquid form, or become sensibly moist.

> Many of the haloid salts, and several of the saline compounds of carbonic and nitric acids, possess this property; *Nitrate of Soda* may be cited as an instance.

Efflorescent Salts are those which, on being exposed to the air, part with their water of crystallisation, and fall into a powder.

> *Carbonate of Soda* and *Sulphate of Soda* (Glauber's Salts), belong to this class.

§ 75.

Many salts display an affinity for each other; the combinations thus obtained belonging to the third order are called *Double Salts*. They consist of two different bases with the same acid; thus

Alum is composed of Sulphate of Alumina and Sulphate of Potash $= K\,O,\ S\,O_3 + Al._2\,O_3,\ 3\,S\,O_3 + 24\ H\,O.$

§ 76.

Connected with chemical combinations and decompositions various phenomena occur, that are partly the result of the previous union of their constituents, and partly the product of their existing combination or decomposition. The most important and most usual of these phenomena are set forth in the following sections.

The most remarkable, and on many accounts the most important, of these phenomena is *Combustion.* It is produced by the chemical combination of the two substances with the evolution of heat and light: it usually takes place in consequence of the body's combining with Oxygen, but in the case of some few substances on their union with Chlorine and Sulphur. Those bodies which may be regarded as the causes of combustion, in consequence of this very property, are called *Supporters of Combustion,* whilst the radicals combined with them are called *Combustibles.*

If the bodies are gases, or if during combustion they become wholly or partially converted into gases, their union with the supporter of combustion generates *Flame;* this is the case, for instance, with Hydrogen, Sulphur, Phosphorus, Coal; if, on the contrary, the combustible does not assume the gaseous form, a *glowing* appearance, as of red or white heat only, is produced; and is confined to the form of the combustible: this occurs with most of the Metals and Carbon.

The application of heat is usually required to commence combustion, but when the process has been begun the heat given out is usually sufficient to maintain the action.

Smoke arises from the imperfect combustion of any body: it consists chiefly of minute particles of Carbon which have escaped oxidation.

Combustion becomes *extinguished* when the substance by growing cool ceases to give out as much caloric as is needed to maintain the process of combustion. — This may be produced by a draft of air, by applying water or other bodies which lower the temperature; or the same end may be attained by excluding the atmospheric air, whose presence is usually required to furnish a supply of oxygen.

Hence the extinguishing of burning missiles, by covering them with damp earth, or hides.

§ 77.

Many solids possess the property, upon coming in contact with a fluid body, of themselves becoming fluids also, and forming with it one homogeneous substance. This condition of the solid is called its *Solution*, and the fluid which brings about the change is called a *Solvent.*

§ 78.

A distinction must here be remarked between *chemical* and *mechanical* solutions.

In *Chemical Solutions* a perfect chemical union of the solid with the liquid is produced, in accordance with the laws of defined proportions established in §§ 62 and 63. Both the constituents of the compound exhibit a change in their properties, and are combined in an entirely new substance formed by their union, which substance, on the completion of the process, generally assumes a solid form, that is, it becomes crystallised. Among the principal and most powerful solvents of this class may be enumerated *Sulphuric, Nitric,* and *Chloric acids.*

§ 79.

Mechanical Solution is the mere union of a solid with a liquid, in such a manner that its aggregate form is changed without any alteration being effected in the chemical properties of either the solid or its solvent; if the latter be separated from the solution, as it generally can be by means of heat, the substance that has been dissolved is obtained again with its chemical properties unaltered. Water acts as a solvent for many bodies, especially for most of the salts; alcohol is a solvent for resins, &c.

§ 80.

The strength of any particular solvent, when employed, varies much according to the properties of the body to be dissolved. A solution is said to be *saturated* when no more of the solid can be dissolved. With respect to the chemical solutions, saturation depends, as has been observed above, on the established laws of chemical combinations; in aqueous solutions, on the contrary, saturation depends greatly on the temperature of the water. In general, the capacity for dissolving increases as the temperature rises: thus, nitrate of potash is soluble in seven times its weight of water at 60°, and in its own weight of boiling water; and in chemical solutions, too, the elevation of the temperature materially affects the result; *chloride of sodium* (culinary salt) is equally soluble in water at all temperatures; whilst *oxide of calcium* (lime) is more freely dissolved by cold water than by hot.

§ 81.

The *crystallisation* of any body that has been dissolved consists in its solidifying, and in the separation from it of its solvent. It is effected either by the gradual cooling of a hot solution, in which case with the fall of the temperature there is a corresponding diminution of the dissolving power, and thus the solid crystallises; or else, by the application of heat, the liquid is driven off up to *the point of crystallisation*, when the surface of the fluid will be seen to be overspread with a thin film.

Water is capable of holding several salts in solution at the same time. When the temperature of the water has been so lowered by cooling, or such a quantity has been driven off by evaporation that it can no longer hold in solution the given quantity of any one of the salts, then that particular salt disengages itself, and crystallisation begins, the other salts continuing in solution till, from a like cause, they successively crystallise. Hence we are able to separate any number of salts from each other.

> Application of this in the purification of saltpetre from the common salt mixed with it.

§ 82.

In the chemical decomposition of solutions, the disengaged body may be either solid, liquid, or aëriform.

In the first case it renders turbid, and often colours the liquid, sinking at length to the bottom, whence it is called a *Precipitate.* Those substances are called *Reagents* which indicate the presence of any body that exists in but a minute proportion in a solution, either by rendering it turbid, by staining it some particular colour, or by depositing a precipitate. Thus, *ex. gr.*, a *solution of nitrate of silver* is a most powerful reagent in detecting *chloride of sodium.*

In the second case the disengaged body generally remains united with the solvent, from which it often happens that it can be separated only by *Distillation.* To effect this, the compound liquid is put into a suitable vessel, and heated, when that element which boils at the lower temperature passes off in the form of vapour, which on being cooled condenses into a liquid and is readily collected.

Lastly, in the third case, the constituent which is disengaged passes off in the form of gas. If it is desirable to secure this, it is allowed to ascend into a convenient receiver, previously filled with water, or occasionally with some other fluid; the apparatus commonly employed for this purpose is called a *Pneumatic Trough.*

SECTION III.

OF SOLID BODIES.

I. PROPERTIES OF SOLID BODIES.

§ 83.

Every *solid body* possesses an independent figure, *i. e.*, by virtue of the agency of its molecular attraction (§ 13.) its form is perfectly defined on every side; this is a consequence of the intensity of its force of cohesion, a force which exists in its highest degree in this class of bodies. The particles constituting the mass of these bodies are, for the most part, not susceptible of any displacement or separation from the mass; to effect this, such an external force must be applied as shall suffice to overcome their cohesion. The portions broken off, on being restored to their original position, no longer show their former cohesion, but merely the adhesion of their surfaces.

§ 84.

The existence of solid bodies may be explained, on the hypothesis that between their atoms or molecules there is an attractive and repulsive force in constant operation, and that the former force is predominant (§ 14.); consequently, the atoms approach so nearly in their aggregate form, that the resultant of these molecular forces represents the condition of the stability of the body's atoms, which have thus lost all power of free motion.

§ 85.

Many solid bodies have a symmetrical form, thus, in the animal, vegetable, and mineral kingdoms, is evinced a tendency in matter, which cannot be mistaken, to combine itself into complete and symmetrical wholes; the principal distinction is, that in organised bodies *spherical* forms predominate, whilst unorganised bodies are mostly *angular.*

Such symmetrical angular bodies as the mineral kingdom presents to us in countless numbers are called *crystals.* Each perfectly formed crystal in inorganic nature is to be regarded as a complete whole.

The formation of crystals appears to be owing to this, that the attractive force of the molecules is exerted only in certain directions. In bodies of a spherical form this force must operate equally in all directions; in those of an irregular figure it acts unequally in many or in all directions.

§ 86.

It is necessary in order to a body's crystallising that its smallest particles should be at liberty to obey, uncontrolled, this internal attraction. This can only happen when it is in a liquid state, whence it may be inferred that all crystallised bodies have at some time been fluids.

Substances capable of crystallisation assume a perfectly crystalline form only when their particles can constantly and undisturbed obey the attractive force, and thus unite together. If the transition from the liquid to the solid state be sudden, the tendency to crystallise is developed only in the interior of the body, and the minute portions which have united together, present the appearance of an irregular mass of imperfectly formed, or very minute crystals. Fusible substances, as the metals, sulphur, and the like, if cooled rapidly, not unfrequently are found in this state.— As many mountains (particularly the Uralian mountains) have this character, if taken as a mass, it may reasonably be conjectured that they formerly existed in a fluid state, and probably in that of fusion.

With the externally symmetrical shape of a crystal there subsists also a great regularity in the arrangement of the particles and molecules composing the interior of the body; this is called its *structure*. The regular formation of the interior of a crystal may be seen by splitting or breaking it, which may be done in certain directions with ease, so as to obtain perfectly smooth surfaces. If the subdivision of the body be continued in these directions, called its *planes of cleavage*, a regular, or at all events a symmetrical, figure is obtained, which is the *primitive form* of the crystal, and which may differ widely from the figure at first presented by the body; should this prove to be the case, the latter is known as the *secondary form* (1). By continuing to subdivide the primitive form we shall procure a number of minute fractions resembling it in figure (2) : we may regard these small and perfectly similar portions as a type of those ultimate atoms of which the body so divided is composed (3).

1) As an illustration of the subdivision of a crystal we may adduce a cube of fluor-spar, which gives an octoëdron for its primitive form; or a hexagonal crystal of calcareous spar, which gives a rhomboid.

Haüy, who first discovered the importance and peculiarity of the subdivision of crystals, obtained the following primitive forms, the *tetraëdron*, *fig.* 22.; the *parallelopipedon*, *fig.* 23.; the *octoëdron*, *fig.* 24.; the *hexagonal prism*, *fig.* 25.; the rhombic *dodecaëdron*, *fig.* 26.; the *dodecaëdron* with *triangular faces*, *fig.* 27. To the parallelopipedon belong also the *cube*, *fig.* 28.; and the *rhomboëdron*, *fig.* 29.

Fig. 22.

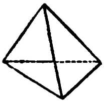


Fig. 23.

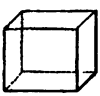


Fig. 24.

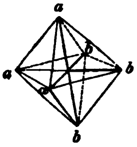
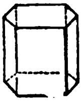


Fig. 25.



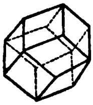


Fig. 26.

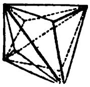


Fig. 27.

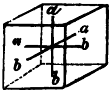


Fig. 28.

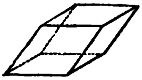


Fig. 29.

Among many instances of natural crystallisation we may mention rock-crystal, quartz, granite, calcareous spar, fluor-spar, copper ore, sulphurous pyrites, the loadstone, &c. Flakes of snow and the glazed frost also are natural crystals.

Of artificial crystallisation, it will suffice to mention that of alum, common salt, saltpetre, sulphate of copper, sugar, sulphur, and bismuth. — Microscopic crystallisation of the above substances.

The crystallisation of tin in the tinning of iron plates becomes visible when acted upon by nitric acid, and produces the well-known Moire antique.

The planes of cleavage are readily distinguished in lead ore, Iceland spar, &c.; also in zinc, antimony, bismuth, &c.

2) A piece of Iceland spar, if broken into its smallest particles, gives perfectly formed rhomboidal crystals.

Haüy regarded the parallelopipedon, the tetraëdron, and the triangular prism, as the forms of the ultimate molecules, which suffice for the production of every other form. He supposed crystals to be composed of these molecules arranged in courses one upon the other. The great variety of crystalline forms derived from so small a number of primitive ones he explained, on the supposition that the strata diminish gradually as their distance increases from the primitive nucleus, and that this takes place either at the angles or at the sides only, or in a position intermediate between them. Thus, *ex. gr.*, from the cube, as the primitive form, may be derived, by means of combination, the *octoëdron*, *fig.* 30.; the *rhombendodecaëdron*, *fig.* 31.; and the pentagonal-dodecaëdron, *fig.* 32.

Fig. 30.

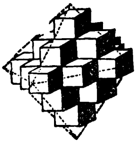


Fig. 31.

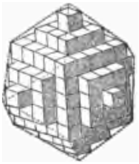
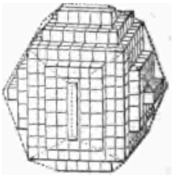


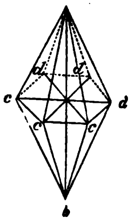
Fig. 32.



It might, perhaps, seem more natural to consider the primitive form in all cases as the basis of the different crystalline bodies; in order, however, to facilitate the description of the various specimens of crystals — particularly in classifying minerals — certain forms are assumed to be *fundamental.* The number and kind of these fundamental forms are variously stated by different crystallographers. Such crystalline forms as may be deduced from one of these fundamental forms are said to constitute a *System of Crystallisation.*

The chief points to be regarded in crystals are the angles and the centres of their edges and sides. The lines joining these points, and passing through the middle of the crystal, are called *Axes;* most bodies of this class have three such axes at right angles to each other, as may be readily seen from their figure.

Fig. 33.



These three axes are either all equal, or two of them are equal, or all three unequal. One, and usually that which is most dissimilar from the other two, is named its *principal* axis, and the others *secondary* or *subordinate* axes.

Thus the cube, *fig.* 28., and the octoëdron, *fig.* 24., have each three axes, either of which may be regarded as the principal axis. The six-sided double pyramid, *fig.* 33., has four axes, of which *a b* is the principal, and the others the subordinate axes.

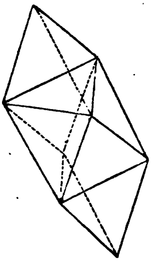
The term *amorphous* is applied to such minerals as are met with in shapeless forms, which forms have perhaps been produced by some external causes that have interfered with the body's assuming its own appropriate form; if their internal structure be also destitute of anything of a crystalline nature, the substance is said to be dense.

Crystalline bodies frequently differ widely in their physical properties from non-crystallised, especially in the greater hardness of the former. The most remarkable instances are afforded in carbon and aluminium, which, when crystallised, become the hardest of known substances, the diamond and the sapphire.

§ 87.

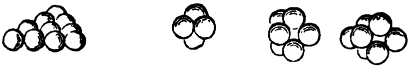
The various modifications of cohesion, expressed by the terms *hardness*, *softness*, *brittleness*, and *ductility*, applied to solid bodies, may be accounted for as arising from the variations in their internal structure.

Those bodies are said to be *hard* which resist a considerable external force, either by pressure or impact.



§ 86. "But," says Mr. Brande in his 'Chemistry,' vol. i. p. 16., "in crystallography we meet with appearances which Haüy's theory but imperfectly explains. A slice of fluor spar, for instance, obtained by making two successive and parallel sections, may be divided into acute rhomboids; but these are not the primitive form of the spar, because, by the removal of a tetraëdron from each extremity of the rhomboid, an octoëdron is obtained. Thus, as the whole mass of fluor may be divided into tetraëdra and octoëdra, it becomes a question which of these forms is to be called primitive, especially as neither of them can fill space without leaving vacuities; nor can they produce any arrangement sufficiently stable to form the basis of a permanent crystal.

To obviate this incongruity, Dr. Wollaston (*Phil. Trans.* 1813) ingeniously proposed to consider the primitive particles as spheres which, by mutual attraction, have assumed that arrangement which brings them as near as possible to each other. When a number of similar balls are pressed together in the same plane, they form equilateral triangles with each other; and if balls so placed were cemented together, and afterwards broken asunder, the straight lines in which they would be disposed to separate would form angles of 60° with each other. A single ball, placed anywhere on this stratum, would touch three of the lower balls; and the planes touching their surfaces would then include a regular tetraëdron. A square of four balls, with a single ball resting upon the centre of each surface, would form an octoëdron; and upon applying two other balls at opposite sides of this octoëdron, the group will represent the acute rhomboid. Thus the difficulty of the primitive form of fluor above alluded to, is done away by assuming a sphere as the ultimate molecule. By oblate and oblong spheroids other forms may be obtained."



Most stones, several metals, and various kinds of wood, possess this quality. Of two bodies that is said to be the harder which causes a disruption of the other; the diamond is esteemed the hardest of all bodies, because all others may be cut by it, but it cannot be cut by any one of them.

Soft is a term applied to all such bodies as may be made to change their form on the application of a trifling force, no disruption of their parts taking place, and the form thus impressed upon them being retained after the force has ceased to be exerted upon them.

Moist clay, tallow, butter, wax, &c. may serve as familiar examples of soft substances.

Those hard bodies whose particles are held in such a state of mutual repulsion, that the separation of one or more of their particles causes the disruption of a large portion of the body, or even its total dissolution, are said to be *brittle*. This property may be induced in many bodies, by raising them to a red heat, and then cooling them suddenly.

Brittle bodies may be pulverised with ease by applying some external mechanical force. Glass that has been cooled quickly, and highly tempered steel, possess this quality in a remarkable degree.

First Exp. If the tail or glass thread be broken off a Dutch-tear (*i. e.* a liquid glass-drop cooled by immersion in cold water), the whole body falls into minute fragments. Let a small sharp piece of gun-flint fall into a Bologna flask, and it will break it into a number of pieces.

Glass cooled slowly does not display this brittle nature; but by the more gradual loss of its heat it acquires a greater or less degree of elasticity, as may be seen in fine threads of glass.

Second Exp. If a Dutch-tear or a Bologna flask be cast in clay, heated with it, and then slowly cooled, it will be found that the brittleness spoken of in the first experiment will have entirely disappeared.

Those bodies are said to be *malleable* whose parts may be extended in all directions, their length and breadth being considerably increased without the continuity being broken.

Several of the metals, in consequence of their possessing this property, are of great value for various purposes, connected with the arts, as, *ex. gr.*, iron, copper, brass, lead, tin, zinc. Gold, silver, and platinum are malleable in a remarkably high degree — gold-leaf, silver-leaf, platinum foil, gold and silver wire; — Wollaston's finest platinum wire.

Those bodies which admit of being bent without breaking, and which retain the form they have thus been made to assume, are said to be *ductile* and *pliable*.

All the malleable metals have this property, copper and iron in a remarkable degree; many other solid bodies possess it too; as, for instance, the roots and twigs of trees, cords, catgut, &c.

These different modifications of solidity are by no means qualities that invariably admit of strongly defined lines of demarcation, nor are they unsusceptible of variation as to degree in the same body. Heat seems to exert a great influence on them, as the accumulation of this agent in any body always diminishes its cohesive attraction. Further details on this subject belong to the doctrine of caloric.

§ 88.

By *strength* is understood the resistance which a solid opposes to any force whatever, tending to produce a separation of its continuity. The following distinctions are made: —

1. The *absolute strength* of materials, or their resistance to tension, *i. e.* the resistance which the rod, wire, or cord, fastened at one end, offers to a force acting at the other end in the direction of the body's length, and *tending to tear it asunder.*

2. Their *lateral strength*, or the force with which a beam fastened or supported at one or both ends resists *rupture*, the stress being applied perpendicularly to the beam.

Their *resistance to compression*, or the force which a body opposes to a weight applied in the direction of its length, with a tendency either to *crush* the body, or by *deflecting* it to break it.

The weight required to effect the particular kind of rupture is, in every case, the measure of the body's strength.

As it is of considerable importance in many matters connected with art, to know the strength of different substances, experiments have been performed to test this point. The metals, woods, and cords have chiefly been tried, these materials being the most extensively used. The general result of these investigations is, that the strength of the metals exceeds that of the woods, and that among different bodies of both these classes a great inequality subsists.

The capacity of any substance for resisting tension is of importance in determining what force of traction it can withstand, applied in the direction of its length; as in suspension-bridges, anchors, cables and the like.

In order to estimate the strength of the material under investigation, whether it be wire, rod, or cord of several strands, let its upper extremity be securely fastened, and then attach weights to its lower end, till rupture takes place. That weight which snaps the body is regarded as the measure of its absolute strength.

The following is a list of the metals, according to their strength beginning with the strongest:—

Steel, wrought-iron, cast-iron, platinum (?), silver, copper, brass, gold, tin, bismuth, zinc, antimony, lead.

It has further been ascertained, that the tenacity of cast metals is less than that of the same metals when forged and drawn into wire; that alloys frequently affect their strength in no small degree; and that metals at a low temperature are much stronger than when heated. — Necessary application of these principles to the boilers of steam-engines; effect of constant use upon the barrels of fire-arms.

In consequence of its great tenacity, iron is the most important metal in a technical point of view. According to Tredgold's and Duleau's experiments, a piece of the best bar-iron 1 square inch across the end would bear a weight of about 77,373 lb., while a similar piece of cast-iron would be torn asunder by a weight of from 16,243, to 19,464 lbs.

It is worthy of remark, that thin iron wires, arranged parallel to each other, and presenting a surface at their extremity of 1 square inch, will carry a mean weight of 126,340 lb.

List of the woods, beginning with the strongest:—

Oak, alder, lime, box, pine (*Pinus sylv.*), ash, elm, yellow pine, fir (*Abies picea*).

All wood of the same name, however, is not of uniform strength; for such trees as grow in mountainous districts are, in this respect, much superior to those which grow in a champaign country; and in the same individual there exists a great difference in the trunk, branches, and roots. A piece of well-dried pine-wood (*Pinus sylvestris*), presenting a section of 1 square inch, is able, according to Eytelwein, to support a weight of from 15,646 lb. to 20,408 lb., whilst a similar piece of oak will carry as much as 25,850 lb.

In order to ascertain the strength of cordage, take specimens of different thicknesses and of various numbers of strands and threads, composed, too, of different materials, and more or less twisted. It has been proved experimentally that ropes, cords, and threads, of equal thickness, are stronger in proportion to the fineness of the strands they are made of, in proportion, further, to the fineness of the hemp or flax of which the strands themselves consist, and the smallness of the degree in which they are twisted. Again, damped hempen ropes were found to be stronger than such as were dry, tarred than untarred, twisted than spun, and unbleached than bleached. Silk ropes have three times the strength of those made of flax, the thickness being the same in each case. Table III. in the Appendix contains the estimated resistance to tension of some of the principal bodies.

In like manner the strength of leather, paper, and other substances is increased by pressure.

Several substances of both animal and vegetable origin which are converted from the liquid to the solid form, such as gums, varnish, glue, &c., are known to possess extraordinary tenacity. Thus Rumford found that a copper plate $\frac{1}{20}$th of an inch thick, rolled into the shape of a hollow cylinder, acquired twice its original strength on being coated over with well-sized paper of double its thickness, and that a cylinder of rolls of paper glued together, presenting at the end an area of 1 square inch, would support a weight of 30,000 lb.; whilst a similar cylinder of hempen strings glued together lengthways would carry 92,000 lb., the latter having a strength exceeding that of the best iron.

It has been further discovered, that the strength of two cords or rods is directly as the area of their ends, independently of the length of the cord or rod, and the shape of its end. Thus a cylindrical cord whose thickness is double that of another (*i. e.* which has a diameter twice the length of the other) will require four times the weight that the latter does to break it; and a rod of cylindrical form is equal in strength to one in the shape of a parallelopipedon, provided that the areas of their ends be equal. Again, a cord twice the length of another has the same strength with it, other things being equal.

The *lateral strength* of materials, or their *power of resisting a cross-strain*, becomes a matter of practical moment, especially in estimating the weight that can be safely borne by beams of wood and bars of metal.

To learn the degree of strength possessed by bodies in this case, they are laid in a horizontal position, supported at their extremities and loaded with weights until they break; or else, one end only is fixed, and a weight is moved from the point of support towards the free end till the body breaks. In both cases the weight causing the fracture serves as the measure of the lateral strength of the material under investigation.

In treating of the laws which regulate the lateral strength of bodies, the *length* of a beam is understood to mean, not its actual dimensions, but merely the distance between its two points of support; or, if one end only be fixed, the distance at which the weight is placed from this point. By a body's *lateral strength* or *power of suspension*, is meant the utmost weight it can be made to carry by one or other of the above-named modes of support, without breaking.

Theoretical investigations and actual experiments alike lead to this result, that in a parallelopipedon of uniform thickness, supported on two points and loaded in the middle, *the lateral strength is directly as the product of the breadth into the square of the depth, and inversely as the length.* Let W represent the lateral strength of any material, estimated by the weight, b the breadth, and d the depth of its end, and l the distance between the points of support; then

$$W=\frac{fd^2b}{l}$$

If the parallelopipedon be fastened only at one end in a horizontal position, and the load be applied at the opposite end, we have

$$W'=\frac{fd^2b}{4l}$$

It is to be observed that the three dimensions, d, b, and l, are to be taken in the same measure, and that b be so great that no lateral curvature arise from the weight; f in each formula represents the lateral strength, which varies in different materials, and which must be learnt experimentally.

Tredgold has further investigated what weights bodies are capable of supporting, without a permanent alteration in their form being induced. In Table IV. in the Appendix will be found the results of his experiments; f representing the weight which a cube, whose side contains 1 square inch, will carry without permanently affecting its form; M is the absolute weight of a cubic foot of the material.

In accordance with these experiments Tredgold says that the preceding formula, when reduced to practice, must be modified as follows:—

For a uniform parallelopipedon, supported at its two ends, the load being placed in the middle—

$$W=\frac{2}{3}\cdot\frac{fbd^2}{l}$$

When a beam is supported at both ends, and the load is placed at distances g and g' from the ends,

$$W=\frac{l}{6gg'}\times fbd^2$$

If the weight be evenly distributed over the whole beam, its power of suspension becomes twice as great as when the whole is placed in the middle; in this case,

$$W=\frac{4}{3}\frac{fbd^2}{l}$$

If the beam be supported at one end, and the weight be placed at the other, it will carry only one-fourth of the weight which it would bear if supported at both ends.

Then we have,

$$W=\frac{fbd^2}{6l}$$

The same laws hold true in regard to cylinders. If a cylinder rest on the two ends, and be loaded in the middle then,

$$W=\frac{\cdot 7854fd^3}{2l}$$

d in this last formula representing the diameter of the cylinder. The other formulæ for cylinders correspond with those already given for beams.

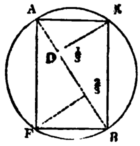
Besides the weight imposed on them, the materials, whether beams or cylinders, have also to support their own weight; an element which has not yet entered into our calculations. This weight is distributed over the entire length of the beam: it enters therefore into the formula in which this is assumed as the load to be sustained, $W=w+w'$, w being the weight to be supported, and w' that of the beam itself; in those cases, on the contrary, in which the load w is placed in the middle or at the extremity of the beam, $W=\frac{2w'+w}{2}$. The amount of w' in each case being found in column M of Table IV. in the Appendix.

From the preceding formulæ and theoretical propositions we obtain the following:—

A beam having a rectangular end, whose breadth is two or three times greater than the breadth of another beam, has a power of suspension respectively two or three times greater than it; if the end be two or three times deeper than the end of the other, the suspensive power of that which has the greater depth exceeds the suspensive power of the other four or nine times: if its length be two or three times greater than the length of another beam, its power of suspension will be ½ or ⅓ respectively that of the other; provided that in each case the mode of suspension, the position of the weight, and other circumstances, be similar. Hence it follows that a beam, one of whose sides tapers, has a greater power of suspension if placed on the slant than on the broad side, and that the powers of suspension in both cases are in the ratio of their sides; so, for instance, a beam, one of whose sides is double the width of the other, will carry twice as much if placed on the narrow side, as it would if laid on the wide one. — Application in the beams of ship's decks, shoring houses, &c.

If it be required to saw out of a tree a beam that shall have the greatest possible power of suspension, its two sides must be to the diameter of the tree as $\sqrt{\frac{1}{3}}$ and $\sqrt{\frac{2}{3}}$ to 1, *i. e.*, as $\cdot58 : \cdot82 : 1$. By a simple construction, the required proportions are thus obtained:—Divide the diameter AB of the trunk, *fig.* 34., into three equal parts, and let fall the perpendicular DE upon D, one of these three points; now draw AE, BE, and the sides AF, BF, parallel to them, AEBF is a section of the beam which will withstand the greatest force; BE being placed vertically. It may be mathematically demonstrated that—

Fig. 34.



$$\begin{aligned} AE &: EB : AB, \text{ as} \\ \sqrt{\tfrac{1}{3}} &: \sqrt{\tfrac{2}{3}} : 1, \\ \cdot58 &: \cdot82 : 1. \end{aligned}$$

In a piece of round timber (a cylinder) the power of suspension is in proportion to the diameters cubed, and inversely as the length; thus, a beam with a diameter two or three times longer than that of another, will carry a weight 8 or 27 times heavier respectively than that whose diameter is unity, the mode of fastening and loading it being similar in both cases.

The lateral strength of square timber is to that of the tree whence it is hewn as 10 : 17 nearly.

The lateral strength of a beam supported at both ends, whatever be the section it presents, is least when the whole weight acts at the middle, and greatest when placed near the ends; hence the rule that considerable weights are to be placed not in the middle but near the ends of their supports. By an equal distribution of the load over the entire length of the timber, it can be made to bear twice as great a weight as it would have borne had the load been placed in the middle, and four times as much as it could have borne had it been fastened at one end only.

A considerable advantage is frequently secured by using hollow cylinders instead of solid ones, which, with an equal expenditure of materials, have far greater strength, provided only that the solid part of the cylinder be of a sufficient thickness, and that the workmanship be good, especially that in cast-metal beams the thickness be uniform, and the metal free from flaws. According to Eytelwein, such hollow cylinders are to solid ones of equal weight of metal as 1·212 : 1, when the inner semi-diameter is to the outer as 1 : 2; according to Tredgold as 17 : 10, when the two semidiameters are to each other as 15 : 25, and as 2 : 1, when they are to each other as 7 : 10.

An extraordinary method of increasing the suspensive power of timber supported at both ends, a method, however, which has been confirmed by repeated experiments, is, to saw down from one-third to half of its depth, and forcibly drive in a wedge of metal or hard wood, until the timber is slightly raised at the middle out of the horizontal line. By experiment it was found that the suspensive power of a beam thus cut one-third of its depth was increased $\frac{1}{19}$th, when cut half through it was increased $\frac{1}{29}$th, and when cut three-fourths through it was increased $\frac{1}{87}$th.

Resistance to pressure is taken into account chiefly when a body, as, *ex. gr.*, a column of stone or wood, has to withstand the pressure of a weight acting in the direction of its longer axis. The consequences of this compression are various as the body is long or short; if the former, the material when overweighted will bend and break; if the latter, the body becomes shorter and thicker, its parts at length separate, the upper pressing through the lower, and by mutual friction reducing the substance to powder.

The force required to crush a body increases as the section

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§ 88. Tredgold, in his "Practical Essay on the Strength of Cast Iron," mentions an experiment by Muschenbröek on a piece of bar iron, which gives 63,286 lb. for the weight that would tear asunder a square inch of the specimen he examined. He refers also to a specimen tested by Captain S. Brown, which tore asunder with a weight of 16,265 lb.; and two experiments made by Mr. G. Rennie, where the bars respectively required a force of 18,656 lb. and 19,488 lb. per square inch to produce rupture.

"The resisting body is supposed in the theory to be perfectly homogeneous, or composed of parallel fibres, equally distributed around the axis, and presenting uniform resistance to rupture. But this is not the case in a beam of timber; for by tracing the process of vegetation, it has been found that the ligneous coats of a tree, formed by its annual growth, are almost concentric; and that they are like so many hollow cylinders thrust into each other, and united by a kind of medullary substance which offers but little resistance: these hollow cylinders, therefore, furnish the chief resistance to the force which tends to break them. Now, when the trunk of a tree is squared, in order that it may be converted into a beam, it is evident that all the ligneous cylinders greater than the circle inscribed in the square or rectangle, which is the section of the beam, are cut off at the sides; and therefore, as Montucla remarks, almost the whole resistance arises from the cylindric trunk inscribed in the solid part of the beam. The portions of the cylindric coats which are towards the angles, add a little, it is true, to the strength of that cylinder, as they cannot fail to oppose *some* resistance to the straining force; but it is far less than though the ligneous cylinder were entire. Hence we cannot, by legitimate comparison, accurately deduce the strength of a joist cut from a small tree, by experiments on another which has been sawn from a much larger tree or block; the latter is generally weak, and very liable to break. As to the concentric cylinders we have been speaking of, they are evidently not all of equal strength. Those nearest the centre being the oldest, are likewise the hardest; which again is contrary to the theory, in which they are supposed to be uniform throughout. After all, however, it is still found that in some of the most important problems the results of the theory and well conducted experiments coincide even with regard to timber: thus, for example, the experiments of Duhamel on rectangular beams afford results deviating but in a slight degree from the theorem of Galileo, that the strength is proportional to the product of the breadth into the square of the depth." — *Gregory's Mechanics*, vol. i. p. 121.

Rennie says, that cast-iron may be crushed with a weight of 93,000 lb., and brick with one of 562 lb.

of the body increases; and this quantity being constant, the resistance of the body diminishes as the height increases.

According to Eytelwein's experiments, the strength of columns or timbers of rectangular form in resisting compression is, as

1. the cube of their thickness (the lesser dimension of their section). 2. as the breadth (the greater dimension of their section). 3. inversely as the square of their length.

§ 89.

Elasticity, as one of the accessory properties of bodies, has already been explained, § 7. It possesses a peculiar importance as respects solids, partly from its being itself a moving force, and partly because it serves to measure the intensity of other forces. This application of elasticity to bodies of this form will now be a subject for our consideration.

All bodies offer a resistance to any force which tends to produce an alteration in their volume. This resistance is the elasticity of each particular body called into exercise by the external agency of the force impressed upon it, and acting in a direction exactly opposite to such force. A *perfectly elastic* body would resume its original volume and form, however great the alteration that had been produced in it, and however long the duration of such alteration. Perfectly elastic solids have no actual existence, this property being always confined within *certain limits*, which, as we have remarked before, § 7., are measured and expressed by the least force that produces a permanent change in the arrangement of the component particles of any body, or even a disruption of such particles.

Glass, notwithstanding its brittleness, is a highly elastic body; but the limit of its elasticity is precisely that point at which it breaks if its form be altered ever so little. An iron wire, on the contrary, may be considerably bent without being broken; the limit of its elasticity, however, is soon reached, for such a wire must be bent but very little, if it is desired that it should again resume its original form.

The elasticity of solid bodies may be affected in many ways. That of metals is increased by hammering, welding, and drawing into wire; it is diminished by heating them. The heating and cooling of steel makes it a highly elastic body; this operation has either no effect at all, or an entirely opposite one on the other metals.

§ 90.

The principal of those solid bodies which possess a high degree of elasticity are, caoutchouc, ivory, tempered steel, the nerves of animals, and the fibres of plants spun or twisted together. The three last-named substances, and steel especially, are employed as moving powers.

The application of steel in the form of springs is various, as in watches and similar machines, door-locks, gun locks, &c. The sinews of animals, or cords, were used by the ancients in their ballistic machines.

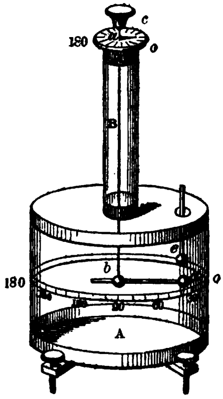
The recoil of cannon balls is owing, in great measure, to the elasticity of the cast-iron of which they are made. Similar effects will be observable in the experiments given § 6.

The resistance which elastic solids offer to those forces that tend to alter the position of their parts, or to diminish their volume, is of considerable importance for many physical purposes. It has been shown already that in highly elastic bodies, the motions, or spaces passed through by those particles whose state of rest has been disturbed, are in proportion to the forces producing such changes; or, in other words, that the resistance which the elasticity offers is always proportional to the displacement of the particles of the elastic body. The application of this force as a measure for other motive powers, depends upon this law of the reaction of a body's elasticity. The most important instruments whose operations depend on this principle are the *Torsion Balance* of Coulomb, the *Dynamometer* of Regnier and Horner; similar to which last-named apparatus are those balances or steelyards which act by means of a *spring;* and, lastly, the older varieties of *powder-testing* machines with springs.

Coulomb's Torsion Balance.

This instrument is for the measurement of small attractive and repulsive forces. The elastic body which serves as the measure of these forces, is a fine vertical dependent metal wire, *a b*, *fig.* 35. (of platinum, silver, or steel). Its upper end *a* is firmly secured, whilst the lower end *b* is stretched by a small weight, and carries a needle *b d*, moving freely in a horizontal direction. The force to be measured is made to act from *e*, on the extremity *d* of the needle. If the force be repulsive, *e* is to be placed in contact with the end of the needle; but if it be attractive, *e* must approach only to such a distance from the needle as the attractive force can act at. The size of the angle,

Fig. 35.



at which the needle is drawn from its position of rest by the force acting on it, serves as a comparative measure of the force's intensity. In order to guard the needle from the disturbing action of currents of air, it is inclosed in a glass cylinder A, and even the wire is incased in a glass tube, which stands on the lid of the cylinder. The upper end of this tube is provided with a brass cap that turns with some friction round the tube, having its smooth upper surface marked off into degrees. The pin *a*, which holds the wire, turns round in the centre of this graduated plate, and carries an index *a c*. In order to estimate the angle of torsion of the wire, the glass cylinder A is graduated at the height of the needle.

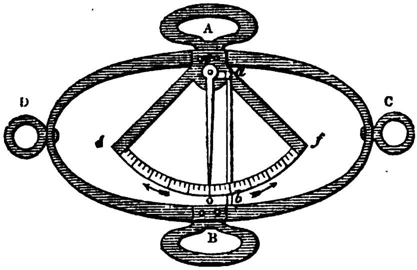
The mode of using this apparatus in measuring the intensity of small forces is for the most part pretty much as follows: — The index *a c* is set at *o* in the upper scale, and the cap is turned round till the end *d* of the needle stands at zero of its scale. Now, let the force which is to be the subject of investigation, act either by attraction or repulsion at *d*, through an aperture left in the cylinder. The needle will now be deflected a certain number of degrees from zero. The force to be measured corresponds exactly to the reaction which the elasticity of the wire opposes to its further extension, or at this distance from each other the effects of both forces are alike.

If, now, it were desired to know to what elastic force of the wire any given force would be equivalent, supposing the distance of the needle were diminished $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. from that point whence the force acts, we should push the index *a c* back in an opposite direction from that in which the needle moves, till the end *d* of the needle was $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., as far from its place of rest, *i. e.* from the point whence the force acts on it; then the number of degrees passed over by the index, multiplied by the number of degrees which the wire has moved in an opposite direction, gives a measure proportional to the two effects produced by the force under review at its two different distances. Suppose, for instance, the first torsion of the wire amounted to 36°, and that it was necessary to put the index *a c* back to 126°, in order to deflect the needle 18° from its original position, it follows that the wire is now turned $126 + 18 = 144°$ from its natural position, and as $144 = 4 \times 36$, this force is four times as great at half the distance. In like manner, the absolute intensity of two different forces may be compared and measured. Suppose that with the force F it is necessary to deflect the wire 70° in order to bring it 10° from its original position, and that the force F′ requires that the wire be moved back 150° to bring it to the same 10°, then the intensities of these two forces F and F′ are to each other as the angle of torsion of the wire in each of these cases, viz. as 70 + 10 = 80 is to 150 + 10 = 160, *i. e.* they are to each other as 1 : 2.

Horner's Dynamometer. Fig. 36.

This is an instrument intended for the admeasurement of greater forces. Its principal part is a hardened steel-spring of great elasticity. It has an oval or elliptical form, and is either compressed or pulled asunder in the direction of the lesser axis, by means of the force that is the subject of measurement. The angular lever *a c o* serves to render the forces which act on the spring, comparable. The fulcrum of the lever is at *c*, the short arm *c a*, moveable about *a*, is connected with the spring by means of *a b*, the larger arm, *c o*, serves as an index to point out by its movements over the scale *d f*, the changes produced by the action of the forces on the spring. A and B are handles by which the action of the force is transmitted to the spring. If the spring be compressed, the index traverses from *o* towards *f*; if it be drawn in different directions it moves from *o* towards *d*. The proportion of the scale is marked by hanging weights into the lower handle, B, towards *d*, and attaching weights in like manner to the upper handle A towards *f*. Still greater forces are made to act on the rings C and D in the direction of the greater axis of the spring pulling it in opposite ways.

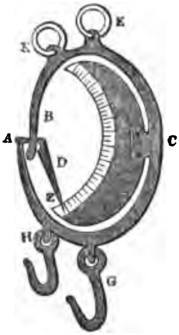
Fig. 36.



The Spring Balance or Steel Yard.

This well-known but not very accurate instrument consists also of a steel bow bent into an elliptical form (*fig.* 37.) open at A, and weaker at the two ends A and B, than in the middle, C. At the lower end is the lever, A D Z, which turning on A serves as an index, and passing through an opening at the upper end near B, moves along the scale marked in the interior of the steel bow with its extremity, Z, referring to the particular degree. There is a ring at E, by which the instrument may be suspended, and a hook at G, to which the object to be weighed may be hung. The weight stretches the spring till its elasticity is in equilibrio with it, and, as may readily be seen, the index will become stationary at some certain point of the scale, which is divided proportionally to the assumed unit of weight. Besides the hook G there is commonly a second, H, and a corresponding ring, K, nearer, to the thin end, for weighing smaller loads, a second scale being graduated on the opposite side of the crescent to express the weights which this hook is meant to carry.

Fig. 37.



Powder-testing Machine.

These machines, intended to test the precise strength of gunpowder, notwithstanding their various modifications, in the main depend on the action of a pretty strong spring, which closes the aperture of a chamber containing a small quantity of gunpowder. When the powder is ignited the spring is driven back to some certain distance by the expansive force of the powder, and is then secured from returning by means of a catch; a measure is thus given of the relative strength of different specimens of this combustible.

§ 90. Perhaps the most interesting application of the torsion rod is for determining the mean density of the earth; the only mode of ascertaining which is by comparing the attractive force of the entire mass of our planet with that of some known part of it. In the Cavendish experiment a large leaden ball is weighed against the whole earth by comparison of their effects upon a pendulum: the nearness of the smaller mass making it produce a sensible effect as compared with that of the larger. The method of solving this problem by observing the oscillations of a horizontal pendulum suspended by a wire was first suggested by Michell, and executed by Cavendish in 1797; as the result of twenty-three experiments, he made the mean density of the earth 5·45 times that of water. M. Reich, of Freyberg in Saxony, in 1837, from fifty-seven experiments, deduced fourteen results, the mean of which makes the density

CENTRE OF GRAVITY.

§ 91.

Every solid body on the surface of the earth may be regarded as an aggregate of atoms gravitating towards its centre, held together by what is called the force of cohesion, and they obey the attraction of the earth, not as isolated particles, but as a certain defined whole. There exists within every solid body on the earth's surface *one* point, around which the atoms equally solicited by gravity are equally distributed in all directions. (As the earth's centre is so remote from the surface, the forces by which the atoms of a body at the surface are solicited towards the earth's centre are nearly parallel, otherwise this would not hold good; now for this infinite number of forces we may substitute a *single one*, which may be regarded as their resultant, and the point at which it is to be applied is the body's *centre of gravity*.)

Bodies of uniform density, whose figure has a centre, have their centre of gravity in such geometrical centre: in bodies of unequal density the centre of gravity will incline to the denser part: in many bodies, as, *ex. gr.*, in rings and other hollow forms (bomb-shells, ships, &c.), the centre is altogether without the mass.

> If a body have a regular geometrical figure, and be of uniform density, the position of its centre of gravity may be found mathematically; if this be not the case, it often happens that actual experiment alone will give its place.

equal to 5·44. The experiment was repeated by Mr. Baily, who began his observations in October 1838, and concluded them in May 1842; the mean of all his experiments (exceeding 2000 in number) gives 5·675 as the mean density of the earth with a probable error of ·004.

The torsion pendulum used in determining this problem is an instrument of such extreme delicacy that it is never at rest; the effect, therefore, of presenting any attracting mass (as a large leaden ball) to it, is not to draw the torsion rod from one position into another, but to change its motion from an oscillation of one extent about one point of rest to another about another point of rest.

The torsion rod, which was made of fine deal, was inclosed in a case of wood, to secure it from currents of air; and precautions were taken to remove every thermal, electric, or magnetic disturbance; and lastly, it was guarded against the effects of radiant heat by wrapping the case in flannel, and gilding the masses, various parts of the frame-work, and the balls at the two extremities of the torsion pendulum. — See *Monthly Notices of the Royal Astronomical Society*, vol. v. Nos. 24. and 28.; or, for a detailed account of Mr. Baily's experiment, vol. xiv. of the Society's *Memoirs*.

§ 91. The truth of the assertion that the centre of gravity corresponds with the centre of figure or magnitude, may be shown, if we suppose a plane or line to be divided into two equal and similar parts, so that its molecules are arranged two by two with respect to the dividing plane or line: take any two molecules similarly situated on opposite sides of this plane, their moments will be equal and opposite; so also of every other pair of molecules (for we have supposed them all to be similarly situated); whence the resultant of the system is in the plane or line, and consequently the centré of gravity is there also. The same reasoning may be extended to the centre of figure or magnitude, *i. e.* the point with respect to which a whole body is symmetrical. Vide *Gregory's Mechanics*, vol. i. p. 50.

§ 92.

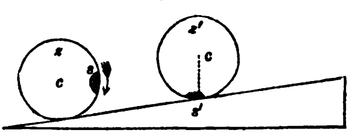
Admitting the above definition of the centre of gravity, it follows that terrestrial attraction may be considered as if it affected only this one point. The next consequence which we deduce is, that the centre of gravity in a body constantly exerts a tendency to descend, and that if a body be free to move, it will not attain a state of rest until its centre of gravity is in this lowest position; for if we conceive of a system in which no other forces than weights are applied, it is clear, if motion arises about the fixed points of support, that it is produced by a part of the force of gravity, the surplus of that force acting on the points. Therefore the sum of the vertical resistances of the fixed points will be less at the first instant of motion than the entire weight of the system: consequently, from those two forces combined, there will result a single force equal to their difference, which will solicit the system downwards. Hence, the centre of gravity will necessarily descend with the velocity due to that difference; and hence, to assure ourselves that several weights applied to any system or machine whatever are in equilibrio, it suffices to prove that if the system be left to itself, its centre of gravity will not descend.

The immediate and universal consequence of this principle is, that if the centre of gravity of any system is at the lowest point possible, there will necessarily be equilibrium, for, if not, the centre of gravity must descend; yet how can it *descend* if it be already at the *lowest* point? (*Gregory's Mechanics*, vol. i. p. 47.)

For this very reason, every body, whether falling *in vacuo*, or through the air, has a tendency to descend with its centre of gravity next to the surface of the earth: if, therefore, at the beginning of its fall it had not this position, the body assumes a rotatory motion about its centre of gravity, which lasts until this point has acquired its natural position.

In consequence of the ease with which spherical bodies are moved, if placed on a plane superficies, they assume such a position that their centre of gravity may be always as near as possible to the surface of the earth.

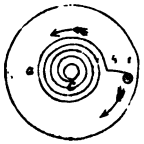
Fig. 38.



Exp. Let Z (*fig.* 38.) be a cylinder whose centre of gravity, *s*, lies beyond its axis, *c*, near the surface; if now it be placed upon an inclined plane, in such a manner that the centre of gravity, *s*, be above the point of contact, and towards the elevated part of the plane, the cylinder will ascend until it arrives in the position z', when its centre of gravity, s', will be vertically below the axis.

If the cylinder be hollow, as in *fig.* 39., and its centre of gravity be represented by a weight, *s*, drawn up by means of a tightly-wound spiral spring, *bac*, such a cylinder will continue to ascend an inclined plane, until the elastic force of the spring can no longer raise the weight, when it will assume a position nearly in the vertical line through the point of contact.

Fig. 39.



§ 93.

If the centre of gravity be supported by any means whatever, then the body will be at rest; if, on the contrary, this point be not supported, the body will incline to that direction in which its centre of gravity is.

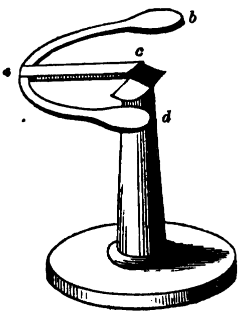
On this simple fact the stable or unstable position of all bodies depends, besides a great number of modifications of these two states, which present themselves in every-day life, either independently of our arrangements or by our own contrivance, though almost without our reflecting about the reason of our action.

The centre of gravity of a body may be supported in three ways, viz. either by that centre itself, or by some point above or below it.

If the body be supported by the centre of gravity itself, it can revolve at pleasure around this point, yet remaining at rest in every position, as happens in the case of a pulley having an axis passed through its centre of gravity, or in that of wheels whose centre of gravity lies in their axle, which is supported by the sockets in which it turns.

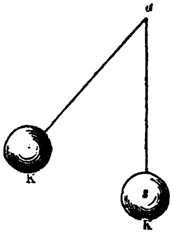
Even those hollow bodies whose centre of gravity is not contained within their volume, may yet be supported by it, if any solid be connected with them, so as to pass through their centre of gravity, and the support be given at that point.

Fig. 40.



First Exp. The centre of gravity of the oval or semi-circular bar *dab*, *fig.* 40., is somewhere between its arms in the line *ac*. Fix a small bar, *vc*, to the bow, and let the centre of gravity, *c*, rest on the edge of the pillar, and the whole apparatus will be balanced. In a similar manner, if two knives be stuck, at any angle, in a piece of wood, the whole may be balanced on the rim of a glass.

Fig. 41.



If the body be supported above its centre of gravity, it is said to be *suspended;* and if then it be free to move, the body will not be brought into a state of rest, till the centre of gravity has attained the lowest possible position beneath the point of suspension, in other words, not till the centre of gravity be perpendicularly below the latter point: thus, *ex. gr.*, when the centre of gravity *s* of the ball K (*fig.* 41.) comes perpendicularly under *a*.

If a body be suspended freely, and a perpendicular be let fall, this line will pass through the body's centre of gravity; if the body be hung up again by some other point, and a second perpendicular drawn, the intersection of these two lines will give the centre of gravity. This experimental mode of ascertaining the position of this point is frequently made use of with regard to plane bodies.

Lastly, the body may be supported below its centre of gravity, and then it will remain at rest so long as the line of direction in which the terrestrial attraction acts upon this point is supported. Accordingly, if a body be supported on a single point, and that point be in the line of direction, that body will be in a state of rest; but the smallest movement, or the slightest deviation of the centre of gravity from its place, would overthrow the body.

Many circumstances with which we are familiar depend upon this fact. A sphere can remain at rest only upon a perfectly horizontal plane; if the inclination be ever so small, the sphere will roll to the lowest part of the plane. In balancing a body, its centre of gravity must be supported. It is readily seen for this cause why a heavy body, as a top, may be supported on a point, if a rapid rotation be communicated to its axis. In the apparent ascent of a double-cone, its centre of gravity is progressively descending to a lower point. On the same principle depends the following neat experiment.

Second Exp. If a watch-glass, on which a drop of water has been placed, be laid with its convex side on a looking-glass or porcelain plate to which a small inclination has been given, it immediately acquires a rotatory motion, if pressed a little sideways on the edge. On account of the adhesion produced by the particle of moisture, it cannot glide straight down the glass; by pressing it a little on the side, its centre of gravity is slightly elevated, and for want of support it falls again, and generates the rotation which is directed from the top to the bottom of the slanting glass.

§ 94.

We generally endeavour to give a body a stable position, by supporting it beneath its centre of gravity; however, an equally firm position is not thus secured under all circumstances. If the centre of gravity in any body be very low, or near the base, the position of such a body is not easily disturbed; and even should it be changed, it readily reverts to its original place; such a body is said to be in a condition of *stable* equilibrium. But if the centre of gravity be considerably elevated above the base, or should the base be narrow, so that a small inclination of the body throws the vertical line through its centre of gravity without the base, the body falls over in the direction in which it was disturbed; such a body is said to be in a state of *unstable* equilibrium. Every body stands more securely, the lower its centre of gravity, the wider its base, and the greater its weight.

An egg set so that its longer axis is parallel to the horizon is in a state of *stable* equilibrium; if placed on its end, in one of *unstable* equilibrium.

A waggon loaded high is more easily overturned than one whose load is placed low: in order to depress the centre of gravity as much as possible, the heaviest goods are therefore put first into a waggon. — Construction of modern gun-carriages. High and low wheels; width of the track.

Every one knows that we can stand more steadily on two feet than on one; in the former case the line of direction falls between the two feet, in the latter it comes under the sole of that foot on which our body is supported, for which reason we incline ourselves to that side. In order to stand very firmly we enlarge the base by placing our feet wide asunder.

Exp. Let a person stand sideways close against a wall with his feet together, he will find it impossible to lift the outer foot, because its sole forms the base of the body.

The leaning towers at Pisa and Bologna (the former of which is 315 feet high, with an inclination of 12¾ feet from the perpendicular, the latter 134 feet high, and inclination 9½ feet) do not fall because their line of direction falls within the base.

§ 95.

If the form of a body be changed, or if its parts be differently distributed, in either case the position of its centre of gravity will be altered; thus, with bodies whose form is variable the position of this point is also changeable.

If a man carry a load on his back, he will stoop forward; if on one shoulder, he will lean to the other side to avoid overbalancing.

Exp. The astonishing movements of the toy known as the Chinese Tumblers are owing to the shifting position of the centre of gravity, their hollow wands containing quicksilver, which always runs to the lowest point.

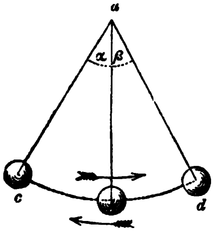
III. THE PENDULUM.

§ 96.

Every solid body suspended freely is called a *Pendulum.*

A distinction is made between the *simple* or *mathematical* pendulum, and the *compound* or *physical* one. The former is regarded as an inflexible imponderable line suspended at one end, *a* (*fig.* 42.), its other end, *b*, being considered as a point possessing weight.

Fig. 42.



a Point of suspension.
b Centre of gravity.
a b Length of pendulum.
c d Arc of oscillation.
$\alpha=\beta$ Angle of elongation.

A simple mathematical pendulum has no actual existence. It is merely conceived of as fulfilling these requirements, in order thence theoretically to deduce the laws of its motion. That which comes most nearly to the mathematical idea of a pendulum is a thread, at the end of which is hung a small metal ball, — one of platinum is best.

All such as actually exist are physical pendulums, every material particle of which acts as the heavy point of the simple pendulum.

§ 97.

The centre of gravity in every pendulum in a state of rest is its lowest point (§ 92.). A pendulum can, consequently, rest only in a perpendicular position. If moved from this position into any other whatever, and then left free to fall, it moves backwards and forwards, going to an equal distance on each side of the perpendicular; ascending on the one side to a height equal to that from which it had fallen on the other. Its descent is owing to the attraction of the earth, whilst its ascent results from its inertia, and the final velocity acquired by its fall. A Pendulum will therefore have an accelerated motion in its descent, and a retarded motion in its ascent.

Gravitation being the cause of the motion of a pendulum, its operation is never suspended, and a pendulum would vibrate for ever were not its oscillations finally overcome by those impediments which it has in common with all terrestrial motions.

The motion of a pendulum is precisely similar to that of a body on an inclined plane, or curved surface (§ 37.); the line, or rod, in the one case offering a resistance to the force of gravitation, corresponding to that opposed in the other by the surface on which the solid body moves. Thus also, as the final velocity acquired by the body in its descent carried it to an equal height up the opposite surface, so the pendulum, on regaining its perpendicular position, performs its ascent.

§ 98.

The motions of a pendulum may be classed under those around a centre, they are in fact *oscillations* (§ 23.). The point of suspension, *a* (*fig.* 42.), is the centre of motion, and the point *b*, in the pendulum on which the resultant effect of gravity is produced, describes the arc *c b d*. The radius of this circle, or the right line, *a b*, is the *length of the pendulum*.

An *Oscillation* is the name given to the motion along the arc *c b d*, through one half of which the centre of gravity of the pendulum descends, while it ascends through the other half; the *angle of Elongation*, α, β, is the deviation of the pendulum from the perpendicular; the *Time* is the space of time employed in performing one entire oscillation.

1. In the mathematical pendulum, the point of motion is imagined to be a ponderous point at the end of the line; in the physical pendulum it is a point whose position between the point of suspension and the end of the pendulum depends on the form of the latter.

§ 99.

These motions obey the following laws: —

1. *Pendulums of equal lengths vibrate at the same place in equal times*, i. e. *they are isochronous*,

It must be observed that the heights to which they are elevated must not differ greatly, and especially that they must not be raised more than 10° beyond the perpendicular.

2. *If two pendulums be of unequal lengths, the shorter one will oscillate more quickly than the longer, and the times are to each other as the square roots of the lengths of the pendulums, or the squares of the times are as the lengths.*

Put l = the length of the pendulum, e its angle of elongation, t its time; then, by the higher mathematics,

$$t=\pi\frac{\sqrt{l}}{g}\left\{1+(\tfrac{1}{2})^2\sin.^2\frac{e}{2}+\left(\frac{1\cdot3}{2\cdot4}\right)^2\sin.^4\frac{e}{2}+\ldots\right\}$$

and if e contain but a few degrees, it is very nearly

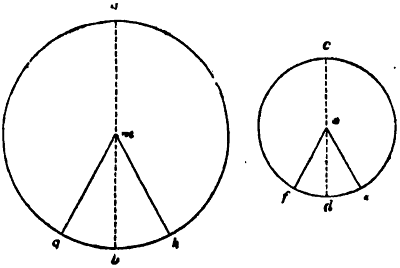
$$(\text{A})\ t=\pi\frac{\sqrt{l}}{g}$$

If the pendulums be of unequal lengths, the place being the same,

$$\begin{aligned} t:t' &= \pi\frac{\sqrt{l}}{g} : \pi\frac{\sqrt{l'}}{g} \\ &= \sqrt{l} : \sqrt{l'} \\ \text{or } t^2:t'^2 &= l : l' \end{aligned}$$

The above law is deduced from the fall of bodies on an inclined plane (§ 36.), where it was shown that all bodies descend along the chords GB, DB (*fig.* 9.), to the lowest point, B, in the same time that they would have fallen through the diameter AB, as follows.

Fig. 43.



If the diameter, AB, of the circle were $16\frac{1}{10}$ feet, a body would occupy one second in falling GB or DB. If

AB = 4 × 16·1 feet, then the body would require two seconds; if AB = 9 × 16·1 feet, three seconds (§ 33. 3.). Let *ab* (*fig.* 43.) be four times as great as *c d*, the body would require twice as long a time to fall through *g b* as to describe the arc *f d*. If *a b* be nine times as great as *c d*, or *m b* nine times as great as *o d*, the time in falling through *g b* will be three times longer than in falling through *f d*, and so on. By means of the higher mathematics, it has been demonstrated, that the times in which bodies move through the small arcs *g b* and *f d* are rather less than those in which they would have fallen through the chords *g b* and *f d* of the same arcs, but that the proportion of the times is the same. Such a ratio now obtains in the motions of pendulums. Let the length of the pendulum, *b m*, be four times greater than *d o*, the bob will require twice as long to move through the arc *g b* as through the arc *f d*; if *bm* were nine times greater, thrice the time, and so on. The same ratio as to time obtains in ascending from *b* to *h*, or from *d* to *k*, as subsisted in falling through the corresponding arcs (§ 37.). From these explanations it follows, that if we wish to make the duration of an oscillation 2, 3, 4, 5, &c. times greater, we must lengthen the pendulum 4, 9, 16, 25, &c. times, *i. e.* the lengths of the pendulums must be as the squares of their times.

§ 100.

A *Seconds' Pendulum* is one whose vibration occupies one second.

As the motion of a pendulum, like the fall of bodies in *vacuo*, or their descent on an inclined plane, is a consequence of the earth's attraction, the length of a seconds pendulum must depend on the intensity of this force (*a*). Where gravitation is equally powerful, all pendulums vibrating seconds must be of equal lengths; and where this force does not vary, the length must continue constant. If the attraction increases, the pendulum will beat more quickly; if it diminishes, its vibrations will be slower. Observation has shown, that a pendulum beating seconds at the equator oscillates more rapidly as it approaches the poles (*b*). Hence it follows that the force of gravity must increase towards the poles, and that the cause of this increment is the flattened figure of our earth, *i. e.* that the axis of the earth is shorter than its equatorial diameter. Under the equator, the length of a pendulum beating seconds is 39·0168 inches; and under the poles 39·217 inches; the entire difference between these two extremes being only ·2003, or little more than ⅕ of an inch; in the latitude of London the length of such a pendulum is $39\frac{1}{8}$, or, more accurately, 39·13929 inches, at the temperature of 62° Fahr.

a. If the time in which a pendulum described a small arc were exactly equal to that in which it would move along its chord, then the length of a seconds pendulum would be exactly the fourth part of the height fallen in the first second of time. But, according to the observation made in the preceding §, 2, it has been shown mathematically that the time is something less. The calculation may be made by formula A in that article: thus,

$$t=\pi\frac{\sqrt{l}}{g},\ \therefore\ l=\frac{t^2 g}{\pi^2},\ \text{if } t=1 \text{ sec.}$$

$$\text{then } l=\frac{g}{\pi^2}=\frac{g}{9{\cdot}8696}=\cdot 1013\, g.$$

b. Richer (1672) discovered that a pendulum beating seconds which he carried from Paris to Cayenne, moved in this latter place rather more slowly. Newton thence deduced, by his second law of gravitation, the flattened form of our earth (§ 29.). The length of the seconds pendulum consequently increases with the latitude; it may be found in inches by the following formula:

$$l=39{\cdot}0168+\cdot 2003 \sin.^2 \phi$$

39·0168 being the length of a seconds pendulum under the equator, and ϕ the latitude of the place.

Difficulty in accurately determining the length of the seconds pendulum in physical pendulums. Kater's Compensation Pendulum. — See *Kater and Lardner's Mechanics,* in the *Cabinet Cyclopædia,* p. 320.

§ 101.

The application of this physical apparatus, and the consequences deduced from its operation, are as important as its construction and mode of acting are simple. The following instances will fully bear out the above assertion.

1. The pendulum is the most accurate and common measurer of time. Its universal application is owing to the fact that pendulums whose lengths remain unchanged always make isochronous oscillations, the time being (nearly) equal in unequal arcs.

The influence of heat on the length of physical pendulums, and the means for counteracting this agent by what are called compensation pendulums, belong more appropriately to a treatise on heat than to that department of natural science now under review.

2. The isochronism in the oscillations of pendulums of equal lengths establishes the constant and unvarying action of gravitation at the same place on the earth's surface.

3. It is clear that all substances whatever are equally attracted towards the earth, from the fact that pendulums perform their oscillations in equal times, however various the materials of which they are composed, if they be but similar in form and equal in length.

Newton's and Bessel's experiments with pendulums whose balls were made of gold, iron, glass, wood, stone, salt, &c. confirm this.

4. It is obvious that gravity diminishes as the distance from the centre of the earth increases, because a pendulum on the summit of a mountain beats more slowly than at its base; also, because as we approach the equator the pendulum's motion becomes retarded.

Bouguer and Condamine carried on these experiments in America under the line during the last century.

5. The absolute intensity of the earth's attraction at any place, or the height a body falls in the first second of time, may be most exactly determined by finding the length in inches of the seconds pendulum at that place, and multiplying that number by 9·8696.

In observation *a*, § 100., it was stated that the length of a seconds pendulum $l = \frac{g}{9{\cdot}8696}$, $\therefore\ g = 9{\cdot}8696\, l$. — For London, therefore, where the length of the pendulum is 39·13929 inches, the velocity acquired by a falling body under the action of gravity for one second of time must be $= 39{\cdot}12929 \times 9{\cdot}8696$ inches $= 32{\cdot}19076$ feet. (§ 33.)

6. The mutual attraction of all bodies and of the earth is evinced by the phenomena presented to us in the pendulum. It has been noticed that a pendulum in the neighbourhood of a large mountain inclines towards that mass.

7. The direction of a pendulum at rest gives us also the line of direction in which gravity acts on all bodies.

IV. IMPACT OF SOLID BODIES.

§ 102.

By Impact or Percussion is meant the collision of two bodies, of which one at least is in motion. The existing motion cannot be destroyed unless a sufficient cause be found; and as both the mass in motion and that at rest, by virtue of their inertia (§ 27.), act upon each other whenever percussion supervenes, the resultant of this motion must depend on the momentum of the moving body (§ 28.). The investigation of the laws of impact refers to those modifications in the motion, both of the impinging body and of that which is impinged on, that arise from their coming in collision.

As regards the direction in which the impact occurs, it is spoken of as *central* and *eccentric*, *right* and *oblique*. It is *central* when the directions from the point of contact to the centres of gravity of the two solid bodies are in a right line, as in *fig.* 44.; *eccentric* when this is not the case, as in *fig.* 45.; *direct* or *perpendicular* when the direction of the moving body stands at right-angles to the side of the body impinged on, as *fig.* 46. *a*; *oblique*, *fig.* 46. *b*, when this is not the case. If the former happen, the whole effect of the moving body passes to the other body; if the latter, a partial effect only is produced. (§ 26.)

Fig. 44.

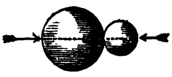


Fig. 45.

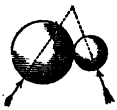
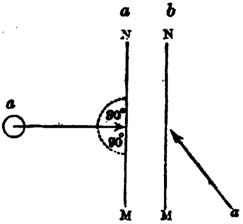


Fig. 46.



In investigating the general laws of motion, the masses only of bodies and their velocities were noticed, without enquiring into the particular properties of the bodies themselves; in examining the effects of Percussion, the latter are principally observed, because the laws of motion are accordingly much modified.—A distinction is made in the percussion of solids, especially between the impact of *hard inelastic* bodies, and that of *elastic* ones, together with the penetration of such as are soft, because the latter, becoming divided, their parts can no longer be followed after collision.

Hardness and *Elasticity* are essentially relative terms, consequently there exists no such thing as an absolutely hard or a perfectly elastic solid. The laws that we are about to develope must, accordingly, be subject to modifications varying in degree; in theory, bodies are assumed to to be absolutely hard or absolutely elastic, according as one or the other of these qualities predominates.

§ 103.

In perfectly hard non-elastic bodies, the momentum must be divided proportionately to the masses. If two such bodies move in one direction, after impact their joint momentum will be equal to the sum of their single momenta. If they travel in opposite directions, it will equal the difference of their momenta. The common velocity of the two bodies, after impact, is found by dividing the sum, or, if they move in opposite directions, the difference of the momenta by the sum of their masses or weights.

Let M and m represent the masses, and V and v the velocities of two bodies; then the motion of the combined masses after impact, if they move in the same direction, is $MV + mv$ (Comp. obs. § 28.)

Let their common velocity after impact be x, then the momentum of the united masses $= x(M+m)$, whence $x(M+m) = MV + mv$; consequently

$$1.,\quad x = \frac{MV + mv}{M + m}$$

Next let the bodies move in opposite directions, and let $mv < MV$, then

$$2.,\quad x = \frac{MV - mv}{M + m}$$

In which case the momentum of the former body after impact will be

$$3.,\quad Mx = \frac{M\,(MV - mv),}{M + m}$$

and that of the second body,

$$4.,\quad mx = \frac{m\,(MV - mv)}{M + m}$$

whence the momentum lost by the first body is

$$MV - \frac{M\,(MV - mv)}{M + m} =$$

$$5.,\quad \frac{Mm\,(V + v)}{M + m}$$

and by the second body

$$mv + \frac{m(MV - mv)}{M + m} =$$

$$6.,\quad \frac{Mm\,(V + v)}{M + m}$$

or the moving force of both bodies is equally diminished.

If one body, *ex. gr.*, m, be at rest, then $mv = 0$, and in this case

$$7.,\quad x = \frac{MV}{M + m}$$

If $M=m$, then

$$8., \ x=\frac{V\pm v}{2}$$

When $V=v$, in which case only equation 2. admits of application,

$$9., \ x=\frac{M-m}{M+m}.V$$

Lastly, if $MV=mv$, and both bodies move in the same direction,

$$10., \ x=\frac{2\,MV}{M+m}$$

and if they move in opposite directions,

$$11., \ x=\frac{MV-mv}{M+m}=\frac{0}{M+m}=0$$

In order to test experimentally the accuracy of these laws, we must set perfectly hard and non-elastic bodies in a state of central motion, so as to produce impact, by which means we may estimate the velocity of the bodies with tolerable accuracy. The following arrangement will be found available from its simplicity: suspend two balls of clay near to each other in the manner of a pendulum, taking care that they shall describe their oscillation in the same arc; when they strike together, their impact will, of course, be central. The velocity may be measured on a scale over which the suspending cord traverses. The results obtained by experiments of this nature can after all be regarded only as approximations to theoretical laws, since no perfectly hard and non-elastic ponderous body has been found.

§ 104.

One of the most important particulars in these investigations depends on the distribution of the *absolute force of the impact*, or the effect produced by the impinging body on the other.

When a hard body impinges upon a mass, and the latter opposes the body's penetrating it, or communicating motion to it, so as to deprive it of equal velocities in equal times, this constant resistance retards the motion in a manner exactly analogous to that in which gravity acts upon a body ascending vertically. It has been shown above (§ 40.) that the heights to which a body ascends are as the squares of the times, and the times are as the velocities; so, in the case now under review, *the effects and* (if the particles recede so as to admit the entrance of the impinging body) *the depths penetrated are as the squares of the velocity, multiplied into the mass of the impinging body.*

Let M and *m* again represent the masses, V and *v* their velocities; put F and *f* for the force of impact, or the depths penetrated; then the preceding law may be expressed in the following proportions:—

$$1.\quad F : f = MV^2 : mv^2$$

if $M = m$

$$2.\quad F : f = V^2 : v^2$$

if $V = v$

$$3.\quad F : f = M : m$$

§ 105.

If the body impinged on be immoveable, or if its mass be very great as compared with that of the moving body, no motion will be imparted, but both bodies will continue at rest after impact (*a*), and the collision will affect only the point of impact. If its force be such as to overcome the cohesion subsisting between the particles of the body impinged on, then the body in motion will penetrate the one which is at rest until the resistance offered by the cohesion of the particles of the latter exceeds the constantly diminishing momentum of the penetrating body (*b*).

a. If, therefore, in the seventh equation of § 103., the mass M of the body at rest be assumed to be infinitely great as compared with *m*, the mass of the body in motion, then

$$x = \frac{mv}{m + \infty} = 0,$$

i. e. the velocity after impact is *nil.*

b. By the principle above stated we may explain the effects produced by pile-driving machines, hammers, battering-rams, &c., as also the effects of shot and cannon-balls discharged against earth, wood-work, or masonry.

To express the absolute magnitudes of these effects in any particular measure, according to the laws laid down in the preceding section, a factor must be found which may serve as the unit for some particular value of V. Beaufoy has determined this not only better than any other philosopher, but up to the present time he alone has done it satisfactorily. The result of his experiments was, that a body of 1 lb. weight falling with a velocity of 5·68 feet in a second would exert a force of 16·141 lb.; hence we may determine what would be its force with a velocity of 1 foot in a second.

$$F = MV^2$$

$$F = M\frac{16{\cdot}141}{(5{\cdot}68)^2} = {\cdot}5003 \text{ lb.}$$

Hence it follows that a weight of Mlb. moving at the rate of 1 foot in 1 second of time would impinge on a body with a force

$$1.\ F = \cdot5003\ M,$$

and therefore, in general terms, for any velocity V in feet,

$$2.\ F = \cdot5003\ M V^2,$$

and approximately for any height h, as perfect accuracy cannot well be attained on account of the resistance of the air.

$$3.\ F = 30\ M\ h.$$

A few examples will serve to show how these formulæ are to be applied.

1. Required the force of percussion with which a one-ounce musket-ball will strike an object at which it is discharged with a velocity of 750 feet in a second.

By the second equation:

$$F = 0\cdot5003\ MV^2 = 0\cdot5003 \,.\, \tfrac{1}{16} \,.\, 750^2 = 18588 \text{ lb.}$$

2. With what force will a 24 lb. cannon-ball strike a wall against which it is fired with a velocity of 1000 feet in a second?

By the same equation:

$$F = 0\cdot5003 \,.\, 24 \,.\, 1000^2 = 1200720.$$

3. What will be the force of percussion with which a bomb-shell weighing 100 lb. will reach the ground, supposing it to fall from a height of 3600 feet?

By the third equation:

$$F = 30 \,.\, 100 \,.\, 3600 = 10800000 \text{ lb.}$$

4. With what force will the ram of a pile-driving machine descend, supposing its weight to be 1000 lb., and that it has a fall of 4 feet?

By the same equation:

$$F = 30 \,.\, 1000 \,.\, 4 = 120000 \text{ lb.}$$

From a consideration of the above examples, the reader will see why shot and balls discharged by means of muskets and cannons produce such surprising effects, notwithstanding the smallness of their masses; it is also evident why by means of artillery the moderns are able to demolish the hardest bodies which opposed an invincible resistance to the cumbrous battering-rams of ancient warfare.

§ 106.

As, in consequence of their inertia, all bodies require a certain time to impart motion to others that are at rest, whilst the motive power communicates itself to all the atoms of the mass; so in like manner it happens that a body at rest capable of being set in motion, may yet, if impinged upon by another body moving with a high velocity, be penetrated or broken at the point of collision without the body as a whole being set in motion.

The following experiments and examples may serve to confirm the statement made above: —

First Exp. A stick, whose ends rest upon two horse-hairs, may be divided by striking it a smart blow in the middle without the hairs being broken.

Second Exp. A tallow candle may be fired through a door that stands ajar, without the door being moved.

Third Exp. A ball if fired through a pane of glass makes a round hole without cracking the parts around.

§ 107.

When elastic bodies impinge on each other, the effects which follow depend not merely on the momentum they communicate to each other, but also on the *reaction* caused by *their elasticity.* For, when two elastic bodies strike together, the parts of each thus brought in contact are compressed with a force equal to the action they reciprocally exert. At the moment when this force ceases the particles fly back to their original position, and impart to each body a percussion exactly opposite to its former motion. If the bodies be perfectly elastic, the loss of momentum sustained by each may be demonstrated mathematically to be twice as great as that of non-elastic bodies.

If the impinging bodies are non-elastic, the loss of momentum sustained by each is

$$\frac{Mm\,(Vv)}{M+m}\ (\S\ 103.,\ 5.\ \text{and}\ 6.).$$

Since in elastic bodies the reaction of both is equal to this loss, the momentum lost by the two bodies is

$$\frac{2\,Mm\,(V+v)}{M+m}$$

The first body, therefore, still retains a force of

$$MV-\frac{2Mm\,(V+v)}{M+m}=$$

$$1.,\ \frac{MV(M-m)-2\,Mmv}{M+m}$$

and the second

$$mv-\frac{2Mm(V-v)}{M+m}=$$

$$2.,\ \frac{-mv(M-m)-2MmV}{M+m}$$

The simplest case that can be conceived, is that in which the masses of the two bodies are equal, as after impact the bodies will exchange both their directions and their velocities: hence is deduced the following:—

1. Both bodies move towards each other directly. After collision they recoil with inverse velocities.

1st. Let $M = m$, then by substituting in equations 1. and 2. the force of the first body after impact =

$$3., \; -Mv$$

that of the second =

$$4., \; -MV$$

whence the first has acquired the velocity $-v$ and the second that of $-V$.

2. Let both bodies move in the same direction, that which travels most swiftly following the other; after it has overtaken the first body, both will continue to move in the same direction, but with inverted velocities.

In this case $-v$ must be changed into $+v$ throughout in the preceding equations; whence after coming in contact the first body will acquire the velocity $+v$, and the second that of $-V$.

3. When one body is at rest, the moving body after impact becomes stationary, and the other moves on with the velocity of the impinging body.

If the second body be at rest, then is $v = 0$, whence the velocity of the first after impact $= 0$, and that of the body at rest becomes $-V$, *i. e.* the velocity of the first is transferred to the second.

The above cases may be experimentally proved by means of the ivory balls in the impact machine, described § 103., ivory being the substance that approaches most nearly to perfect elasticity.

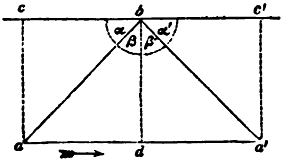
If 100 elastic balls were suspended in such a manner that they just touched, their centres being in one horizontal line, and each successive ball containing a mass equal to half that of the preceding, then the velocity of the last ball would $= (\frac{4}{3})^{99}$ or 23385000000 times that of the first. This velocity would amount to more than 97 million miles in a second, supposing the first to move at the rate of 1 foot; in other words, the last ball would describe a space nearly equal to 5 times the earth's distance from the sun in one second of time; again, if the first ball have a diameter of but 1 inch, the largest would have a diameter 20 times greater than that of the earth, and its solid content would be 8000 times greater than that of the earth.

§ 108.

If an elastic body in motion strike an elastic plane in a state of rest, the former will recoil with a velocity equal to that with which it advanced. If the impact be perpendicular, it returns in the same direction; but if it be inclined, the body returns under the same angle, but in the opposite direction; this is expressed thus: the *angle of incidence* = the *angle of reflexion.* Let $a\,b$ (*fig.* 47.) be the direction in which the body approaches the plane, $b\,a'$ will be the line of its return, so

$$\angle\alpha = \angle\alpha', \text{ or } \angle\beta = \angle\alpha'$$

Fig. 47.



$d\,b$ Perpendicular.
β ∠ of Incidence.
β' ∠ of Reflexion.

Resolve the velocity in the direction $a\,b$ into two, in the directions $c\,b$ and $d\,b$. The reaction of the plane being in the direction $b\,d$ changes the velocity in $d\,b$ into an equal and opposite velocity in $b\,d$, leaving the velocity in $c\,b$ unaltered. By compounding the two velocities, the motion after impact will therefore be in the line $b\,a'$, making the angle of reflexion $d\,b\,a'$ = angle of incidence $d\,b\,a$. The velocity after reflexion will also be the same as that before incidence.

V. EQUILIBRIUM OF SOLID BODIES, AND OF THE MECHANICAL POWERS.

§ 109.

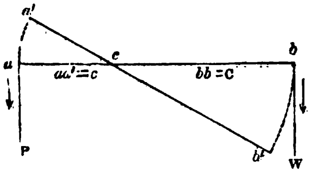
If two or more forces act in opposite directions on a solid body which is fixed, but capable of being moved about a certain point of its mass, such body will either continue in a state of rest, or it will be set in motion.

If the moments of the forces that tend to move the body in opposite directions be equal, it will continue at rest: this condition is called the *Equilibrium* or *Statical Momentum of the Forces.*

If, on the contrary, the moments of the forces in one direction exceed those in the other, the body will be set in motion in the direction of the greater moment.

Let *a b* (*fig.* 48.) be the solid body capable of being turned about the point *c*. Suppose the mass *m* to act on *b*, and M on *a*, then if the line turns about *c* the momentum at *b* (§ 27.) is $m . bb' = m\,C$, and at $a = M .\ a a' = Mc$. If these products are equal, but the directions in which they tend to excite motion are opposite, their effects will be neutralised, and the line will remain at rest; if they are unequal, the line will move in the direction of the greater momentum.

Fig. 48.



Now, since the moments are the products of the forces or masses, M, *m*, into their velocities, C, *c*, in the condition of equilibrium we obtain,

$$MC = mc$$
$$\text{or } M : m = c : C$$

i.e. *the forces or masses must be to each other inversely as their velocities at their points of contact with the solid body.*

Hence is deduced the most important law on which rests the whole doctrine of the equilibrium of solids: *To increase any motive power a proportionably greater velocity must be imparted to it, or it must be made to describe so much the longer space; to give a great velocity to any mass, which it is desired to set in motion, it will be necessary to increase the force acting on the mass.*

§ 110.

On this general law of the action of motive powers on solid bodies depend those various contrivances known by the general name of *machines*.

Machines in general are employed, either

1. by means of a small force to hold *in equilibrio*, or to set in motion a great weight, or

2. by means of a considerable force to impart a great velocity to a small body.

There are six simple *machines*, viz. the *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Wedge*, and the *Screw*. All the different *compound* machines which have been or that can be constructed, consist only of various combinations of these six simple ones, generally known as the *mechanical powers*.

a. The Lever.

§ 111.

A *physical* or *compound lever* is any inflexible rod or solid body whatever, supported on one point, and acted on at two other points, by two forces tending to move it in opposite directions about the point of support. If, instead of such a body, the reader will conceive of an inflexible imponderable line, he will have the ***mathematical*** **or** ***simple lever.***

§ 112.

The two forces acting on the lever are generally distinguished by the names of *Power* and *Weight;* and the point on which the lever is supported, or about which it turns, is called the *Fulcrum.*

§ 113.

Levers are commonly divided into three orders, according to the position of the fulcrum with regard to the power and weight.

In levers of the First Order, or two-armed levers, *the fulcrum is between the power and the weight;* a lever of this kind may have its arms of equal length, the fulcrum *c* being equidistant from the points at which the P and W act, as in *fig.* 49. A., or it may have unequal arms, as in *fig.* 49. B, where the fulcrum is nearer to one of the forces than to the other.

In one-armed levers the power and weight are both on the same side of the fulcrum, which is at the extremity of the lever; in those of the Second Order, *the weight is between the fulcrum and the power,* as in *figs.* 50. A. and B.

In levers of the Third Order, *the power is applied between the fulcrum and the weight,* as in *fig.* 51.

Fig. 49. A.

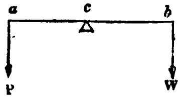


Fig 49. B.

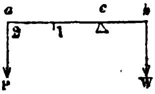


Fig. 50. B.

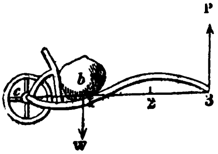


Fig. 50. A.

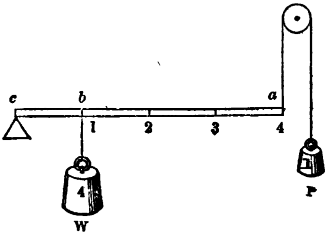


Fig. 51.

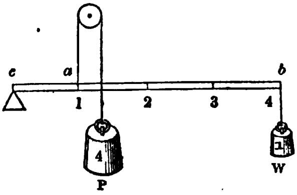
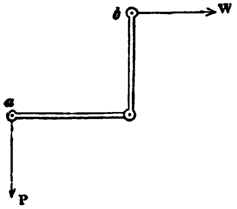


Fig. 52.



The arms of a lever may be either right or curved lines, or they may even form an angle, in which last case the fulcrum will be at the vertex. *Figs.* 49. A. and B. are instances of *rectilinear* levers, *fig.* 50. B. of a *curvilinear*, and *fig.* 52. of an *angular* lever.

§ 114.

The following law holds true with levers of every class:—

1. ***Every lever is in a state of equilibrium when the power and weight are to each other inversely as their distances from the fulcrum*** (a): or,

2. ***Equilibrium obtains in a lever when the moments of the forces acting on the lever are equal.*** (§ 109.)

Thus, according to *figs.* 49. A. and B., 50. 51. 52., if

$$P : W = cb : ac, \text{ or}$$
$$P.\ ac = W.\ cb.$$

a. The distance of a force from the fulcrum is the perpendicular that may be drawn from that point to the line of direction of the force. If the forces act perpendicularly on the arms of the lever, the arms will themselves be the distances; if they act obliquely, a perpendicular drawn from the fulcrum to the line of direction of the force gives the distance. $P a'$, is the direction of the force P (*fig.* 53.), and the perpendicular $c a'$, its distance; $W b'$, is the direction of the weight W, and $c b'$ its perpendicular or distance.

If the angles α and β are known, which the directions of the forces make with the arms of the lever, then by trigonometry it may be found that

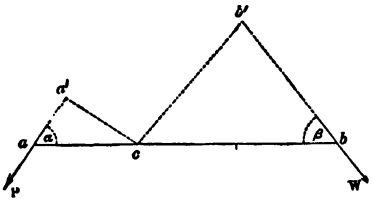
$$ca' = ac\ .\ \sin.\ \alpha, \text{ and}$$
$$cb' = cb\ .\ \sin.\ \beta:$$

in this case, therefore, the statical momentum is

$$P\ .\ ac\ .\ \sin.\ \alpha = W.\ bc\ .\ \sin.\ \beta$$

The accuracy of the first of the above laws follows immediately from what has been explained in the observations to § 109. The line *ab*, *fig.* 53, is to be considered as a mathe-

Fig. 53.



matical lever, the masses M and *m* may represent the power P and the weight W, and the velocities C and *c* must be to each other as the distances *ac* and *bc* from the fulcrum *c*, because they are the radii of the circles in which the power and weight move.

From the law of the equilibrium of the lever it follows, further, that when the forces act perpendicularly on the arms of any lever whose arms are of equal length, as *fig.* 49. A., that equilibrium obtains when the power equals the weight; and when the arms are unequal, equilibrium will be attained when the power acting on the longer arm has to counterbalance a weight as many times greater than itself as the arm on which it acts exceeds the arm which supports the weight.

Influence of the centre of gravity of a physical lever on the law of equilibrium.

Where must the fulcrum be placed beneath a rod of uniform thickness, so that 1lb. may counterbalance 2, 3, 4, &c. lbs., the power and weight acting at the extremities of the rod? — Where must a weight of 2, 3, 4 lbs. be attached to a one-armed lever, so that a weight of 1 lb. at the end of the lever shall maintain the equilibrium?

All the above-named cases, with regard both to the lever of one and that with two arms, may be experimentally verified; an extremely simple apparatus will answer this purpose, the mode in which it is fitted up may vary. The chief point is the lever itself, which should be a prismatic rod having its centre of gravity exactly in the middle. As a lever of two arms, it must be suspended in the middle, and made to turn as upon an axis; if one-armed, it must be fixed at one of its extremities. The forces to act on it must be weights, which are to be hung at different distances according to the scale by which the rod is graduated.

§ 115.

If this law is not fulfilled, the lever will move; the weight will either rise or fall, according as the power is too great or too small to produce equilibrium.

During the continuance of this motion the points on which the power and weight are acting (a and b, *fig.* 48.) will describe arcs of circles around the fulcrum c, and according as the arms of the lever which constitute the radii of the circles are equal or not, those points themselves will move with equal or unequal velocities. The arcs $a\,a'$, $b\,b'$ represent the velocities, and these are to each other as $a\,c : c\,b$, that is, as the arms of the lever; or in general terms as the distances of the power and weight, a and b, from the fulcrum. Thence it follows that *the spaces which the power and weight have to describe are to each other as the length of the arms on which they act.*

§ 116.

The most important applications of the lever are the following: —

1. *To change the direction in which forces are acting.* For this purpose the angular lever is used, as, for instance, in the common bell-pulls, in the trigger of a musket, in a claw-hammer for drawing nails, &c.

2. *With a small power to hold in equilibrio or set in motion a great weight.* To effect this both the single and double-armed levers are employed, the power acting on the longer arm, as, *ex. gr.*, crow-bars, pump-handles, rudders, wheel-barrows, shears, tongs, &c.

3. *To magnify motions so minute as not otherwise to be perceptible.* This is accomplished by means of a lever of unequal arms, the power acting on the shorter arm, the longer arm showing the movement of the shorter by traversing a larger scale, as, for instance, in the dynamometer (§ 90. *fig.* 36., the angular lever $a\,c\,o$), the pyrometer, hygrometer, wheel barometer, &c.

4. *To estimate the amount of pressure, or the mass of the body.* This is unquestionably the most important and general application of the lever. The equal-armed lever is commonly employed, as in the *pair-of-scales;* sometimes the unequal, as the *steel-yard.*

Various modes of constructing balances; adjustment of the common balance; sensibility of the balance; hydrostatic balance; assaying balance; weigh-bridge.

Another kind of lever is that known as the *knee-lever;* it consists of a metal rod ($c\,a$, *fig.* 54.), having a fixed point of

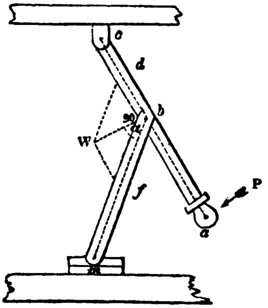


Fig. 54.

support, c, around which it moves. It is connected in the middle by a hinge with another rod, $b\,m$, which when the power P, acting in the direction of the arrow, presses on a, the rod $b\,m$ is driven out from b on m. If $c\,a = 2\,c\,b$, then the effect of the lever on $b = 2\,P = b\,w$. The force then acts in the direction $b\,w$; it may, therefore, be resolved into the two components $b\,d$ and $b\,f$ (§ 24.), of which $b\,d$ is cancelled by the resistance from the point c above, the force then acting on m is $b\,f$. These components acting in the direction of the rods $b\,c$ and $b\,m$ will be greater in proportion as the angle at b is obtuse, and consequently an intense pressure can be produced at small distances. It is proved mathematically that the pressure $b\,f = \dfrac{b\,w}{\cos\,\alpha}$ and $\alpha = c\,b\,m - 90°$. At $60° = 2\,b\,w$, at $70° = 3\,b\,w$, at $78°\ 27' = 5\,b\,w$, at $84°\ 15' = 10\,b\,w$, &c. Thus, suppose a force of 50 lb. to act at a, then m would be pressed on with a force at 60° of 200, at 70° of 300, at 78° 27′ of 500, at 84° 15′ of 1000 lb.

b. The Pulley.

§ 117.

The *Pulley* consists of a wheel called a *sheave*, fixed in a block and turning on pivots; round its rim is a groove over which a cord may pass. Pulleys are either *fixed* or *moveable*.

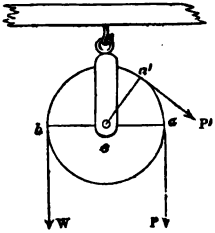
§ 118.

The *fixed Pulley* (*fig.* 55.) revolves about its axis without changing its place. The weight, W, is suspended at one end of the cord which passes over the sheave, whilst the power, P, acts on the other end of the cord.

From c, which may be considered as the fulcrum, draw a and a' radii; these lines will represent the arms of a lever, on which the forces are acting perpendicularly. Hence it follows, that the fixed pulley acts as a lever with equal arms, and will therefore be *in equilibrio* when the Power and Weight are equal.

From what has been just shown, it follows that pulleys of this order do not economise power, their only use is to permit the application of it in any direction that may be desired. The spaces traversed by the power and weight are equal.

Fig. 55.



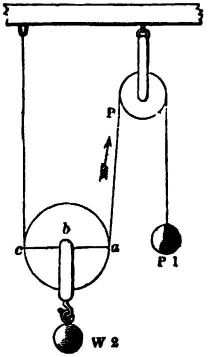
$$bc = ac = a'c$$
$$W \,.\, bc = P \,.\, ac = P' \,.\, a'c$$
$$W = P = P'$$

> *Exp.* Pass a cord over a pulley of this kind, attach an equal weight to each end of the cord, and the weights will be at rest: if one be made to ascend any height, as 1 foot, the other will descend through an equal space.

§ 119.

The *moveable Pulley* (*fig.* 56.), besides its motion round its axis, has also a progressive one. The weight, W, hangs from the axis of the pulley, whilst the cord which passes round it is secured at one end, and is acted upon at the other by the power, P.

Fig. 56.



c Fulcrum

$$ac = 2\,cb$$
$$cb \,.\, W = ca \,.\, P$$
$$1\ W = 2\ P$$
$$P : W = 1 : 2$$

Whether the cord runs parallel on both sides of the pulley, or if the power act vertically upwards, in either case this pulley acts on the principle of a lever of the second order, whose arms are to each other as 1 : 2. For, draw the diameter *c a*; then *c* is the fulcrum, *b* the point of the lever on which the weight, and *a* that on which the power, acts; consequently the distance, *c b*, of the weight from the fulcrum = radius = 1, and the distance, *c a*, of the power = diameter = 2. The statical momentum, therefore, requires a force which shall be equal to half the weight, or, in other words, *the moveable pulley effects a saving equal to half the power: i. e.*—

$$\begin{aligned} P : W &= cb : ca \\ &= r : 2r \\ &= 1 : 2 \\ \therefore P &= \tfrac{1}{2} W. \end{aligned}$$

These pulleys are used where the economising of power is an object in raising weights to any required height.

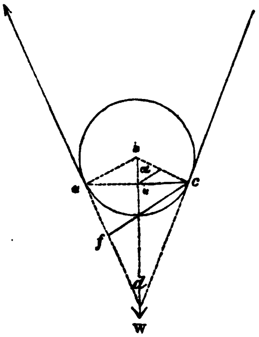
The distance traversed by the power is twice that gone over by the weight; for, in order to elevate the weight one foot, both sides of the cord must be shortened by that amount; that is, the power must pass through two feet.

Exp. These laws of equilibrium and motion are easily verified: hang to the axis of a moveable pulley, see *fig.* 56., any weight, as 1 oz., W, the other end of the cord on which the force is to act must run over a fixed pulley and have attached to it as the power, P, a weight of ½ oz. A condition of equilibrium will now be obtained. Next draw the power downwards any distance, as 1 foot, and it will be seen that the weight has been raised ½ a foot.

If, however, the power does not act vertically on such a pulley, but obliquely, as P, *fig.* 57., so that the ends of the cord, if produced in the directions *c* and *a*, would form the angle *c d a*, then in this case

$$P : W = ce : cf$$

Fig. 57.



and if the angle *c b d* be put = α

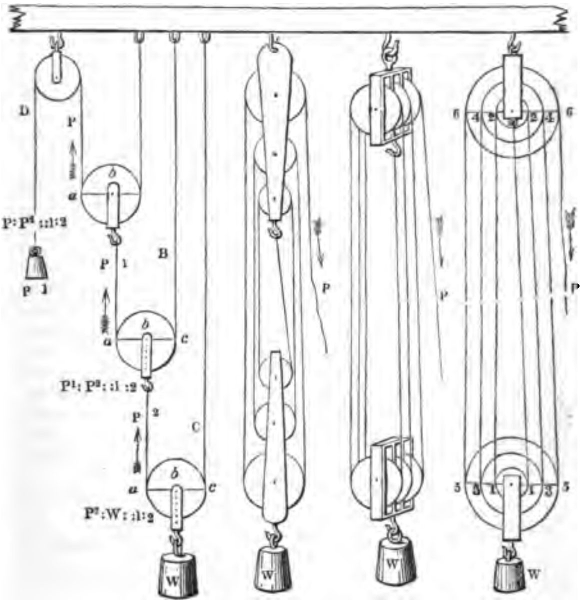
$$\begin{aligned} P : W &= cd.\ \cos.\ \alpha : cd.\ \sin.\ (\pi - 2\alpha) \\ &= \cos.\ \alpha : \sin.\ 2\alpha \\ &= 1 : 2\sin.\ \alpha \end{aligned}$$

Thus, in proportion as α is diminished P must be increased. If $\alpha = 30°$, then must $P = W$, and if $\alpha < 30°$ $P > W$, if $\alpha = 0°$; *i. e.* if the direction of the cord be horizontal P will be infinite. Hence it is evident why no force whatever can stretch a long cord perfectly tight.

§ 120.

A *system of pulleys* is a number of pulleys either all moveable, or both moveable and fixed acting in concert.

Fig. 58. *Fig.* 59. *Fig.* 60.



$$P : W \quad 1 : n; \quad P = \frac{W}{2n}$$

$$\begin{array}{l}
P \;\, : P' = 1 : 2 \\
P' \,: P'' = 1 : 2 \\
P'' : W = 1 : 2 \\
\hline
P \;\, : W = 1 : 2^3
\end{array}$$

$$P = \frac{W}{2^n}$$

A *system* may consist, as in *fig.* 58., of a number of single moveable pulleys, in which case one pulley only acts on the others. *A condition of equilibrium will be obtained when the forces are to each other as* 1 *to that power of* 2 *which equals the number of moveable pulleys*, or, the power may be found which will hold any given weight *in equilibrio*, by dividing the weight as many times by 2 as there are moveable pulleys in the system.

First Exp. Procure three moveable pulleys, such as are represented in *fig.* 58., arrange them so that the weight W may act on the lowest pulley, I, and the power P on the highest, III, in a vertical direction, and a weight of 1 oz. will prove a counterpoise to 8 oz. In this figure we have

$$\begin{array}{lrcl}
\text{in pulley III,} & P : P' & = & 1 : 2 \\
\text{—} \quad \text{II,} & P' : P'' & = & 1 : 2 \\
\text{—} \quad \text{I,} & P'' : W & = & 1 : 2 \\
\therefore & P : W & = & 1 : 2.2.2 \\
 & & = & 1 : 2^3 \\
 & & = & 1 : 8
\end{array}$$

If there be n pulleys,

$$P : W = 1 : 2^n$$

$$\text{and } P = \frac{W}{2^n}$$

In the ordinary arrangement, as shown in *fig.* 59., both moveable and fixed pulleys are employed, and generally of each an equal number: they are arranged in blocks, either side by side, or one above another. The weight is attached to the moveable block, the fixed one serving only to apply the power in the desired direction. The former alone diminishes the power required, dividing the weight equally among all the pulleys; every moveable pulley requires the application of a force equal to half the weight in order to produce equilibrium (§ 119.), consequently the following statical law obtains in this system of pulleys: *In a state of equilibrium the power equals the weight divided by twice the number of moveable pulleys.*

Let there be n pulleys in the moveable block, then each bears $\frac{W}{n}$ of the weight, and as the power need be only half the weight in order to bring about equilibrium, it follows that

$$P = \frac{W}{2n}$$

$$\text{or } P : W = 1 : 2n$$

i. e. a power of 1 lb. will counterbalance a weight of $2n$ lb.

Second Exp. Procure such a block as is represented in *fig.* 59., carrying three pulleys, then each supports $\frac{1}{3}$ W, and the power which will retain this in equilibrio, by the above law, $=\frac{W}{2.3}=\frac{1}{6}W$, *i. e.* 1 oz. power will counteract a weight of 6 oz.

A new kind of pulley may here be mentioned, invented by an Englishman of the name of White (*fig.* 60.). It consists of several cylinders of unequal diameters, but all having one common axis and performing their revolutions in the same time. In the lower block the diameters of the cylinders must be to each other as the odd numbers 1, 3, 5, &c., whilst those in the upper or fixed block are as the even ones, 2, 4, 6, &c. In these machines the universal law of mechanics holds good, that as force is diminished velocity increases; in other words, that the power must pass over a space greater than that gone over by the weight by as many times as the weight exceeds the power.

Call the velocity of the weight, or the height through which it is to be raised, h, then the power will pass over a space

in *fig.* 58. $= 2^n h$
in *fig.* 59. $= 2n\,h$.

Both classes of systems are in common use, to elevate great weights by the exertion of a small power.

Why is the arrangement in *fig.* 59. more general than that in *fig.* 58.?

c. The Wheel and Axle.

§ 121.

The *Wheel and Axle* (*fig.* 61.) consists of a cylinder A, which revolves about its axis, having a wheel, R, of larger diameter fixed to it. The weight, W, is suspended by a cord, which is wound round the axle; the power, P, exerted by men, animals, or inanimate matter, acts on the periphery of the wheel. Instead of the whole wheel, one or more radii may be used as arms, on which the force may work perpendicularly. A single crooked arm, like that in *fig.* 62., is called a *winch*, several are called spikes.

Fig. 61.

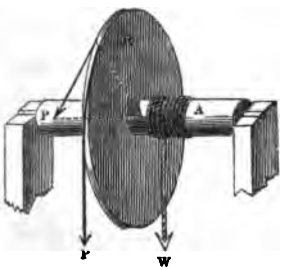
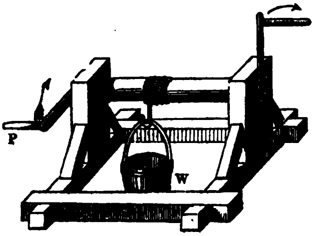


Fig. 62.



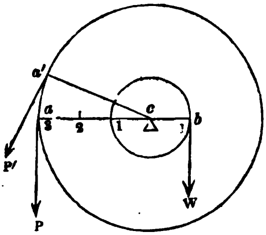
If the wheel turn horizontally, it is called a *Capstan*; if vertically, a *Windlass.*

§ 122.

Equilibrium obtains in this machine between the power and the weight, when the former is to the latter as the radius of the axle to that of the wheel.

Draw from the fulcrum c (*fig.* 63.), right lines, *c b* and *c a* or *c a′*, to the points on which the weight and power act, and you obtain a lever of two unequal arms, in which the distance of the weight, W, from the fulcrum, *c*, is the radius, *c b*, of the axle, and that of the power P or P′, is *c a* or *ca′*, the radius of the wheel. Now, the statical moments of such a lever are,

Fig. 63.



$$P : W. = bc : ac$$
$$P.\ ac = W.\ bc$$
$$P = \frac{bc}{ac}.W$$

$$P.\ ac = W.\ bc,$$
$$\text{or } P'.\ a'c = W.\ bc\ ;$$
$$\text{whence } P : W = cb : ca,$$
$$\text{or } P' : W = cb : ca'.$$

In this machine the power acts perpendicularly, as in the fixed pulley; its direction, however, need not necessarily be parallel to that of the weight.

Exp. Let there be a small model of a wheel-and-axle, like that represented in *fig.* 61., then hang a weight to the cord coiled round the axle, and to that which passes in an opposite direction, over the wheel, suspend another weight; and equilibrium will be established when the power is such a multiple of the weight that acts on the axle as the diameter of the axle is of the diameter of the wheel; thus, *ex. gr.*, if the diameter of the axle be 1 inch, and that of the wheel 10 inches, a power of 1 lb. will be an equipoise to a weight of 10 lb.

When the Power has turned the wheel round once, the axle has revolved once also. The Power has gone over a space indicated by the circumference of the wheel, whilst the Weight has been raised a height equal to the perimeter of the axle; hence it appears that the velocity of the Power is to that of the weight as the circumference of the wheel to that of the axle; *i. e.* as their radii; or, as before, the space described by the Power exceeds that described by the Weight as many times as the former is contained in the latter.

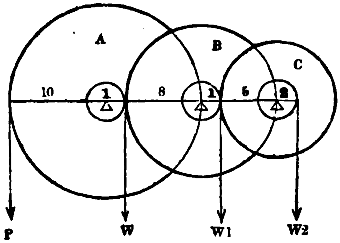
§ 123.

When several wheels work one upon another, we have those various modifications of *Wheel-and-Pinion Work* that are to be met with in compound machines. To aid this action, the surfaces of the wheels are furnished with corresponding elevations and indentations called *Teeth*, or *Cogs*.

The statical moments of the forces are inversely as the product of the radii of the axes to the product of the radii of the wheels in which they work; and as the number of teeth which the wheels and leaves that the pinions carry must be as the peripheries of the wheels and axles, their statical moments are also *inversely as the product of the leaves of all the pinions to the product of the teeth of all the wheels.* — The spaces passed over by the power and weight are proportional to their products.

Combinations of wheel-work are used either by means of a small force, to set heavy loads in motion, or else by the application of considerable power, to attain a high velocity. In the former case, the power must be applied to the circumference of the first wheel, its pinion must transmit the motion to the teeth of the second wheel, and so on, in such a manner that the weight shall be raised by the axle of the last wheel. In the other case this arrangement must be reversed, and the power must be exerted on the first pinion, in order to communicate a greater velocity to the last wheel.

Fig. 64.



$$\begin{array}{l} P : W = 1 : 10 \\ W : W' = 1 : 8 \\ W' : W'' = 2 : 5 \\ \hline P : W'' = 2 : 400 \\ \qquad = 1 : 200 \\ P = \dfrac{W''}{200} \end{array}$$

The following example may serve as an illustration: —

	In.	Teeth.
Let the wheel A (*fig.* 64.) have a radius	= 10 and	100
its pinion - - -	= 1	10

Then $P : W = 1 : 100$
$= 10 :$

Wheel B a radius - -	= 8 and	64
Its pinion - - -	= 1	8

Then $W : W' = 1 : 8$
$= 8 : 64$

Wheel C a radius - -	= 5 and	50
Its pinion - - -	= 2	20

Then $W' : W'' = 2 : 5$
$= 20 : 50$

Supposing all the wheels to act on each other as represented in the figure, it follows that —

$$\begin{array}{rl} & P : W = 1 : 10 \\ & W : W' = 1 : 8 \\ & W' : W'' = 2 : 5 \\ \hline \text{whence} & P : W'' = 2 : 400 \\ & \qquad = 1 : 200 \end{array}$$

A force of 50 lb. acting on P would therefore sustain a weight W″ of 200 × 50 = 10000 lb. In order to raise this load a height of 1 foot, the power must describe a space of 200 feet.

On the contrary, if the motive power be applied at W″, in the time C makes one revolution, B would perform 6¼, and A 40 revolutions. The circumference of wheel A is 5 times as great as that of C; the circumfernce of A must therefore revolve with a velocity of $5 \times 40 = 200$ times that of C.

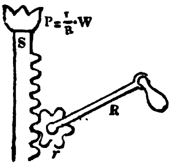
The applications of such compound wheel-work are extremely numerous and familiar; it will suffice to instance mill-work, watches, odometers, &c.

A wheel or pinion may also be made to work in a straight bar carrying teeth on its edge, called a *rack*, to which it will thus communicate a progressive motion.

A simple combination of this kind is represented in *fig.* 65.; it consists of a handle R having a pinion r fastened perpendicularly to its axis, the teeth of which work in the rack S. The cogged wheel, on being pressed by the handle, raises the rack on which the load is placed. The operation of this machine obeys the same laws with the wheel and axle. For let R represent the length of the handle, and r the radius of the wheel, then

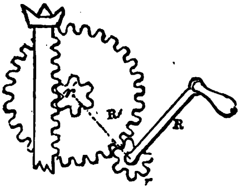
$$P = \frac{r}{R} . W.$$

Fig. 65.



In *fig.* 66. the pinion r of the winch R does not communicate immediately with the rack, but first turns a large toothed wheel, R′, carrying a pinion r', which works the rack S, and thus elevates the load. In this case let R be the length of the winch, R′ the radius of the wheel, r that of the first pinion, and r' that of the second; then,

Fig. 66.



$$P = \frac{r r'}{R R'} W.$$

$$\begin{array}{l} P : W' = r \ : R \\ W' : W = r' \ : R' \\ \hline P : W = r r' : R R'; \end{array}$$

$$\text{whence } P = \frac{r r'}{R . R'} . W$$

Let $r=r'=1$ inch, $R=10$ inches, and $R'=5$ inches, then $P=\frac{W}{50}$ *i. e.* 1 lb. power would be a counterpoise to 50 lb.; and by exerting a force of 100 lb. weights of less than 5000 lb. may be raised.

d. The Inclined Plane.

§ 124.

An *inclined plane* may be used as a mechanical power. It serves either to support, or to move upwards or downwards any weight that may be placed upon it with the exertion of a force less than the absolute weight of the resisting body.

The power P may be applied, either:

1. In a direction parallel to the longer side A C of the inclined plane, as *fig.* 67.

Fig. 67.

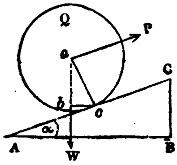


Fig. 68.

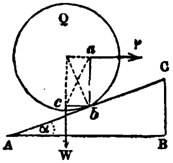
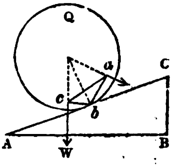


Fig. 69.



$P : W = BC : AC$ $P : W = CB : AB$

$P = W . \sin. a$ $P = W . \tan. a.$

2. or parallel to its base A B, as *fig.* 68.
3. or parallel to neither of these lines, as *fig.* 69.

In all these three cases the statical momentum between the power and the weight may be deduced according to the laws of the lever.

§ 125.

On the inclined plane, A C, *fig.* 67., let there be any body Q. By virtue of terrestrial gravitation it has a tendency to descend perpendicularly in the direction *a* W, *i. e.* its absolute weight acts as the resistance to be overcome in the direction *a* W. But the body Q rests on the point *c* in the plane; this point then becomes the fulcrum on which the weight presses.

Next let a power P act on Q in the direction *a* P, parallel with the plane A C, and we obtain the action of a right-angled lever, if we draw from c its fulcrum, the arm $c\,b$ of the lever, perpendicular to the line of direction of the weight W, its other arm $c\,a$ being drawn at right angles to the direction of the power P. The statical moments then are

$$P.\,ac = W.\,bc$$
$$\text{or } P : W = bc : ac$$
$$\text{and since } \Delta\, abc \text{ is sim. to } \Delta\, ABC$$
$$P : W = BC : AC$$
$$\text{whence } P = W\,.\,\frac{BC}{AC} \text{ or } = W\,.\,\frac{\text{perp.}}{\text{hyp.}}$$

i. e. when the power works in a direction parallel to the inclined plane, the power and weight will be in equilibrio when the former is to the latter as the perpendicular of the plane is to its hypothenuse.

Expressed trigonometrically the law stands thus,

$$P : W = \sin.\,a : 1$$
$$\text{or } P = \sin.\,a\,.\,W$$

The space described by the power is to that described by the weight as the hypothenuse of the plane to its perpendicular.

Next, suppose the power P, which keeps the body Q on the inclined plane AC, *fig.* 68., to be exerted in the direction a P, parallel to the base A B; the action of a right-angled lever will be at once recognised if you draw from b, the fulcrum, the arm $b\,c$ of the lever at right angles to the line of direction of the weight, and the other arm in like manner to the direction of the power P. In this case the statical moments become

$$P\,.\,ab = W\,.\,bc$$
$$\text{or } P : W = bc : ab$$

and from the similarity of the triangles $c\,b\,a$ and C B A,

$$P : W = BC : AB;$$
$$\text{whence } P = W\,.\,\frac{BC}{AB} \text{ or} = W.\,\frac{\text{perp.}}{\text{base}}$$

i.e. when in an inclined plane the power acts parallel to the base, equilibrium will obtain if the power be to the weight as the perpendicular of the plane is to its base.

This law expressed trigonometrically is

$$P : W = \tan.\,a : 1.$$
$$P = \tan.\,a\,.W.$$

In this case the distance passed over by the power is to that passed over by the weight as the base of the inclined plane to its perpendicular.

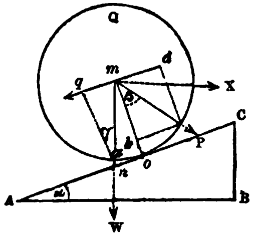
Lastly, let the power P, *fig.* 69., act in a direction parallel neither to the hypothenuse A C, nor to the base A B, but in the direction a P, then the statical moments will be, according to the form of the right-angled lever $c\,b\,a$,

$$\mathrm{P}\,.\,ab = \mathrm{W}\,.\,cb$$
$$\text{or } \mathrm{P} : \mathrm{W} = cb : ab$$

As in this case $\triangle\, c\,b\,a$ is not similar to $\triangle$ C B A, the proportion between the arms of the lever can no longer be expressed by the sides of the plane.

All the three sides may be generally expressed as follows:

Fig. 70.



$$\mathrm{P} = \mathrm{W}\,\frac{\sin.\,\alpha}{\sin.\,\beta}$$

Represent the weight W of the body Q, *fig.* 70., by the line $m\,n$, which being considered as a force may be resolved into the components $m\,o$, $m\,q$ (§ 25.), $m\,o$ is lost as being the pressure on the plane, and $m\,q$ is the remaining weight or force with which Q tends to move in the direction $m\,q$; but $m\,q = m\,n\,.\,\sin.\,\gamma$, and, from the similarity of the triangles $m\,n\,q$, CAB, $\gamma = \alpha$; whence the relative weight $mq = \mathrm{W}\,.\,\sin.\,\alpha$; whilst the pressure which the body exerts on the point o of the inclined plane $= m\,o = \mathrm{W}\,.\,\cos.\,\alpha$.

Now let a power $\mathrm{P} = m\,a$ act on the centre of gravity m of the body in the direction $m\,a$, at any angle $\mathrm{P}\,m\,o = \beta$ against $m\,q$; resolve P or $m\,a$ into two forces, one of which $m\,b$ coinciding with $m\,o$ is perpendicular to the hypothenuse of the plane, and is consequently lost in the pressure $m\,o$ that is exerted on the plane; the other component md acting parallel with the plane, and $= \mathrm{P}\,.\,\sin.\,\beta$. In a state of equilibrium, therefore,

$$\mathrm{W}\,.\,\sin.\,\alpha = \mathrm{P}.\,\sin.\,\beta\,;$$

$$\text{whence } P = W . \frac{\sin. \alpha}{\sin. \beta}$$

If P be parallel to A C, as in the direction *m d*, then $\beta = 90°$, and $\sin. \beta = 1$, in which case

$$P = W . \sin. \alpha = W . \frac{\text{perp.}}{\text{hyp.}} \quad \text{(Case I.)}$$

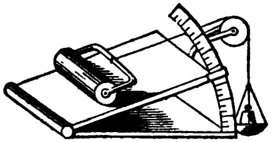
If P be parallel to the base A B, as *m x*, $\beta = 90 - \alpha$, so $\sin. \beta = \cos. \alpha$, whence

$$P = W . \frac{\sin. \alpha}{\cos. \alpha} = W . \tan. \alpha = W . \frac{\text{perp.}}{\text{base}} \quad \text{(Case II.)}$$

From what has been shown above it follows as a general rule, that the saving of power effected is greater the less the perpendicular height of the plane is as compared with its hypothenuse; that the maximum saving is when the power acts parallel with the inclined side; and that the power must be increased in proportion as the direction in which it is exerted approaches a perpendicular to the inclined plane.

Exp. The laws of equilibrium in the inclined plane may be experimentally verified by laying a ball or cylinder, of known weight, upon a smooth board inclined at different angles; a cord passes from the cylinder over a pulley in such a manner that it may act parallel to the inclined side, parallel to the base, or at any angle that may be required; the other force is represented by a weight placed in a scale attached to the other end of the cord. See *fig.* 71.

Fig. 71.



The chief use of the inclined plane, considered as a mechanical power, is in the construction of convenient ways for the transmission of heavy loads, which are to be raised to some particular height by but a small expenditure of power.

The ladders used by brewers to lower casks into a cellar; the inclined plane down which a ship is launched. Application in fortification, in the inclines by which you ascend a rampart. In what direction is a force applied most advantageously upon an inclined plane? When must the horses exert the greatest force in drawing a load up-hill?

e. The Wedge.

§ 126.

The *Wedge*, *figs.* 72. and 73., may be resolved into two inclined planes BGC and AGC, having a common base GC. BC, AC are called the *sides* of the wedge, BA the *back*, and GC the *length*.

Fig. 72.

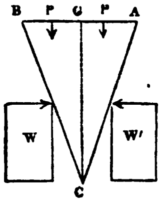
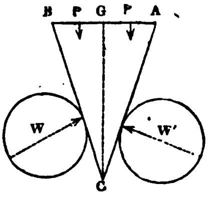


Fig. 73.



As a mechanical power it is used thus: the force P acts on A B, the back of the wedge, so as to drive it between the two bodies w and w', and overcome the resistance they offer.

This resistance may present itself perpendicularly to the length of the wedge, as *fig.* 72., or perpendicularly to its sides, as *fig.* 73.

In the former case the statical momentum of the wedge resembles that of an inclined plane when the power is parallel to the base; therefore,

$P:(w+w')$ or $W=(BG+GA)$ or $BA:GC$, *i. e.* in this case equilibrium obtains when *the power is to the resistance of the bodies as the back of the wedge is to its length.*

In the other case the wedge resembles an inclined plane, on which the power acts parallel to the longer side: its statical momentum will therefore be:

$P:(w+w')$ or $W=(BG+GA)$ or $BA:BC$ or AC, *i. e. when the power is to the resistance of the bodies as the back of the wedge is to its side.*

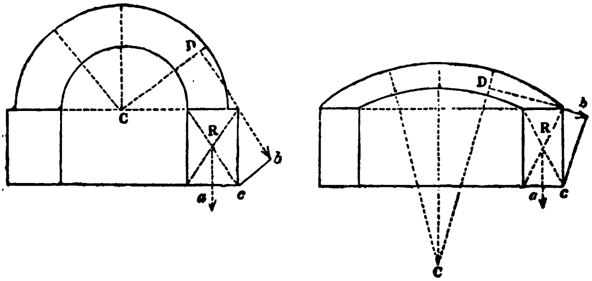
The universal law of mechanics holds good with regard to the wedge, that the spaces described by the weight and power are inversely to each other.

The difference between the inclined plane and the wedge is, that the former is fixed, and the power is exerted in communicating motion immediately to the weight; whilst the latter, being itself set in motion by the power, overcomes the resistance, or raises the weight with a smaller expenditure of force than would otherwise be requisite. The remark made with respect to the inclined plane holds true as to the wedge also, that, as a general rule, its effectiveness increases as the length of the back compared with that of the side is diminished.

Besides its use in cleaving bodies, this mechanical power is also employed to press substances together, and to raise loads to small heights. — The coins used for adjusting cannon; knives, axes, chisels, nails, cutting-tools, &c. act on the principle of the wedge; so also do the stones in an arch, which are merely truncated wedges.

That the stones of an arch act on this principle may be thus shown: — The centre of gravity D (*fig.* 74.) of half the arch presses at right angles from the side D C of the wedge, *i. e.* in the direction D *b*, against the buttress R, producing a tendency in the mass R to revolve about the point of support *c*. The effect, therefore, resembles that of a crooked lever *a c b*, on the arm *a c*, of which, and at the point *a*, the weight of the buttress acts vertically downwards from its centre of gravity R, in the direction R *a*; the pressure of the arch is, meanwhile, exerted in the direction D *b*, at the point *b*, of the other arm *b c*. Now, that the mass of the buttress shall resist the pressure of the arch, the following condition of equilibrium must be secured between these two forces, viz.: —

Fig. 74.



$$R . ac = D . bc;$$

and as *a c* is half the breadth, B, of the buttress, it follows further, that

$$R . \tfrac{1}{2} B = D . bc.$$

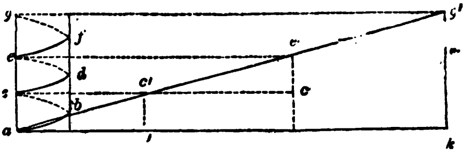
On comparing the dimensions of the arms of the two crooked levers in *fig.* 74., it is obvious that, with equal weights, or masses of arched-work to support — even if the stones of which such arch is constructed be longer and more acute wedges — the pressure on the buttress will be much greater when the arch is low than when it is high.

f. The Screw.

§ 127.

A *screw-line* is generated by winding a right-angled triangle about a cylinder, so that the longer of the two sides containing the right angle shall be perpendicular to the cylinder (see *fig.* 75.).

Fig. 75,



The hypothenuse will describe the screw-line about the exterior of the cylinder. Every part of this line which goes once round the periphery of the cylinder, as *abc*, *cde*, *efg*, is called a *spire* of the *worm* or *thread*, and each such part requires an equal portion of the triangle, $aa' = cc' = ee'$. The distance of one spire from the next, $ac = ce = eg$, and which coincides with the height of the corresponding portions of the triangle, $a'c' = c'e' = e'g'$, is called the *breadth* of the screw.

If the screw-line consists of one continuous elevation, the cylinder forms a *male screw;* if, on the contrary, the interior of the cylinder contains a winding cavity, it is called a *nut* or *female screw.* The combination of both these parts constitutes the mechanical power known as the screw.

§ 128.

The working of the screw may be deduced from that of the inclined plane, the weight resting on the inclined face of the thread, whilst the power acts on the surface of the cylinder parallel to the base of the screw. This will exactly correspond with that case of the inclined plane in which the power acts parallel to the base. Equilibrium, therefore, obtains when *the power is to the resistance as the distance between two threads of the screw is to the circumference described by the point to which the power is applied;* thus,

$$P : W = h : 2r\pi$$

$$\text{and } P = \frac{h}{2r\pi} . W$$

W being the weight or resistance to be overcome, h the height or distance between the threads, and r the radius of the screw.

The space described by the power is to that described by the resistance as the circumference of the screw to the distance between two of the threads.

Power is therefore economised the nearer the threads of the screw approach each other, *i. e.* as the distance between them is diminished as compared with the perimeter of the cylinder. In applying this mechanical power, either the *screw* or the *nut* by its progressive motion may be made to exert the power or overcome the resistance, according as the other revolves about its axis.

The purposs to which the screw is applied are very various; it serves,

1. *To compress bodies closely together;* as, for instance, in the different kinds of presses, machines for stamping coins, vices, &c.

2. *To raise weights;* as when it is used in adjusting guns; and,

3. *To produce uniform and delicate movements,* as in the case of divider compasses, and the micrometer screws of different instruments.

§ 129.

The screw is occasionally used to set in motion sun-and-planet wheels, the worm of the screw working into the teeth of that wheel which drives the gear, and by its revolution compelling the wheel to turn about its axis. Such a combination is termed an *endless screw.*

This machine is employed partly to raise loads—as the adjustment of cannon—but principally to show delicate and uniform circular movements in various instruments; *ex. gr.*, Ramsden's Dividing Machine.

§ 130.

With the screw we conclude the catalogue of the mechanical powers. On comparing the remarks made on each of these simple machines, it will be found that all the laws which regulate equilibrium in each, either depend immediately on what was established with regard to the lever (§ 114.), or may be deduced from the principal laws of motive powers (§ 109. Obs.); we are consequently constrained to regard *the lever* as the *fundamental machine.* Again, as all compound machines are merely combinations of the simple ones, it follows that *the lever furnishes the fundamental law for the entire theory of machinery.*

The simple machines do not meet the requirements of actual life, because their dimensions would of necessity be cumbrous and colossal if great weights were to be maintained in equilibrium, or set in motion by their means; in most cases, too, no material could be found possessing the strength requisite for such enormous machines; and lastly, their very unwieldiness would present an insuperable obstacle to their utility. To meet this exigency the mechanical powers are variously combined, several being used in a single piece of machinery.

In every machine, however complex, the proportions between the power and the weight may be so adjusted as to secure equilibrium, by applying the law which holds good as to every simple machine, *viz.* that *in a condition of equilibrium between the power and the weight, the distances described in equal times by these forces must be inversely as the forces themselves*, or, in other words, *their velocities will be inversely to the forces.* If the machine be set in motion and the spaces measured which the weight and power respectively pass over in equal times, the ratio of the one to the other is immediately known.

> Suppose, for example, that a certain weight is raised one inch by means of a machine, whilst the power goes over 16 inches, either in a right line or a circle, or, in fact, in any direction whatever, it follows that the power must be the sixteenth part of the weight to maintain equilibrium.

Thus have the principal laws of both simple and compound machines been developed and illustrated; the reader is, however, reminded that we have supposed the machines themselves to be destitute of weight, mass, &c., and that no impediment stands in the way of motion other than the exact forces by which equilibrium is brought about. In practice it is found that, in addition to the mere weight of the material composing the different parts of the machinery, there are many other causes that check motion; viz. the *various Impediments to Motion.*

VI. OF THE IMPEDIMENTS TO MOTION.

§ 131.

Under this head are included those existing causes which counteract all terrestrial motion whatever, opposing to it some certain resistance which ultimately overcomes and annihilates it. By experience we learn that these causes are partly the properties of the bodies themselves, partly the resistance of the medium through which they act, and partly, also, the adhesion of those surfaces that come in contact whilst the machine is in action. There are two, distinguished as the principal impediments to motion, which are frequently found acting conjointly; viz. *Friction*, and *the Resistance of Air* or *Water.*

These impediments are frequently very observable, exerting a material influence on the moving of great weights, and especially affecting the performances of machinery. In practice it is of high importance that we should be able to estimate accurately the amount of these obstacles, as also that we should know by what means they may be best overcome. Although we may be acquainted with the chief causes that impede the actions of our machines, it will yet happen that, owing to the varied modes in which these causes co-operate, we shall have to rely almost solely on the results of empirical investigations, without being able to draw accurate deductions from theory.

§ 132.

Friction is principally of importance in the moving of solid bodies on one another.

It arises from the nature of the surfaces coming in contact, and the adhesion excited between them; these are never perfectly smooth, consequently the eminences on the one surface sink into the declivities of the other. Friction is said to be *sliding* when the parts of one surface move parallel with the other, as when the pin of a wheel runs in its socket, or an axle in the nave of a wheel; it is known as *rolling* when a round body turns about the surface of some other body, so that fresh points of both the surfaces are continually brought in contact with each other.

The force which it requires either to abrade the elevated parts or to raise them from the depressions in which they had become lodged is the *measure of the friction*. This is usually found by trying what part of the weight of the moving body must be exerted to maintain its equilibrium, or to overcome the resistance arising from this source. The fraction expressing this ratio is denominated the *co-efficient of the friction*.

> The co-efficient expressing the friction of any body is most readily found by laying the body which is under investigation upon a horizontal plane; a cord passes from the body parallel to the plane, and over a pulley, having at its other end a scale in which weights are to be placed until the body begins to move. The weight that sets the body in motion gives the numerator; that of the body itself will be the denominator of the fraction representing the co-efficient required.
>
> Or thus: Lay the body on the plane, and incline it gradually until the body begins to slide or roll. The co-efficient of its friction will be equal to the tangent of the angle of the plane's inclination. Whence, by § 125, a power W. sin. a, will hold the sliding body *in equilibrio;* whilst the pressure which it exerts on the inclined plane = W., cos. a: put c for the co-efficient, then,

$$c.\ W.\ \cos.\ \alpha = W.\ \sin.\ \alpha$$

$$\text{whence } c = \frac{\sin.\ \alpha}{\cos.\ \alpha} = \tan.\ \alpha$$

Amontons, Muschenbrök, Coulomb, Morin, and other philosophers, have experimentally investigated the various degrees of friction in different substances.

§ 133.

According to investigations and experiments made on this subject the following results have been arrived at:—

1. The amount of friction in the case of surfaces of the same material is very nearly proportional to the pressure perpendicularly exerted on such surfaces.

2. With equal pressure and similar surfaces, friction increases as the dimensions of the surfaces are increased.

3. It increases in proportion to the time that contact continues, a maximum, however, being attained after a certain time. (a) For this reason, friction is greater when a body is passing from a state of rest to one of motion, than even when it is in the latter state.

(a) When metal works upon metal the maximum is gained almost immediately, with wood upon wood, after the lapse of a few minutes, but with wood on metal not for some days.

4. The rougher the surfaces in contact the greater will be their friction.

5. A regular velocity has no considerable influence on friction; if the velocity be increased the friction is certainly greater, but that depends only on secondary causes; viz. the generation of heat, and the resistance of the air.

6. Similar substances excite a greater degree of friction than dissimilar; a considerable difference, however, may be remarked between the latter. If the weights be small, the hardest bodies excite the least friction (a); in most cases the co-efficient for sliding friction will range from $\frac{1}{7}$ to $\frac{1}{3}$.

(a) The steel pivots of pendulums or balances generally move on hard polished stones.

Iron and steel produce the least friction on brass or bronze.

Cast-iron on bronze - - -	$c=0{\cdot}147$
steel on bronze - - - -	$c=0{\cdot}152$
iron on bronze - - - -	$c=0{\cdot}172$
bronze on iron - - - -	$c=0{\cdot}161$
bronze on cast iron - - -	$c=0{\cdot}217$
cast iron on cast-iron - - -	$c=0{\cdot}152$

Bronze sockets for the iron axles of wheels.—Application of these experiments in the pivots and pivot-rests of machinery.

7. Friction is diminished by smearing the surfaces of bodies with some unctuous matter, by which means the inequalities are made slippery, and dissimilar bodies are brought into closer contact.

In the choice of unguents, such as are of a *viscous* nature must be selected for those bodies that have large inequalities to fill up, and such as are *fluid* for substances of greater density. When wood slides on wood, tallow, tar, or solid grease of some kind or other is used, but for metals, liquid oil is preferred. Of late finely pulverized plumbago has been employed, as being a superior agent in lessening friction, particularly in carriages with iron axles, and in large machinery. By the aid of suitable unguents, friction may be reduced to one-half or one-fourth of its original amount.

8. A rolling motion produces far less friction than a sliding one, partly because the surface in contact is but a line, or a point if the bodies be of a globular form, and also because the irregularities of the moving body fit like the teeth of a wheel in those of the other body; the progressive motion is thus accelerated without that abrasion taking place which occurs when the materials slide one upon the other.

Babbage adduces the following experiment to show the constant decrease of friction. A roughly chiselled block of stone of 1080 lb. weight was drawn from the quarry, on the surface of the rock, by a force of 758 lb.; it was then laid on a wooden sledge and drawn about upon a wooden floor, the force required for this purpose being 606 lb.; when both the wooden surfaces were smeared with tallow, 182 lb. sufficed; and, lastly, when the load was placed on wooden rollers of 3 inch diameter, the force was reduced to 28 lb. In these different cases the co-efficient was accordingly, 0·7, 0·56, 0·17, 0·026, or the proportion was, as

$$1 : 0·8 : 0·25 : 0·037$$

In carriages there exists a double friction, viz. the rolling friction of the felloes on the road, and the slipping friction of the axle in the nave. Let r be the radius of the axle, R that of the wheel, P the resistance to the axle, p the power acting in the opposite direction, equilibrium will be obtained if $p\,\mathrm{R} = \mathrm{P}\,r$, so $p = \frac{\mathrm{P}r}{\mathrm{R}}$. Whence it follows that r should be as small and R as large as possible; with a minimum p, however, R must not be made of too great dimensions. The carriage follows most easily when the axle is almost level with the breast of the horse in the position of drawing his load, *i. e.* when the traces make with the horizon an angle of little more than 11°.

The most serious obstacle to be overcome in draught is invariably the friction of the wheels upon the road, and this evil is at its maximum when the carriage travels on such bad road that the wheels sink so that not merely the lower surface of the felloes but even their sides produce friction. Add to this that when the wheels sink into the soil, it becomes much compressed by them, this can be compensated by the felloe of the wheel being tapered in the form of an inclined plane; it is therefore important that the wheels should have such a breadth that they may not readily sink deep into loose earth. According to Babbage, on well-paved roads the friction amounts to $\frac{1}{71}$st part of the load; on gravelled roads it is nearly double, or $\frac{1}{35}$th; when a fresh layer of earth has been spread it is $4\frac{1}{2}$ times as much, or $\frac{1}{16}$th. According to another set of experiments, an ordinary stage-waggon, with its load weighing together 1000 lb., requires in loose sand a force of traction of 250 lb. ($\frac{1}{4}$); on a fresh made road, 140 lb. (about $\frac{1}{8}$); on a common by-road, 106 lb. (between $\frac{1}{9}$—$\frac{1}{10}$); on hard dry meadow ground, 40 lb. ($\frac{1}{25}$); and on a good dry high road, 25 lb. ($\frac{1}{40}$). According to Bevan's experiments, the force of traction required on different roads expressed in parts of the load Q will be for hard macadamised roads, 0·0305 Q.; on such a road in a muddy state, 0·0345 Q.; on hard dry meadow ground, 0·04 Q.; on hard clay, 0·053 Q.; on common, not macadamised, roads, 0·106 Q.; on roads fresh covered with stones, 0·143 Q; and in loose sand, 0·204 Q.

These experiments, and the efforts of modern times to promote industry by transporting its products from one place to another with the greatest possible speed, giving wings as it were to commerce, led to the important invention of railroads, the full effects of which it will require another generation to develope. The principal object aimed at is the greatest practicable diminution of friction; this has been sought by laying down iron rails, on which the iron wheels of the carriages are made to run. Flachat's experiments give the latest results that we are yet in possession of. According to him, friction on a railroad varies from $\frac{1}{180}$th to $\frac{1}{240}$th, whilst on common roads it may be estimated at from $\frac{1}{15}-\frac{1}{20}$, so that a horse upon a railway can draw a load from 10 to 12 times as great as on common roads. Gerstner reckons the force of traction of 1000 lb. on common roads at from 40 to 120 lb., *i.e.* $\frac{1}{25}-\frac{1}{8}$; and on railroads 6 lb. or $\frac{1}{166}$.

If now we collect in one view what has been advanced with regard to the different cases of friction, the most important point will be seen to be,

1. That friction increases,
    - *a.* As the pressure or weight is increased.
    - *b.* As the roughness of the surface is increased.
2. That friction may be diminished,
    - *a.* By lessening the weight of the body in motion.
    - *b.* By mechanically reducing the asperities of the sliding surfaces.
    - *c.* By converting a sliding motion into a rolling one.
    - *d.* By applying some suitable unguent.

In order practically to determine the amount of friction in any particular machine, hang weights to it until the machine just begins to move. In this way the amount of friction is obtained when motion commences, and (by 3) it has been shown that this always exceeds the friction which obtains after all parts of the machine have for some time been in play.

§ 134.

Whilst, as has been shown above, friction is one of those natural evils which invariably consume some portion of the power expended in producing any motion whatever, and by consequence in many points of view it may be regarded as highly prejudicial, yet if viewed in another relation we shall find that it is of incalculable advantage. Were it not for friction, nothing could be nailed, screwed, or tied together, or even securely held in the human hand. All solid bodies, including ourselves, and all other animated beings, would occupy a stable position only upon a perfectly level plane, and all matter would present a smooth surface, like water, whilst the entire mass would endeavour to descend to the lowest possible point and there settle. — Instance the unsteady gait of a man walking on smooth ice. The bricks ranged one upon the other forming a wall that defies the violence of the storm, nay, the solid rock itself, which the hurricane fails to shake, are indebted to friction for their firm position ; deprive their materials of this property, and the slightest push would level them with the ground. — The revolution of a cart wheel is owing to the rolling friction excited by the inequalities of the road; in descending a hill it is frequently necessary to lock the wheel in order to increase the friction.

§ 135.

In machines there exists a second impediment to motion, which we have now to notice, and one which has a considerable analogy to friction, viz., the *rigidity of ropes;* this exerts considerable influence, particularly in the working of pulleys. The degree in which this cause affects the motion of machinery has been but partially ascertained by experiment, whilst many external circumstances, such as the dampness or dryness of ropes, the quality of their material, the mode in which they are prepared, &c., all which exert great influence on the working of the ropes, do not admit of being estimated with any great degree of accuracy. That which is most important with regard to this property may in general terms be set forth as follows:—

The resistance which the stiffness of the rope offers to motion is directly proportional to the weight that acts on it, and also to the thickness of the rope; it is further in inverse proportion to the diameter of the wheel or axle about which the rope is coiled; *i. e.* the greater the weights to be set in motion by means of the machine, and the stronger the ropes, the greater will be the resistance which this stiffness offers; but the greater the wheel, axle, or roller over which the rope runs, the less will this impediment be.

It has been experimentally proved that a weight of 1lb., hanging on an axis of one inch in diameter by a cord 1 line thick, requires a resistance of $\frac{1}{2}$ an oz. or $\frac{1}{32}$ of a lb., on account of the stiffness of the rope. Assuming this law as the basis of our calculations, and keeping in view the proportions named above, we may find the friction of the ropes in any machine. Suppose, for instance, that from an axle 8 inches in diameter there is suspended a weight of 1600 lb. by a rope 16 lines in thickness; then, by the above rule, if the rope were 1 line thick, and the roller 1 inch in diameter, the resistance would be $\frac{1600}{32} = 50$ lb.; but since the former is 16 lines thick, it would, if wound round a one-inch axle, be $50 \times 16 = 800$ lb.; the axle, however, is 8 inches in diameter, whence the exact resistance is $\frac{800}{8} = 100$ lb. Or, in general terms, if d be the diameter of the axis in inches, d' that of the rope in lines, and W. the weight to be moved, the resistance of the friction will be

$$R = \frac{d' \, W}{32d} \text{lb.}$$

The friction of new ropes exceeds that of old ones which have been some time in use; moisture also affects it considerably, greatly increasing the resistance of the ropes.

§ 136.

Nearly all the motions of solid bodies take place either in the air or water. A body moving in either of these fluids must consequently thrust before it, or move aside out of its path, those particles that it meets. From this source there arises a perpetual loss of motive power, the motion of the mass decreases, and will at length cease altogether. Universal experience has proved that this resistance increases with the velocity of the moving body, and with the extent of surface that it presents to the fluid; the figure of the body can however be made to counteract materially the resistance offered by the particles of the fluid.

The greatest mathematicians have laboured to calculate the amount of this resistance, and have discovered that it depends:

1. *On the density of the medium in which the motion takes place.*

The resistance met by a body moving in water is therefore greater than that sustained by a similar body traversing the air; and since water is about 800 times denser than air, the resistance of the former exceeds by 800 times that of the latter. Again, as the atmosphere is not at all times and in all places of uniform density, this resistance will occasionally vary.

2. *On the extent of surface of that section of the body which is perpendicular to the direction of its motion; the external figure of the surface materially affects the results obtained by this rule.*

The same body moving with uniform velocity in the same fluid sustains a resistance proportional to the surface opposed to the fluid. The resistance is consequently greater when the body moves with the broad end foremost than when the taper end goes first.—The management of a ship's rudder. The blade of a sword moves more easily through the air if turned edgewise than if the flat side be presented.

Wedge-shaped and pointed bodies of equal section with bodies presenting a flat surface meet with less resistance than the latter from the medium through which they pass, because, acting on the principle of a wedge (§ 126.), they separate and move aside the particles of the fluid. This principle is clearly exhibited to us in the forms of birds and fishes, from whose figure we have borrowed the shape of our ships. A double-convex form is given to the bob of a pendulum to lessen the resistance of the air. Balls meet with about one-fourth less resistance from the air through which they are fired, than cylinders of equal diameters to them would sustain: this is owing to the cause just assigned.

3. *On the velocity with which the body moves, since the resistance increases as the square of the velocity.*

Suppose, *ex. gr.*, a body moves with a velocity of 2 feet in a second, it must then drive forward a column of the fluid 2 feet long, with a velocity of 2 feet, and consequently overcome a resistance which is equal to the product of the mass into the velocity (§ 28.), viz. $2 \times 2 = 4 = 2^2$, *i. e.* a resistance proportional to the velocity squared. Again, let the body have a velocity of 3 feet, it must then push on a column of the fluid 3 feet in length with a velocity of 3 feet; the resistance, found as before, amounts to $3 \times 3 = 9 = 3^2$, which is proportional to the square of the velocity. Or, expressing it generally; put M = the mass of the fluid moved aside in 1 second of time, and v the velocity with which this is effected, the resistance will be $M\,v$. Now obviously M becomes greater the greater the velocity. Let s be the surface of the plane moving against the fluid, the mass of the fluid driven on by such plane in 1 second will therefore be $M = s\,v$. Substituting this value of M as the measure of the resistance $s\,v\,.\,v = s\,v^2$.

Bodies of similar figure and of the same material overcome the resistance of the medium through which they move the more easily, or, in other words, with a smaller expenditure of motive power, the greater their size. Cannon-balls of the same material will, accordingly, sustain a diminished resistance from the air the more massive their dimensions. Now, suppose the diameter of an iron ball to be 1 inch, that of another 3 inches, their masses will be $1^3 : 3^3 = 1 : 27$, and if they have equal velocities their motive power will be in the same ratio. By 2. in this §, the resistance of the medium in which the motion occurs is proportional to the dimensions of the plane that moves against it; in spheres this plane varies as the squares of their respective diameters. In the present example the 1-inch ball meets a resistance of $1^2 = 1$, whilst the 3-inch ball sustains one of $3^2 = 9$. It is further plain that the ratio of $1 : 9$ is less than that of $1 : 27$; the greater ball, therefore, is proportionally less retarded in its motion than the less, in fact only one-third as much in the instance now under review. Suppose that both the balls had a velocity of 1000 ft., and the resistance of the air lowered the velocity of the lesser 300 ft., the greater ball would have lost only 100 ft. of its velocity.

A solid and a hollow ball (or two solid ones of different substances, as iron and lead) of equal diameters with equal velocities experience an equal resistance from the air. As, however, the motive powers of two such balls are as their masses or weights, and the weight of the hollow ball is less than that of the solid one, the loss of motive power sustained by the former will exceed that sustained by the latter. Consequently, in artillery, one object to be attained is to make the missiles of the densest possible substance.—The pressure of bullets in a mould, and the forging of cannon-balls.

The laws laid down under the three preceding heads coincide, however, with experience only under certain mean velocities; with very high velocities, as with extremely low ones, the resistance decreases; in the former case, chiefly because the compressed fluid resumes the place made void by the moving body with a velocity less than that of such body, so that a vacant space exists behind any mass that is projected with extreme velocity. This is the case with the motion of any body through the atmosphere with a velocity exceeding 800ft.

§ 137.

The resisting forces that have been explained in the foregoing paragraphs cause the actual motions of bodies to deviate materially from the laws established in the earlier part of this volume, and in accordance with which these very forces themselves have been illustrated.

Thus it has been said, that every body on which a momentary force acts will move in a right line, and persevere in such motion; actually, however, this does not happen. The impediments with which we have now become acquainted oppose themselves to this motion, lessen the velocity, and, after retarding the body for some time, bring it to a state of rest.

The descent of bodies falling freely is not a uniformly accelerated motion, but is one constantly approximating to it. When the resistance of the air equals the force of the motion produced by terrestrial gravitation, this is fully seen; hence it follows, that all bodies do not fall from the same height to the earth in equal times: the denser bodies more easily overcome the resistance offered, because, with the same volume as others of less density, and with the same amount of resistance from the medium, they have a greater motive power.

A coin, for example, falls more quickly to the ground than a scrap of paper; but in an exhausted receiver (as will be shown presently), where the resistance of the air is nearly destroyed, both bodies descend with equal rapidity.

The motion of a body on an inclined plane must evidently deviate still more from accelerated motion, since friction is added to whatever impediment may arise from the medium in which the body is placed.

A pendulum or a ball on a concave circular superficies (§ 37.), which bodies, but for these modifying causes, would be a kind of *perpetual motion*, lose this property solely through the operation of these forces.

A pendulum is found to oscillate much longer *in vacuo* than in the ordinary atmosphere.

The resistance of the air is further the occasion of the paths of projectiles deviating from the line, which theory leads us to expect. It is always extremely difficult accurately to calculate all the impediments that present themselves, both because the density of the atmosphere itself varies from time to time, and also because the agitated air, or wind, exerts a considerable influence on the path of any projected body. Experience has generally confirmed, what is asserted in the theoretical rules laid down in § 136., that the height or distance to which any body is projected invariably falls short of what calculation had promised, and that the deficiency increases as the velocity: thus balls propelled by force of gunpowder, especially bomb-shells, whose path approximates most nearly to the parabolic curve, fail in accomplishing the distances that theory prescribes to a greater extent than missiles projected by a smaller force.

According to Vega, a 4-pound cannon-ball, which passes over a space of 6437 ft. in the air, would *in vacuo* traverse 23,226 ft.; and Hutton states that he had found the resistance of the air to amount to 100 times the weight of the ball, when a 6-pound ball was projected with a velocity of 2000 feet.

§ 138.

Having considered the resistance offered by that fluid medium, in which all motion transpires, as deducting from power, and prejudicial in various ways, we must now look at the other side of the question, and shall find that, as in the case of friction, this very peculiarity is of most important advantage, nay, it may even be reckoned amongst our greatest physical benefits. On this property depends the possibility of swimming—both the natural swimming of fishes and the more artificial movements of man; without this property in water the rudder would be useless to the ship and the paddle-wheels to the steamboat. But for the resistance of the air, not a bird could fly; the parachute of the air-balloon and the Congreve rockets would alike be of no service; nay, lastly, but for this property of the atmosphere the gentlest showers would descend upon the earth with the violence of hail storms, with a constantly accelerated velocity, causing ceaseless havoc. The following experiment may serve as an illustration:—

Exp. The *water-hammer* is used to show the force with which water strikes when *in vacuo*. This instrument consists of a glass tube containing a small quantity of water, free from air and closed at both ends. Generally speaking, it is 10 or 12 inches long, and about half an inch wide, tapers off to a point at the upper end, and expands at the lower into a hemispherical form. Hold the tube with the small end downwards, then invert it briskly, and the water will strike with a report as if a solid body had hit the glass, and would indeed break it unless it were made of a suitable thickness.

SECTION IV.

NON-ELASTIC FLUIDS.

I. PROPERTIES OF BODIES IN THE FLUID STATE.

§ 139.

The characteristic of *fluids* is, that their particles constitute a whole held together with so slight a degree of cohesion, that a very small force suffices to separate these particles, which reunite as soon as the force that parted them is withdrawn. No arrangement or structure of the component particles, such as obtains in solids, has been discovered in bodies of this class. (§ 86.)

§ 140.

This particular condition of matter is explicable on the assumption that the atoms of a fluid are at such a distance from each other, that their attractive and repulsive forces are held *in equilibrio*. (§ 14.) The molecular forces of each individual particle are therefore equally exerted around the centre of its mass; whence it follows that their particles are spherical; an opinion which experience confirms, as the smallest observable portions of fluids, *i. e. drops*, are of a globular form.

First Exp. If a mixture of quicksilver and diluted sulphuric acid be shaken, the former will divide itself very distinctly into small globules.

Second Exp. Drops of water on a stick covered with *semen lycopodii*, or with dust will assume a form obviously spherical.

Drops of rain, mist, hail — shot-casting.

§ 141.

Whilst the cohesion in the interior of a fluid may be estimated at *nil*, on account of the extreme facility with which it may be displaced in all directions, yet on the surface the cohesive principle is more strongly developed; *i. e.* the particles of a fluid are less easily displaced at its surface than elsewhere. This fact may be thus explained: below the surface the attraction and repulsion of the particles are mutual, whilst those on the surface retain only their repulsive force, thus opposing a resistance to the entrance of any foreign body.

First Exp. A needle will float on the surface of smooth water, but sinks if but a portion of it be covered with the fluid.

Second Exp. Gold and silver leaf, iron filings, and similar substances, will swim on the top of smooth water.

§ 142.

Bodies of this class are compressible in so low a degree, that very great force alone will produce any sensible reduction in their bulk; for all mean pressures they may therefore be regarded as incompressible. If the power by which they have been compressed be withdrawn, they immediately resume their original dimensions; thus proving that in a low degree they are both compressible and elastic bodies.

Compare § 7., the second experiment under which § may show that bodies in the fluid state sometimes possess a high degree of elasticity.

§ 143.

All fluids do not possess in an equal degree the primary distinction of their aggregate form, viz. perfectly free motion among their particles. As there exist various degrees of solidity, so do there of fluidity also. That fluid which is both the most abundant and of the greatest value—viz. water—possesses a mean degree of fluidity; sulphuric acid and ether are among the bodies possessing the property under review in the highest degree, whilst the fatty oils are much less fluid than water. Many bodies are in a medium state between solids and liquids; such are honey, syrup, tar, &c. This condition depends on such a juxta-position of the particles that the inequality between the molecular forces begins to be observable. These facts show that there exists a regular gradation from the solid to the fluid state. (§ 14.)

II. PRESSURE OF FLUIDS.

§ 144.

Fluid bodies, equally with solid, are subject to the operation of gravity. The latter, in consequence of the great cohesion of their parts, obey as a whole the attraction of gravitation, descending towards the earth in a mass; whereas each individual atom of a fluid is acted upon by this force, without the force of cohesions impeding its yielding thereto.

The next peculiarity to be observed in fluids as resulting from this property is, that all their particles press on those immediately below them. This pressure must consequently vary as the depth.

§ 145.

On account of the extreme facility with which the particles of a fluid move in any direction, the lower ones, unless prevented by some other body, yield to this pressure, *i. e.* they have a tendency to flow in all directions. Hence, on a horizontal plane, there would result a uniform extension of the fluid; on an inclined plane, all the particles would move, agreeably to the universal law of gravity, directly towards the lowest point.

> To this cause is to be attributed the movement of all the water on the surface of our earth, and the collection of it which is formed at its lowest parts.

§ 146.

If a fluid be contained in any vessel, or if it be surrounded by solid walls, the tendency of the lower particles to flow in all directions is counteracted by the resistance which the solid boundary offers. Hence fluids always assume the figure of the vessel that contains them.

> Ponds, seas, lakes. — On this property depends the founding of different metals, *ex. gr.* cannon and musket balls.

§ 147.

Every fluid contained in a vessel will exert a pressure on the sides of such vessel, in consequence both of the force of gravity, and also in consequence of its readiness to yield to pressure in all directions, *i. e.* not merely in the direction in which gravity acts, but also sideways and upwards: this pressure is always equal to that which is exerted by the particles of the fluid perpendicularly over it, *i. e.* to their weight.

§ 148.

It appears, therefore, that a fluid in a vessel exerts a twofold pressure, viz. on the base, and on the sides of the vessel.

The *pressure on the base* is in the direction of gravity. The amount of this pressure, as was shown in the preceding §, is not always to be estimated according to the mass of the fluid, but every particle on the base exerts a pressure proportioned to the height at which the fluid stands in the vessel.

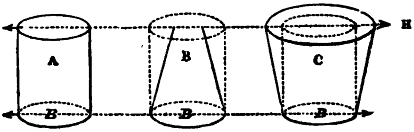
> Suppose a fluid to stand in a vessel at such a height, that 100 drops are arranged vertically one below the other, the lowest drop will exert on the base a pressure equal to the weight of the 100 drops.

The pressure of a fluid on the base of the containing vessel is equal to the weight of a column of the fluid, whose base is equal to the base of the vessel and its height to the distance of the surface of the fluid from the base.

From this law it follows, that it is only in vessels of uniform width that the pressure on the base will be exactly equal to the fluid's weight; in such as taper upwards the pressure will be greater, whilst in such as diminish downwards it will be less than the weight.

Hydrostatic paradox. Let the three vessels, A, B, C, *fig.* 76., have equal bases, B, and let the fluid stand at an equal height, H, in all the three; let the absolute weight of one cubic unit of the fluid be $= w$, then the common pressure on the base $= BH \cdot w$.

Fig. 76.



The following experiment may serve to confirm the truth of this hydrostatic paradox.

Exp. Weigh a cylindrical vessel accurately in a balance. Fill it up to a certain point with water and weigh again. The additional weight will give both the weight of the mass and its pressure on the base. Now pour out some of the water, and immerse a body of such a size as will again raise the water to its original level, such body being suspended by a string, so that its weight may not press on the scale. The fluid will be found capable of counterpoising the same weight as at first, proving that its pressure on the base is not lessened by diminishing its bulk, provided that the base and height remain unaltered.

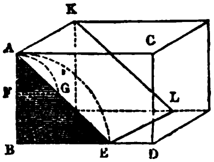
§ 149.

The pressure of a fluid on the *side* of the containing vessel, in like manner, depends not on its mass, but on the height at which it stands. Every drop touching the side of the vessel presses laterally on the point of contact with a force equal to the weight of all the drops above it to the surface of the fluid. The lateral pressure must therefore vary as the depth.

The pressures on all the sides of a vessel are together equal to the weight of a column of fluid whose base is equal to the sum of the sides, and depth equal to the depth of the centre of gravity of the surface. In the case of an upright vessel, it is therefore equal to the weight of half the fluid contained. On any portion of the side the pressure is equal to a column of fluid having that portion of the side *for its base, and the depth of the centre of gravity of the surface for its height.*

As every particle of the fluid exerts a pressure horizontally on the sides proportioned to its depth below the surface (§ 148.), this may be represented for every particular point in the side by a horizontal line equal to the depth of the point below the surface. Let *fig.* 77. represent the vertical section of a vessel filled with fluid to the brim, AB the depth of the fluid, then the lateral pressure at F = FG or FA, at B = BE or BA. In like manner the pressure may be deduced for any point in AB; whence the horizontal pressure for the whole line AB may be expressed by the Δ ABE. But every line in that side parallel to AB sustains an equal pressure; whence the entire horizontal or lateral pressure borne by the perpendicular side ABKG = the weight of a triangular pyramid ABELKG of the fluid. Put d = the depth AB, its breadth b = AK, and the weight of a cubical unit of the fluid = w, then the pressure P on the side ABGK will be

Fig. 77.



$$P = \frac{d^2 . b}{2} . w.$$

If the side be a square,

$$P = \frac{d^3}{2} . w.$$

If the side be no rectangular figure, let A represent its area, and the horizontal pressure on any one of its sides, whatever the form will be

$$P = \frac{A . d}{2} . w = \cdot 5 \ A d w.$$

It is required to express the entire lateral pressure on all the sides of a cube filled with a fluid?

If the base of a rectangular vessel be a square, whose side = 1 foot, the height of the vessel = 2 ft. express the weight of the water such a vessel would contain when full: what will be the pressure on the base? what the pressure on one side? what the entire pressure on the base and sides?

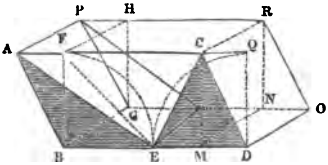
§ 149. 1 cubic foot, or 1728 cubic inches, of water weighs 62 lb. or 1000 oz. in round numbers.—If greater accuracy be desired, it may be as well to add that the weight of 1 cubic foot of water at a temperature of 62° Fahrenheit is 62·3232 lb.

What will be the amount of pressure exerted by water on a flood-gate 3 ft. wide, the water standing at the height 6 ft.?

Important application of the above formulæ in the case of flood-gates, sluices, locks in canals, &c.

Let a vessel have slanting sides, as shown in the sectional plan, *fig.* 78.: the horizontal pressure on the sides may be estimated in like manner as when the sides are perpendicular. The pressure on the line $AB = \triangle ABE$, and that on $CD = \triangle CDE$, but both triangles have equal altitudes and bases with the right-angled $\triangle FBE$, and are therefore superficially equal to it.

Fig. 78.

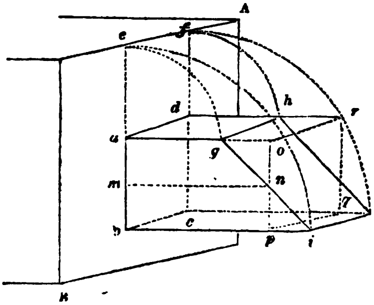


The side A B, making an obtuse angle with the base, sustains a vertical pressure downwards equal to the weight of the prism A F B G H P of the fluid, whilst the side C D, which stands under the acute angle, has a similar pressure, upwards, equal to the weight of the prism C M D O R N. Now the pressure and counter pressure, or resistance, of the side of the vessel are equal to each other; therefore the base sustains a pressure on the line M D represented by the square M C Q D. Thus an intelligible explanation is afforded of the phenomena presented by the hydrostatic paradox in the preceding §; in the vessel, spoken of above, that widened gradually upwards, the overplus of the mass of the fluid was supported by the sides, whilst in that which tapered upwards the resistance of the sides compensated for the deficiency in the fluid's mass.

Lastly, to express the pressure on any portion, *abcd*, of the side A B (*fig.* 79.), find it first for the line *ab*. According to the law above, the pressure on $a = ag = ae$, that on $b = bi = be$; therefore the pressure on $ab =$ trapezium *agib* $=$ rectangle *aopb*, whose height $mn =$ the distance of the middle of the line *ab* from the surface of the fluid. This is further true of every line in the side *abcd*, parallel to *ab*; consequently, the total pressure sustained by *abcd* will be equal to the weight of a column of the fluid *abcdrqpo*. If now the surface of this portion of the side $abcd = A'$ the distance of the surface of the fluid $= me = mn = d'$ and w again be put for a cubical unit of the fluid, then its horizontal pressure will be,

$$P = A'd'w.$$

Fig. 79.



That point in the side of a vessel through which the resultant of the pressures of the fluid passes is called the *centre of pressure.* If the side of the vessel be perfectly rigid, a sufficient pressure applied inwards at this point, will balance the internal pressures of the fluid, just as a support applied at the centre of gravity of a body will alone sustain the body. The tendency of the forces above and below, or on either side, to produce rotation about this point are equal in opposite directions.

Importance of this principle in the construction of flood-gates and of casks.

§ 150.

If the particles of a fluid are prevented from yielding to the downward pressure exerted upon them, as is always the case when a fluid is contained in a vessel, then, in addition to the pressure on the sides, every particle *presses upwards with a force proportioned to the number of particles above it.* The following experiment may serve to demonstrate this fact:—

> *Exp.* Provide a glass tube, cut off smooth at the ends, both of which are open: let a thin metallic disc, that fits accurately, down the tube, and attach it by means of a thread to the lower end, so that it may serve as a valve; now immerse the tube in water, and at a certain depth it will be found that the metal fails to retain its closed position,—because the upward pressure of the fluid overcomes the downward pressure of the metal.— This pressure is equal to that of a column of water having for its base the metal disc and

for its height the distance of the disc from the level of the fluid, *i.e.* it is equal to the weight of water that insinuates itself into the tube. — Fill the tube with water to the same height as that of the water in which it is plunged; and the pressure from beneath on the metal being thus counteracted, the tube will sink in the water, on account of the weight of the glass.

§ 151.

Bodies immersed in a fluid are pressed by it in all directions with a force increasing as the depth.

Exp. Tie a small bladder to a glass tube, fill it, and also a small portion of the tube, with some coloured fluid; plunge the apparatus into a vessel of water, and the fluid will be observed to ascend in the tube, and that in proportion to the depth of the bladder below the surface of the water.

This pressure is equal to the weight of a column of the fluid having for its base the surface of the body, and for its height the distance of the centre of gravity of the body's surface from the surface of the fluid.

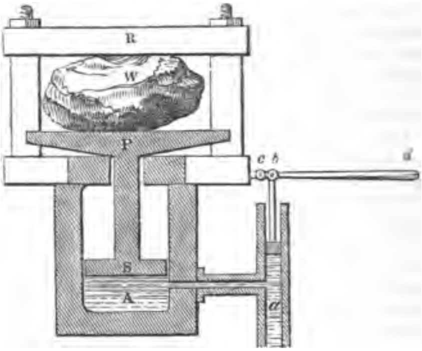
Empty bottles, if closely corked and sunk to a considerable depth in the sea, are sometimes compressed by this means, and sometimes have the corks thrust in. Captain Scoresby relates that the wood of a boat, which had been drawn by a whale 1000 feet below the surface of the sea, in consequence of the pressure, became so much more dense that it would not float again, even after it had been dried. If a cubic foot of sea-water weigh 72 lb., required the pressure on a surface of 1 sq. foot at a depth of 1000 feet.

§ 152.

If a pressure be exerted by means of any external force whatever, against the upper surface of a fluid, it becomes immediately communicated throughout the entire mass; *i. e.* every particle of the fluid transmits the same pressure in all directions to the contiguous particles. This effect is owing to the almost absolute incompressibility of fluids. (§ 142.)

The Hydraulic or *Bramah's Press* is a practical application of this principle to the useful arts. The main features of this apparatus are the following (*fig.* 80.): *a* is a narrow and A a large metallic cylinder having communication one with the other. Water stands in both the cylinders. The piston S carries a strong head P, which works in a frame opposite to a similar plate, R. Between the two plates is placed the substance to be compressed, W. In the narrow tube *a* is a piston, *s*, worked by a lever, *cbd*, its short arm *cb* driving the piston, while the power is applied at *d*. The pressure exerted by the small piston *s* on the water at *a* is transmitted with equal force throughout the entire mass of the fluid, whilst the surface at A presses up the piston S with a force proportioned to its area. For instance: if the cylinder *a* of the force-pump has a diameter of ½ an inch, while the great cylinder has a diameter of 200 inches, then the pressure of the water in the latter on the piston S will be 2 x 200 = 400 times that on *s*. Next suppose the arms of the lever to be to each other as 1 : 50, and that at *d*, the extremity of the longer arm, a man works with a force of 50 lb., the piston *s* will consequently descend on the water with a force of 2500 lb., and the head P will rise with a force of 400 x 2500 = 1,000,000 lb.; if, according to Nicholson's calculation, we deduct ¼ for loss of power caused by the different impediments to motion, by means of this machine one man would still be able to exert a force of ¾ of a million lb. This press is used in pressing paper, cloth, hay, gunpowder, &c.; also in uprooting trees, testing the strength of ropes, &c.

Fig. 80.



Exp. In a glass full of water, immerse the end of a Dutch-tear. As soon as the glass thread of the tear which projects above the level of the water is broken off the tail recoils and the tumbler is broken. This proves how instantaneously the concussion is transmitted through all the particles of the water.

III. PROPERTIES OF FLUIDS IN A STATE OF EQUILIBRIUM AND IN MOTION.

§ 153.

A fluid can be in a state of complete repose, or of hydrostatic equilibrium, only when the free motion of its atoms is checked, by all those on the surface of its mass being at an equal distance from the centre of the earth. Since gravity acts with equal force on all the particles of a fluid that are at equal distances from the earth's centre, the motive forces thus generated counteract each other, and the surface is of necessity *horizontal*, or parallel to the surface of the terrestrial spheroid. Hence is deduced the following law in hydrostatics: —

The surface of every fluid in a state of rest or equilibrium is invariably horizontal.

In accordance with this first law of equilibrium, which all fluids obey, they can accumulate or form masses only in such spaces — cavities or vessels — as by the resistance of their sides counteract the tendency of the fluids to extend their particles laterally.

The convex figure of the sea; artificial horizons; influence of adhesion to the sides of the vessel. (§ 53.)

If there be several fluids that do not mix in one vessel, they will arrange themselves, even if they be shaken, one above the other, as their specific gravities diminish; or, in other words, the densest fluid will be at the bottom of the vessel, and the rarest at the top, the strata when in a state of rest being all perfectly horizontal.

Exp. In a tall narrow glass, of cylindrical form, pour some quicksilver, water, oil, alcohol, and turpentine: shake together these ingredients, and on settling they will present the appearance spoken of above.

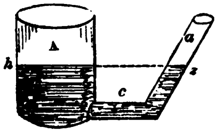
§ 154.

If two or more vessels containing fluid communicate with each other, equilibrium obtains when the fluids in both or all the vessels stand at the same level.

This hydrostatic law also rests on the fact, that the pressure of the atoms at equal depths is equal in all directions. — If the fluid stand at a higher level in one vessel than in the other, the particles in the former exert a greater lateral pressure on the channel of communication than those in the lower vessel can; these particles, therefore, are continually thrust upwards, till they exert an equal and opposite pressure, which does not happen till the columns are of equal heights.

First Exp. Let A and *a*, *fig.* 81., be two glass vessels communicating by the tube *c*. Pour water into one of them, and it will stand at an equal height in both, the surfaces lying in the horizontal line *h z*. Incline the apparatus, the position of the fluid will be changed, and equilibrium will

Fig. 81.

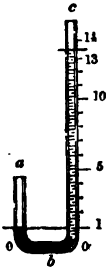


not again be obtained till the surfaces once more form one horizontal plane.

On this property depend several applications to the purposes of the arts of life. The level at which a fluid stands in opaque vessels, as boilers of steam engines, gasometers, stills, vats, and the like, is ascertained by means of a glass tube communicating with the larger vessel, and called a *gauge*. — The subterraneous water obtained by digging for springs; the water-conduits in towns; the flooding of water in cellars at high-water mark.—The effect of adhesion is perceptible on unequal tubes when one of them is of very fine bore.

When different fluids are in communicating vessels their surfaces will be horizontal, but they will stand at unequal heights. — *The levels of the different fluids will be inversely as their densities or specific gravities*, because the forces by which they are pressed down are proportional to these densities.

Fig. 82.



Second Exp. Into one end *a* of a glass tube (*fig.* 82.) pour some quicksilver, and at the opposite end *c* pour water. Equilibrium will obtain when the column *c* is about 13½ times higher than the column of mercury *a*.

§ 155.

If two vessels communicate with each other, and the height at which a fluid stands in one of them exceeds the height of the second vessel, then the fluid cannot stand at a level in these two vessels, but will overflow the shorter one until there remains in the taller vessel only so much of the fluid that its height shall be equal to the height of the shorter.

To this fact we trust in conveying water by means of conduits for the supply of cities; to this hydrostatic pressure also are to be attributed the springs and subterranean waters met with in mining operations.

In consequence of the pressure exerted by the column of fluid in the taller vessel upon the particles that overflow, they have a tendency to rise to a height equal to that of the column. If the aperture through which the fluid escapes from the lower vessel be contracted, it will be seen to rise in a jet to nearly the same altitude with the fluid in the tall vessel; the descent of the falling particles and the usual impediments to motion lessen the height to which the jet would otherwise reach.

Our artificial springs, as well as the artesian wells and natural springs, are but varied applications of this one law. If the aperture of a spring be in a thin stratum, according to Marriott's experiments, the squares of two jets J and j will be to each other as the difference of the columns by which they are pressed and the jets,

$$J^2 : j^2 = (C-J) : (c-j)$$

If, therefore, J and C are known, by means of this proportion we may calculate the altitude of a column c that would produce any required jet, viz.

$$c = j + \frac{j^2}{J^2}\left(C - J\right)$$

and if the column c be given, and the jet j sought,

$$j = \frac{J}{2(C-J)}\left(\sqrt{J^2 + 4\,c(C-J)} - J\right)$$

It has been found experimentally, that to produce a jet J, 5 ft. in height, C must be 5 ft. 1 in., and a jet 100 ft. high requires a column of 133⅓ ft.

The great fountain on the Wilhelmshohe at Cassel is nearly 200 ft. high; it is formed by a union of several pipes by which the jet acquires a diameter of nearly a foot. The fountain at Herrenhausen is 120 ft., and that at Nympfenburg 80 ft. high. The most remarkable natural fountains are the Geyser and the Strock, in Iceland, the former sometimes attaining the height of 212 ft. and the latter that of 150 ft.; volcanic agency is, however, in operation in the last-named instances.

The greater part of the crust of the earth, as far as our knowledge goes, consists of various strata of mountains ranged one above another. Many of these layers are of a loose and porous nature and are penetrated with clefts, whilst others are dense and free from flaws. Through the former the atmospheric moisture, rain, melting snow, and similar bodies find their way till they arrive at some solid stratum or subterranean channel where further progress is stayed. These impervious strata are generally arranged between those of an opposite character; so, lying below some more porous, they catch the water that permeates through the upper strata; when this has accumulated it finds vent, sometimes on the sides of hills, forming *springs* which not unfrequently begin trickling down from a considerable elevation. If, on the contrary, the harder and impervious rocks form the outer coating of the hill side, they will retain the water that has already found its way below the surface and compel it to take an underground course through the porous strata which form the mass of the hill. Such subterranean bodies of water are often of considerable extent. If the fluid makes its escape at some low point of these cisterns, wherever an opening or flaw presents itself a *natural jet d'eau* will be formed; if a channel be provided for it by means of boring, care being taken that the course thus made be continuous, so that the water may not escape laterally, but be led to a point below its original level and then brought to the surface, the water will spout forth in a jet; this constitutes an *artesian well;* the height to which the fluid rises will be equal to that of its source, allowing of course for the drawbacks mentioned above. *Fig.* 83. will perhaps render the above explanation more intelligible. It represents a vertical section of a portion of the earth's crust: let *a*, *c*, and *e* be strata, either porous or full of cracks, which permit the water to flow through, whilst *b*, *d*, and *f* are impervious to water; at *q* there will be a *natural spring*, at *i* a *natural jet d'eau*, and at *g*, *p*, and *l* artificial jets or *artesian wells*, rising, according to the laws of hydrostatics laid down above, to the respective heights *g h*, *p k*, and *l m*, the water not being allowed to come in contact with the loose soil through which the bore is made, but being brought in pipes to the surface; at *n*, on the contrary, there will be no fountain, but the water will ascend in the tube to about *o*.

Fig. 83.

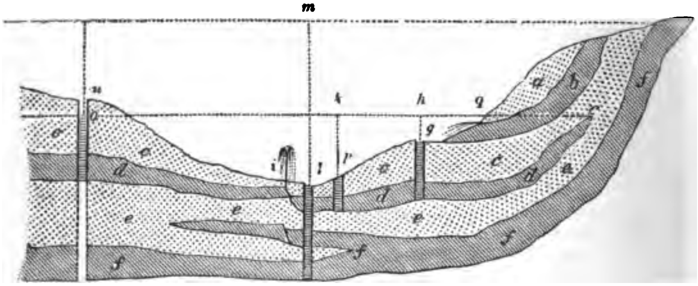
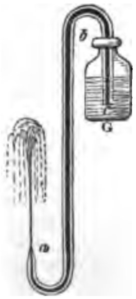


Fig. 84.



Exp. The following is the most

simple method of making a fountain: — Immerse a glass syphon tube, *a b c* (*fig.* 84.), in a vessel of water; exhaust the air by applying the mouth at *a*, and a jet will be produced proportioned to the fineness of the bore and the length of the tube.

§ 156.

If the shorter of two vessels that communicate with each other be closed at top, and both be filled with water, it is obvious that in the lower vessel it will not mount higher than the solid covering. In consequence of the law of equilibrium, explained § 154., the fluid has a tendency to rise to the same level as it stands at in the longer vessel, and this tendency is made apparent by the upward pressure against the lid or covering. *This pressure is equal to the weight of a column of the fluid (water), whose base is equal to the superficial content of the lid, and its height equal to the difference of the levels of the fluid in the two vessels.*

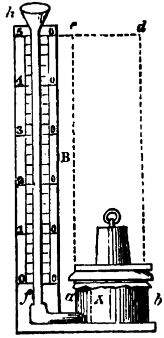
This pressure would be counteracted by placing above the lid a column of the fluid, such as has been just described.

This pressure increases as the horizontal surface of the shorter vessel and the difference of the levels, whatever may be the area of the base of the larger vessel. Hence it is possible, by means of a small quantity of fluid, to exert a very great pressure.—On this principle the *Hydrostatic Bellows* and the *Hydrostatic Press* act, and to some extent *Bramah's Hydraulic Press* (§ 152.), as in this last-mentioned machine the action of the lever is substituted for the hydrostatic pressure.—Most of the phenomena of *earth-falls* are referable to a similar cause.

1. *The Hydrostatic Bellows* consists of two strong discs of seasoned oak connected together by flexible leather in the same manner as the ordinary bellows, and a pipe *h f*, which communicates through the lower disc: if water be poured down the pipe *h f* the upper disc will be gradually elevated by the pressure of the water, which flows in until the leather becomes tense; after the bellows are filled with liquid, continue to pour into the pipe so that the water may stand in it at various heights in succession above the level of the upper disc; the amount of pressure exerted by the column of water may be tested by loading the bellows with weights.

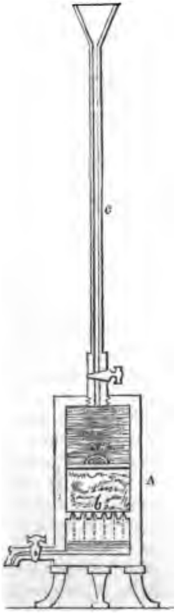
As 1 cubit foot of water weighs 62 lb., it follows that there will be a pressure of 62 lb. on a surface of 144 square inches if the column of water be 1 foot high. Suppose a section of the cylinder A = 144 inches, every foot of water in the column *hf* above the level *a b* presses against the under side of the upper disc with a force of 62 lb.; thus, if *a e* were 60 in., and the tube were filled up to the level *e*, the fluid would exert a pressure of 310 lb., which is equal to the weight of a column of water *abde* standing on A and reaching to the level *ed*. The experiments that can be performed by means of this apparatus will prove the accuracy of some of the laws given above.

Fig. 85.



2. The *Water-press* consists of a strong and wide metallic cylinder of but low dimensions, A, *fig*. 86., into the top of which a long funnel-shaped tube *c* is screwed. The cylinder is divided into three compartments by means of two sieve-like plates *a* and *b*, the upper being movable. Between the two plates is placed the body to be pressed. The compartment above *b* contains the fluid, water, spirit of wine, &c. that is to form the solution. If water is poured into the tube *c*, by the law before referred to the pressure on the fluid in the upper division of the receiver will be increased in the same proportion as the column of the fluid in *c* is increased; the fluid above *b* will then be driven through the body that lies between the plates.

Fig. 86.



3. *Earth-falls* are those natural phenomena which take place when portions of the earth's surface are elevated, then cleft asunder and let fall again, the space once occupied by solid ground becoming covered with water. These occurrences are explicable by the laws of hydrostatics, on the supposition that there exists a hollow space below the surface of the ground which is elevated; that underground channels communicate between this spot and higher regions; through these channels water is conveyed till the subterranean reservoir is filled, when such a hydrostatic pressure is exerted as the surface of the earth is unable to resist. This phenomenon often happens on a small scale; one of the greatest earth-falls on record was that which occurred at Arpino in 1801, when a tract of land 200 Neapo-

litan acres in extent sank to such a depth that the tallest trees were completely lost.

§ 157.

The most important motive power that acts on fluid bodies, and one which is always in operation, is the attraction of gravity.

This force produces in fluids a tendency to flow *in all directions* over a horizontal plane, and *downwards* on an inclined plane (§ 145.). In the latter case the particles in a body obey the laws of bodies falling on an inclined plane (§ 36.), which motion, however, like that of solids, is affected by various impediments.

The velocity of a stream is dependent principally on the *fall*, or inclination, of the bed over which it flows.

A pressure is likewise exerted by the upper particles on the lower, whence these latter, by virtue of the readiness with which the particles of a fluid move among each other, acquire a velocity greater than that of the particles on the surface: this velocity varies with the depth.

§ 158.

Fluids issue from orifices in the base or sides of vessels with a velocity proportional to the altitude; *i. e.* the force and velocity are greater, the higher the surface of the fluid from the orifice.

There exists a remarkable similarity between the motion of a fluid issuing from an orifice and the descent of solids *in vacuo*, as regards the absolute velocity acquired by these two classes of bodies.

The velocity with which every particle of a fluid issuing from an orifice, whether sideways, upwards, or downwards, arrives at the surface of the earth, is equal to that which it would have acquired by falling perpendicularly from the level of the fluid to that of the orifice. Hence it follows, that *the velocity of fluids issuing from an orifice is as the square root of the altitude.*

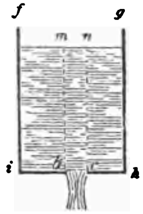
This law is true in practice only as far as it expresses the *ratio* of the velocity; the velocity corresponding to any particular altitude will not be equal to that thus calculated; it merely gives us the constant proportion subsisting between the actual and the theoretical velocities of fluids.

The greater or less density of a fluid does not affect the velocity with which it issues from an aperture, *i. e.* all fluids issue with equal velocities at equal altitudes, because they are solicited by gravity, which acts with equal force on the particles of all bodies.

Thus *ex. gr.*, quicksilver and water issue with equal velocities at equal altitudes, although the former is 13½ times heavier than the latter.

When the orifice is in the base, the height of the level above the base will give the pressure; when the orifice is in the side of the vessel, it will be the distance of the centre of the aperture from the surface of the fluid.

Fig. 87.



If the fluid be constantly maintained at the same level in the vessel $fihg$ (*fig.* 87.), supposing also that the particles of the fluid had no adhesion either to the sides of the vessel or to each other, then the portion mn, which is perpendicularly above the aperture $b\ c$, would fall towards $b\ c$ without any impediment, and would consequently, on arriving there, have acquired a velocity proportional to the height $m\ b$.

According to the laws of falling bodies (§ 33.), the final velocity acquired by a body falling freely is $v = \sqrt{2gh}$; now let v be the velocity with which the fluid issues, and $d = h$ the depth of the orifice below the surface of the fluid, and we obtain $v = \sqrt{2gd}$.

The velocities v and v', at different depths d and d', will be to each other as

$$\begin{aligned} v : v' &= \sqrt{2gd} : \sqrt{2gd'} \\ &= \sqrt{d} : \sqrt{d'} \end{aligned}$$

i. e. the velocities of their efflux are as the square roots of the depths.

Thus, suppose an aperture, a, at a depth of 9 feet, and another, b, at a depth of 4 feet, then the velocity at a will be to that at b as $\sqrt{9} = 3$ to $\sqrt{4} = 2$.

According to Eytelwein, the proportion between the theoretical and the actual velocities, when the thickness of the containing vessel is small, is as 1 : ·619 ; according to more recent experiments, as 1 : ·621 to ·645. Whence, changing the above formula for v, and adopting the mean ratio,

$$v = {\cdot}621 \times \sqrt{2gd}$$

For example, if the altitude of the water above an aperture were 16 feet, then the velocity would be

$$v = {\cdot}621 \times \sqrt{64 \times 16} = {\cdot}621 \times 32 = 19{\cdot}872 \text{ feet.}$$

§ 159.

If the vessel is not filled up as the water runs out, the level must necessarily descend gradually, and the velocity diminish at the same rate. Hence, *the velocity diminishes at the rate of a uniformly retarded motion.*

For instance, let the level at which a fluid stands be 25 inches above the aperture; observe the time it takes to descend to 9 inches, *i. e.* to fall through a space of 16 inches; in the second equal portion of time it will descend 7 inches, and arrive at the 9 in.; in the third portion of time it will fall 5 in., and arrive at 4 in.; then it will descend 3 inches, and arrive at 1 in. from the base, which it will reach in the next equal time.

If the fluid is not maintained at the same height, it will require twice the time for a vessel to empty itself that would have sufficed for the same mass to have flowed out, had the altitude remained unchanged.

§ 160.

To find the quantity of water discharged at an orifice, some particular time, usually one second, must be assumed as the unit. If now the velocity with which the water issues be known, *the quantity discharged in each second may be obtained by multiplying the velocity into the superficial content of a section of the aperture.*

Let a be the size of an aperture, v the velocity in 1 second, then the quantity of fluid discharged in that time is $M = a\ v$, and if the height h of the fluid be known, we have

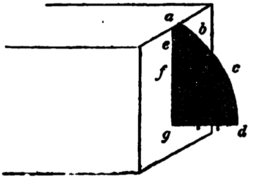
$$M = \cdot 621 \times a \sqrt{2\,g\,d.}$$

Let there be an aperture $\frac{1}{8}$th of a square foot, and let a column of water be maintained constantly at the height of 16 ft., it is required to find how many cubic feet of water will be discharged in a second.

$$M = \cdot 621 \times \tfrac{1}{8} \times \sqrt{64 \times 16} = 2\cdot 312 \text{ ft.}$$

Let $a\,g$, *fig.* 88., be an aperture in the side of a vessel, extending from the surface to a certain depth, or even to the base, in a perpendicular direction, it is clear that the water must issue thence with unequal velocities. According to § 158. the velocity increases with the depth, in the ratio of the square roots of the columns above the aperture. Put the velocity $e\,b$ at e, v, and its depth $a\,e = d$, the velocity $f\,c$ at f, v', its depth d', the velocity $g\,d$ at g, v'', and its depth d'', then it follows

Fig. 88.



$$v : v' : v'' = \sqrt{d} : \sqrt{d'} : \sqrt{d''}$$
$$v^2 : (v')^2 : (v'')^2 = d : d' : d''$$

The ratio thus obtained between the abscissæ d, d', d'', and their ordinates v, v', v'', constitutes the parabola (§ 41.), whence it appears that the water flows out in the figure *a g d c b a*, which is the section of a parabola, whose content is $\frac{2}{3} d \sqrt{d}$, d being the depth of the lowest point of the opening below the surface. Multiply this surface into the width of the opening, and the product will be the quantity of water discharged in 1 second.

Experience, however, invariably shows that a less quantity of water flows from an aperture than calculation by the above theorem would lead us to expect. The difference between the results may in part be accounted for by the mutual adhesion of the particles of water, and also that which they show towards the sides of the vessel; partly also by the theoretical laws of the friction of the fluid against the sides of the orifice; by the resistance of the atmosphere; but most of all by the contraction which takes place in the stream shortly after it issues from the aperture: to this the name of *vena contracta* has been given. This diminution in the circumference of the stream admits of the following explanation; not only, as was at first assumed, does the water perpendicularly above the orifice issue thence, but that which is round about it presses obliquely in all directions towards the aperture, endeavouring to escape from the vessel.

The water in flowing out attains its smallest section at a distance equal to about half the diameter of the opening, at these dimensions it continues for a short time, and then again expands. The degree of this contraction depends on the size and form of the aperture, the altitude of the fluid, and the thickness of the side of the containing vessel. For an altitude of from 4 to 12 ft., the sides of the vessel being also thin, the diminution in the section of the stream is about $\frac{5}{8}$ths, with thick sides $\frac{13}{16}$ths. If tubes of a conical form be employed, whose figure approximates to that of the stream, a much larger quantity of water will issue from a vessel in a given time. Venturi gave to a tube of this kind a length of 11 lines, the aperture at the mouth being $14\frac{1}{2}$ lines in diameter, whilst the end that fitted into the vessel had a diameter of 18 lines, and he then found the quantity of water that issued through the tube was $\frac{30}{31}$ of that which was required by theory. The quantity may be still further increased, by fixing to the tube already described a second widening outwards. With such an arrangement both Venturi and Eytelwein found that the quantity of water thus obtained, was half as much more as would have flowed in the same time without a tube.

In laying on water-pipes it is of extreme importance that we should be able to measure exactly the quantity conveyed in a given time by pipes of certain dimensions. The velocity in such a case will be retarded not only by the causes above specified, but also by the friction of the fluid against the sides of the conduit-pipes. According to Langsdorf, the velocity of water issuing from a pipe is

$$v=\sqrt{\frac{2gh}{1{\cdot}538+{\cdot}625\frac{l}{d}}}$$

l being the length and d the diameter of the pipe in inches, h the altitude of the column, and $2g$ the known measure of falling bodies in feet.

§ 161.

If a fluid issue from a lateral orifice in a horizontal direction. or, as is sometimes the case with springs, at any angle whatever upwards, then every particle of such fluid is to be regarded as a body projected in that direction with a velocity equal to that with which it issues from the orifice. The stream consequently describes a parabola.

As instances we may adduce the jets formed by fountains projected obliquely, and also the form of a stream issuing from an ordinary cistern.

§ 162.

A fluid in motion, water most commonly, *impinges with a certain force* on any body that may lie in its way. The general laws of impact hold good with respect to fluids, at least thus far, that their effects depend on the impinging mass and its velocity, *i. e.* on the quantity of water that strikes in one unit of time (*ex. gr.*, 1 second) against the surface of the opposing body, and on the velocity with which the collision takes place.

If the impact happen in a direction vertical to the surface of the body, its effect is equal to the pressure of a column of water, having for its base the surface impinged on and an altitude equal to that of the column which generates the velocity of the stream.

If the water impinges obliquely on the surface, the force may be resolved into two others, one parallel to the side of the body, and the other perpendicular to it. The latter alone is effective, and is proportioned to the square of the sine of the angle of incidence.

Let S represent the surface impinged on, v the velocity of water, then (§ 33.) the column generating this velocity $=\frac{v^2}{2g}$, so the force of impact $= S.\frac{v^2}{2g}.x$ lb. S, v, and g being expressed in pounds, and x standing for the weight of 1 cubic foot of water; *i. e.* thus, divide the square of the velocity of the water by four times the space fallen in the first second, and the quotient gives the altitude, which multiplied by the surface struck, gives the column of water whose weight is equal to the force of the impact.

But if the water impinge on the surface at any angle a, then by the same formula its effect may be calculated,

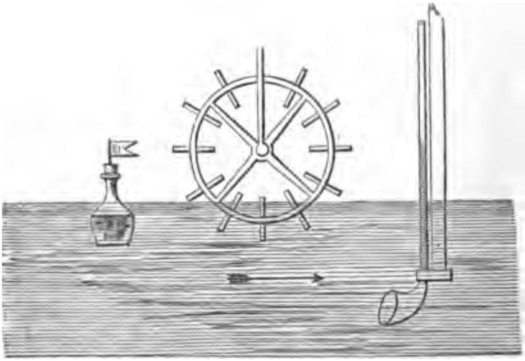
$$\sin.\, ^2a . \left(S. \frac{v^2}{2g} . x \right) \text{ lb.}$$

In flying-bridges the actual force against the sides may be resolved into two components, one of which coincides with and is cancelled by the cable that holds back the vessel, whilst the other is perpendicular to the direction of the rope by which it is drawn towards the bank. This component is most effective when the angles which the side of the boat makes with thé direction of the stream and the rope are complements the one to the other. It is found in practice, that a flying bridge goes most quickly, *cæteris paribus*, when the length of the rope is one and a half times the width of the stream.

The motive power of water is plainly a matter of extensive practical import from the number of machines driven by *water-wheels*. In the *undershot* wheel the water acts only by means of impact, whilst in the *overshot* its weight also contributes to create power,

To measure the velocity of a stream at its surface, hollow floating bodies are used, *ex. gr.*, a small glass bottle filled with water, that sinks just below the level of the current, and provided with a small flag that projects above the water (*vide fig.* 89).

Fig. 89. *Fig.* 90. *Fig.* 91.

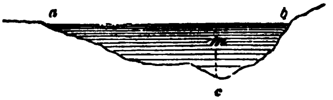


Or, instead of this contrivance, a small wheel may be used, furnished with float-boards (*fig.* 90.); this is placed at that part of the stream whose velocity it is desired to ascertain; the wheel must be immersed to such a depth that the whole surface of the boards may be perpendicularly struck by the stream. In this simple apparatus the friction of the axle is assumed to be *nil*, so the circumference of the float-boards will revolve nearly with the same velocity as the current flows; its amount is readily ascertained by observing the number of revolutions performed in a given time by the circumference of the float-boards.

To estimate the velocity at different depths, various instruments, called ***stream measurers***, are used. One of the simplest is *Pictot's tube*, *fig.* 91.; it consists of a tube bent nearly at right angles, expanding in a funnel shape; this is placed in the direction of the stream; the upper part above the water must be of glass. If the water be in a state of rest, the level within and without the tube will be equal; but if it be in motion, the impulse of the stream will cause a column of the fluid to ascend until the weight of this column becomes a counterpoise to the force with which the water is impelled. Since a column whose altitude is d indicates a velocity $= \sqrt{2gd}$, the velocity of the stream may hence be deduced at any desired depth.

By means of the stream measurer it has been discovered that the velocities vary at different points in the same transverse section of a stream. The velocity is found to be greatest in the middle, where the water is deepest, somewhere in m below the surface, *fig.* 92.: thence it decreases with the depth towards the sides, so that it is least at a and b. In order to learn what quantity of water will flow through such a section in one second, we must first find the mean velocity by the aid of a stream measurer, and then multiply it by the superficial content of the section.

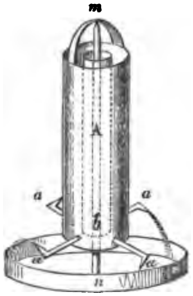
Fig. 92.



§ 163.

If any fluid, as water, issues from an aperture in the side of a vessel, the pressure on that side is diminished, being consumed in the motion of the water. A pressure is, however, generated in a horizontal direction against those parts of the sides that are opposite to the hole by which the water has egress. If the vessel whence the fluid issues is capable of being moved in the direction of this pressure, and the force of the stream be sufficient to overcome the inertia, a rotatory motion will be imparted to the vessel in an opposite direction to that in which the water flows. *Segner's Machine* and *Barker's Mill* are applications of the power thus obtained.

Fig. 93.

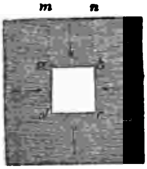


Exp. Segner's Machine consists of a hollow cylinder, A, *fig.* 93., which turns about a vertical axis, *mn*, and is kept full of water. At its base are several pipes for the egress of the water, whose mouths *a* are all in one direction, tangential to the circumference of the cylinder. The pressure against the sides of the tubes produces a motion of the vessel in an opposite direction *b*. The motion ceases if the water flow in an opposite direction from an equal number of tubes.

IV. EQUILIBRIUM OF SOLID AND FLUID BODIES.

§ 164.

Fig. 94.



Let *abcd* (*fig.* 94.) represent a portion of a fluid contained in a vessel, then, agreeably to the laws of hydrostatics, it will be in repose, or in a condition of equilibrium, because the lateral pressures mutually cancel each other, and its weight, together with the pressure of the column above it, is equal to the pressure upwards, *i. e.* the pressure of the column *m a b n* of the fluid, together with the weight of the portion *a b c d* downwards, is equal to the pressure on the base *d c* upwards. Now let any solid body occupy the room of that portion of the fluid represented by *a b c d*, and it will become subject to precisely the same pressures: its upper surface, for instance, will be forced downwards by the weight of the column of fluid *mabn*, while its base, *dc*, is urged upwards by a force equal to the weight of a column of the fluid, *mdcn*. The upward pressure is evidently in excess, by the difference of the weight of a mass of the fluid, equal to *abcd*.

If the weight of the body be equal to that of the mass of fluid which it has displaced, in other words, if its density or specific gravity be equal to that of the fluid, it will float as the mass did whose room it occupies in the fluid; if its weight exceed that of the fluid it has displaced, *i. e.* if it be denser or have a greater specific gravity, it will sink with a force equal to the excess of its weight; lastly, if it be lighter, bulk for bulk, than the fluid, then the upward pressure of the particles will force it up to the surface with a force equal to the difference between the weights of equal volumes of the solid and the fluid.

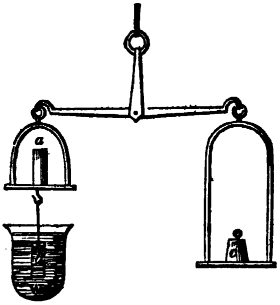
When a solid body floats on a fluid, it exerts a pressure in consequence of its weight on those particles of the latter which are beneath it, and sustains, in return, a pressure equal to the weight of the mass of fluid that it displaces. On this truth depends the following law of equilibrium between solid and fluid bodies:— *Every body immersed in a fluid loses such a portion of its own weight as is equal to that of the fluid it displaces.*

§ 165.

Every body as it sinks in a fluid displaces a volume of the latter equal to its own, and loses, consequently, as much of its own weight as is equal to the weight of a similar volume of the fluid.

First Exp. Place a hollow metal cylinder, *a*, in one of the scales of a balance, *Fig.* 95., suspend to the scale a second cylinder, *b*, of solid metal, exactly fitting the former, and in the opposite scale put a weight, *c*, that shall restore the equilibrium of the balance. Now plunge *b* in a vessel, W, of water, and the scale *c* will descend, but if *a* be filled with water the two scales will again be at rest.

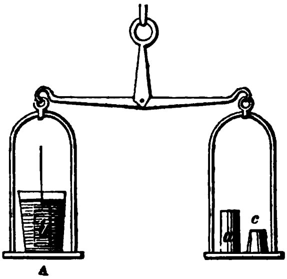
Fig. 95.



Hence it may be understood why bodies can easily be raised below the surface of water that cannot be moved when in the air.

The weight lost by the solid on immersion is gained by the fluid into which it is plunged; *i. e.* the water in the vessel is as much heavier as if a quantity of water had been poured in equal to that which the solid displaces.

Fig. 96.



Second Exp. Place a glass of water, A, in the scale of a balance, *fig.* 96., bring it into equilibrium by means of a weight, *c*, in the opposite scale. Hold the cylinder *b* by a thread, and immerse it in the water in A, and the scale will instantly descend. Fill the cylinder *a* with water, place it in the other scale, and equilibrium will be restored.

§ 166.

If a solid be immersed in a fluid, having the same density with the latter, it will be seen to float in the fluid (§ 164.), and is in every respect similar to a mass destitute of weight, because the fluid which it displaces is of exactly the same weight as itself. — The slightest difference, however, in their densities will cause the solid either to sink, or rise to the surface.

Exp. Wax, whose density differs little from that of water, presents the appearance described above, if spirit of wine be mixed in certain proportions with water.

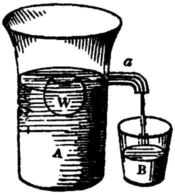
An egg, the specific gravity of which but little exceeds that of water, will float if put in water in which bay-salt has been dissolved.

§ 167.

If the body immersed be specifically lighter than the fluid, it will sink till the volume of fluid displaced equals its own weight. Having lost all its weight, it will be borne up by the counter-pressure of the fluid, *i. e.* it will *swim.*

First Exp. Fill a glass, A (*fig.* 97.), having an opening, *a*, with a cock, with water, until it begins to run out at *a*; by this means an exact level will be obtained. Now lay a ball of wax, W, or any other floating body, on the surface. — This body will be seen to sink to a certain depth, and displace thereby a volume of water, which may be caught in the vessel B. If the water that has overflowed be weighed, it will be found that its weight equals that of the floating body.

Fig. 97.



Hence it necessarily follows —

1. That in a homogeneous fluid a solid body will sink to a greater depth in proportion as its density or specific gravity approximates to that of the fluid.

> *Second Exp.* On the surface of a glass vessel containing water lay a ball of wax, another of deal, and another of cork. — The first will sink almost to the bottom, the second abou half-way, and the last will float on the surface.
>
> In like manner, place an iron and a wax ball on the surface of some quicksilver, and the former will sink half the depth, whilst the wax will swim on the surface.

2. That in fluids of various densities, or specific gravities, the same body will sink deepest in that which has the least specific gravity.

> *Third Exp.* Deal and cork sink deeper in a mixture of alcohol and water than in water alone — wax sinks in alcohol — whereas if either of these bodies be placed on salt-water, they scarcely fall below the surface.
>
> This accounts for the necessity which exists for throwing over some of a ship's ballast when she enters fresh water.

§ 168.

It is not absolutely necessary that the material of which a body is composed should be specifically lighter than water in order that it may float, it will suffice if the entire mass of the solid be lighter than an equal volume of the fluid. Hence such substances may float on water, as are much denser than it, if they be made hollow, or if united to other substances whose specific gravity is less than that of water.

> A glass bottle will swim on water, whilst a glass-stopple sinks — iron vessels and pontoons — life-boats lined with cork; life-preservers for swimming. Almost all animals are specifically lighter than water, and swim more easily than man; he is generally, as Muschenbrök says, heavier than fresh water, his swimming is therefore a mechanical art. Men of ordinary bulk soon sink if unable to swim, but such as are unusually fat float on the surface of water like the Neapolitan *Paolo Moccia*—Why, as a general rule, does not the human body sink in the sea?—Effect of respiration; the swimming-bladder in fishes.

§ 169.

If a body whose specific gravity is less than that of the fluid in which it is placed, be put at any distance below the surface greater than that at which it will swim, it will be borne upwards

(§ 164.) by the particles of the fluid beneath it with a *force equal to the difference between the weight of the body and that of the mass of the fluid which it displaces.* The solid will consequently be lifted above the level of the fluid until the upward force becomes equal to the body's absolute weight, *i. e.* (§ 167.) until only so much fluid is displaced as is equal to the entire weight of the body.

The particles of fluid below any floating body oppose its total submersion with a force equal to the difference between the weight of the body and that of the fluid it displaces. Consequently, before a floating body can be made to sink it must be loaded with a weight equal to this upward pressure; this weight is the measure of a body's *buoyancy*, as it is called; further, the plane of a body's floatation may be made to vary between the two extremes by imposing on it loads proportioned to the depth to which it is desired that it should sink.

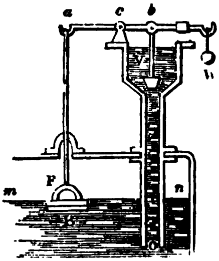
This buoyant force of water is frequently made use of in raising bodies immersed in it; it is still more often employed to regulate the influx and efflux of water, so as to maintain a constant level; and, lastly, on it depends the capacity of vessels for carrying freights; by it we estimate the weights that can be safely borne by floating bridges, bridges of boats, pontoons, &c.

This pressure upwards is thus applied in elevating bodies that have sunk; other floating bodies are loaded till they descend to the former, to which they are then attached; their load is discharged, and the whole rises to the surface. Thus, for instance, one or more boats may be loaded till the water almost touches the gunwale. If, while thus freighted, they are fastened by means of ropes to any body at the bottom of the water, it is plain that when they have discharged their cargoes they will lift the sunken bodies with a force equal to the weight of which they have been relieved. Just in this way the *Camel* acts: it consists of long, decked boats, so formed that they can be applied on each side of a ship. The camel, being filled with water, is lowered to the vessel it is intended to raise, and afterwards the water is pumped out. By this means the largest vessels may be raised several feet.

To regulate the influx and efflux of water, a body is used that will float upon its surface, thus rising and falling with every change in the level of the fluid, it is contrived that it shall close or open the communication between the cistern and its waste and feed-pipes.

The supply of water to the boiler of a steam-engine is controlled by means of a *ball-cock.* This apparatus is represented in *fig.* 98. Let B be the boiler, to which the water

Fig. 98.

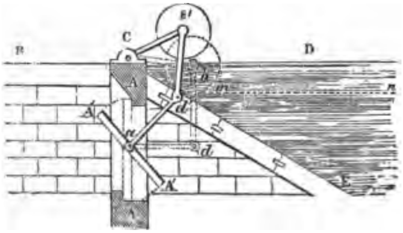


is conveyed by the *feed-pipe* V O, let *m n* be the level at which the water is to be kept, and F the *float* that is to regulate the supply. This part of the apparatus is connected by a rod with the lever *a c b*, *c* being the fulcrum; its other arm at *b* works the *influx-valve*, which opens and closes the feed-pipe. As the water in the boiler becomes converted into steam, the level *m n*, and by consequence the float F, sinks; the arm *a c* of the lever is drawn down, *c b* rises, and in so doing lifts up the valve V, water is admitted by the feed-pipe at O into the boiler, and the original level is restored.

For various reasons plates of stone are preferred as floats in boilers,—they are made to swim by hanging a weight W to serve as a counterpoise at the other end of the lever.

In *Farey's Self-acting Floodgates* the upward pressure of water is very ingeniously made use of. Let A A, *fig.* 99., be a vertical section of such a gate closed, moveable about a horizontal axis, *a*, placed over the centre of pressure, so that the gate has always a tendency to shut. The arm *a d* is fixed at right angles to this door, its end *d* being connected with an iron rod to the cask *b*, which floats on the water before the dam D C E. The gate will retain the position A A till the water has reached its highest level *m n*: if it exceed this, the floating cask *b* will rise with the water, and by means of the rod *b d* it will lift up the lever *d a*, and so open the flood-gate A A. Had, *ex. gr.*, the water attained

Fig. 99.



the height B D, the cask would be at b', and the gate in the position A′ A′, when the water would escape between A A′. As the level falls the cask b' likewise descends, and the gate A′ A′ returns to its former place, till the water again rises to *m n*.

The *absolute weight* that any floating solid *can carry, i. e.* the *buoyancy* of such a body, is most readily found thus:—First, reckon how much 1 cubic ft. of the body weighs, which subtract from that of 1 cubic ft. of water, and the remainder will be the weight that 1 cubic ft. can carry; multiply this by the content of the body in cubic ft., and the product will be the amount required.—How much can a block of dry oak carry without sinking, its content being 4355 ft., supposing 1 cubit ft. of oak to weigh 57·8 lb., and the same quantity of water to weigh 62 lb? — The cubic content of different bodies must be found according to the principles of solid geometry, the practical rules may be found in any work on mensuration; the mode of calculating the weight of 1 cubic ft. of any body whatever will be found in § 176.

The buoyancy of hollow floating bodies—such as ships and pontoons—is generally obtained by reckoning the solid content of their space from the outside to the water-line when empty, and again to that which obtains when laden. The weight of a mass of water of the same solid content will give the vessel's relative buoyancy. Suppose this space in a pontoon amounted to 150 cubic ft., then its maximum weight would be 150 × 62 lb. = 9300 lb. Having the dimensions of the part below the water, when the vessel is empty, and knowing the weight of 1 cubic ft. of water, it appears we can determine the weight she will carry at any particular depth.

§ 170.

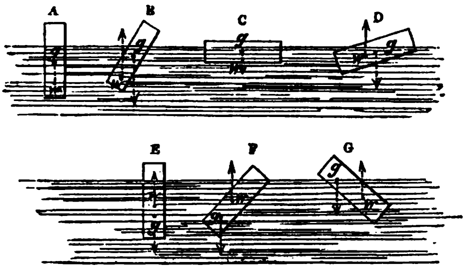
A body floating on a fluid is not equally steady in every possible position in which it may be placed; a distinction is hence made between the different positions it may assume as *stable* and *unstable.* In a position of stable floatation a body is not upset by the exertion of small forces, but invariably returns to its former position; this is not the case when its position is unstable, as small impulses induce a change of place which commonly produces stable equilibrium.

This difference depends mainly on the positions of the body's centre of gravity with regard to that of the column of the fluid it displaces. The former of these points invariably retains the same place in the solid, whereas the latter shifts with the changes that occur in the position of the part immersed. When both these points are in the same vertical line the solid is in a state of equilibrium or repose. If the centre of gravity of the body be below that of the fluid, then the body floats absolutely stable, but if this arrangement be inverted it is unstable, excepting when it is so placed on the fluid that its centre of gravity is above its greatest surface. Hence it is seen that in such bodies the greatest surface is parallel to the water when they float, and that they fall over if this surface be made perpendicular to that of the fluid.

The floating of a plank, a piece of timber, and similar objects, may serve to illustrate the above remarks.

A floating body is maintained *in equilibrio* by two forces parallel to each other, and equal in their intensities; viz. gravity, which solicits the body downwards with a force equal to its weight, and the pressure of the fluid upwards, which latter force acts on the centre of gravity of the column of fluid displaced by the solid. If the points on which these two forces act be vertically one above the other, they counteract each other; if this is not the case, motion must ensue. The diagrams A to G, *fig.* 100., will perhaps serve to illustrate this more clearly: *g* represents the centre of gravity of the floating body in each case, and *w* the centre of gravity of the column of fluid it displaces. When the body is in the position A, it is in one of unstable equilibrium; if it be moved ever so little, as B, both the forces tend towards the position shown in C, when stable equilibrium is gained, as the body floats on its largest surface. If through the oscillations occasioned by its motion it assume the position D, it will again by the operation of the two forces, resume its equilibrium of stability C. Lastly, E represents a body whose centre of gravity lies below that of the column of the fluid; E is *in equilibrio.* However, it may be moved from this position; should even the solid's centre of gravity be raised above the level of the

Fig. 100.



fluid, as G, the operation of the two forces would restore it to that of stable equilibrium, E.

In ship-building it is of importance so to place the centre of gravity that the vessel may not be upset, even by violent concussions, but that it may invariably return to its original position of stable floatation. — Use of the keel in effecting this. — The due proportion and arrangement of the ballast, though important on other accounts, is especially so as furthering this object.

There are some bodies which will swim in more positions than one; nay, some that will swim in any position whatever. To the former class belong the cube and the cylinder, when the centres of gravity of the solid and fluid the body displaces coincide. A cube will float in a position of stable equilibrium if any one of its sides is perfectly horizontal, and a cylinder if it be placed lengthwise on the fluid. If the centre of the solid's gravity in these bodies be not also their middle point, *i. e.* if they be not of uniform density, then the centre of gravity will continually descend to the lowest place, when they will be in a state of stable floatation. Hence it is clear why in the construction of rafts it is advantageous to unite the timbers as they float. Lastly, a globe is a body, supposing its mass to be of uniform density, that floats in stable equilibrium in every possible position; if this be not the case, the centre of gravity of the mass will arrange itself vertically below the geometrical centre, and so the body will swim steadily. On this principle depends the experimental proof, lately adopted, for determining the centre of gravity in bombshells.

Exp. Lay a hollow metal sphere of unequal thickness on the surface of some water, or if it sinks in this fluid, on quicksilver. The ball will roll and oscillate till its centre of gravity has descended to its lowest position. After it has come to a state of rest, mark that point of the sphere's surface that is highest above the level of the fluid; the centre of gravity will lie in the diameter drawn perpendicularly to the horizon from this highest point.

V. ON DETERMINING THE SPECIFIC GRAVITY OF BODIES.

§ 171.

It has been already (§ 31.) observed that by the term *Specific Gravity* is meant the ratio between the weights of equal volumes of different bodies, that of distilled water being taken as unity. In order to find the specific gravity of any substance whatever, we must know not only its weight, but also that of an equal bulk of water. The most simple and ready mode of effecting this is to weigh the body in water. It will always be found to displace a volume of water equal to its own, whence the weight lost by the solid (§ 164.) is equal to that of the fluid displaced. *The specific gravity or density of a body is, consequently, equal to the quotient obtained by dividing the body's weight by the weight it loses in water.*

Thus, expressing it in general terms, let W be the weight of a body, and w the weight lost in water, the specific gravity of that substance will be

$$s = \frac{W}{w}$$

To obtain the specific gravities of solids by weighing them when immersed in water, we use the *Hydrostatic Balance,* which differs from the ordinary one only in having a hook attached to the under side of one of the scales; to this the substance under investigation is attached by a fine thread, whose weight, compared with that of the solid, is so small as to be neglected. A horse-hair, or fine platinum wire, is commonly used, and it remains fastened to the scale whilst the absolute weight of the body is found.

The water in which the body is weighed ought to be perfectly clean, and at some particular temperature. It is usual to take either that at which the density of water is at its maximum, viz. 40° (Stampfer says + 3¾° C., or 3° R., which would answer exactly to 38¾° F.), or else a mean of 60°.

§ 172.

The following is the method for determining the specific gravity of a solid immersed in water. Its weight is first obtained, it is then weighed in water, and the excess of the former above the latter will be the weight lost on immersion, and as was said in the preceding §, the quotient obtained by dividing the weight of the solid by the weight lost will be the specific gravity sought.

A piece of platinum weighing 52·15 gr. = W
lost on immersion - 2·50 gr. = w
whence the specific gravity of the platinum,

$$s = \frac{W}{w} = \frac{52\cdot15}{2\cdot50} = 20\cdot8.$$

§ 173.

If the body under examination be specifically lighter than water, in order to immerse it, it must be connected with some substance of greater specific gravity than water, the weight and also the weight lost upon immersion by the second substance being previously known. From the weight lost by the combined bodies deduct that lost by the heavier, and the remainder will be the weight of the volume of fluid displaced by the lighter body. The quotient found by dividing the weight of the light substance by the weight of the fluid it displaces will be its specific gravity, which must of necessity be less than unity.

A piece of white wax weighed 50 gr. $= W$
" copper " 388 gr. $= W'$

Weight of the two substances $= 438$ gr. $= W + W'$
lost on immersion by the two $= 95{\cdot}8$ gr. $= w + w'$
" " copper $= 44{\cdot}2$ gr. $= w'$

" " wax $= 51{\cdot}6$ gr. $= w$

∴ the specific gravity of the white wax is

$$s = \frac{W}{w} = \frac{50}{51{\cdot}6} = 0{\cdot}969$$

If the substance be of such a nature as readily to imbibe the water on account of its porosity, as, for instance, most species of wood, the following is the course adopted:—The body is first weighed in air, it is then saturated with water and weighed a second time in air. Suppose the first weight to amount to 800 gr., and the second to 920; it has then imbibed 120 gr. of water. If next it be immersed in water, and be found to lose 300 gr., it is clear that it will have displaced a volume of water weighing 300 gr., and in its present state, saturated with water, it has a density of $\frac{800}{300} = 2{\cdot}66$. Subtract 120 gr. from the 300 gr. lost, and there remain 180 gr., and this will be the quantity actually displaced by the body; whence the specific gravity of the insoluble substance is $\frac{800}{180} = 4{\cdot}44$.

Such bodies as dissolve in water, as most of the salts, are weighed in spirits of wine or oil, the ratio of their densities found in these fluids as in water, and the results thus obtained multiplied into the specific gravity of the fluid. Thus suppose the density of a body to be 4·3 times that of the oil, that of the oil being ·9, the body's specific gravity will be $4{\cdot}3 \times {\cdot}9 = 3{\cdot}87$.

§ 174.

To obtain the specific gravities of various fluids, it is necessary merely to determine the weights of equal volumes of each. This is accomplished by immersing the same solid in each of the fluids in question, and noting the loss of weight in each case; the weight lost by such solid in water should be previously known, and its specific gravity must exceed that of any of the fluids under investigation. *Divide the loss of weight consequent upon immersion in each fluid by the loss sustained by the body when it was immersed in water, and the quotients will be their respective specific gravities.*

For this purpose a piece of solid glass is commonly used, suspended by a fine platinum wire to the scale of the balance.

Allow, for instance, that
the glass lost in water 150 gr. $= w$,
„ sulphuric acid 277·5 gr. $= w'$,
then the specific gravity of sulphuric acid is

$$s = \frac{w'}{w} = \frac{277 \cdot 5}{150} = 1 \cdot 85.$$

In like manner the weight of water at different temperatures may be estimated, its specific gravity varying with every change in the heat. Use some heavy body whose volume is accurately known, and notice the weight lost upon immersion; divide this by the number of units of volume contained in the body—*ex. gr.*, cubic inches, cubic feet, &c. — and the quotient will be the weight of one such unit of water at that particular temperature. Thus it is seen why in all attempts to investigate the specific gravity of bodies the water must be used at some normal temperature. (§ 161., Obs.)

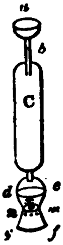
Specific gravity of liquids estimated by means of a small glass bulb, commonly called the *Weighing-Bottle.* — The specific weight of mercury is generally found by this mode only, as glass does not sink in this metal.

§ 175.

As the specific gravities of bodies cannot always be calculated with sufficient convenience by means of the hydrostatic balance, as described in the last two sections, other instruments have been constructed for this purpose called *Hydrometers*, *Areometers*, or *Gravimeters;* they are floating bodies which mark the specific gravity of any substance by the depth to which they sink in a fluid, the amount being shown either by weights, or else upon a scale attached to them. Hence they are distinguished as *Weight* or *Scale-Hydrometers.* The former are used chiefly for finding the specific gravity of solids and fluids whilst the latter serve only for finding that of fluids, after which, indeed, they are frequently named.

Nicholson's Weight Hydrometer, fig 101., consists of a hollow cylindrical body, C; the best are of glass, provided at top with a thin handle, having a fixed mark, *b*, and furnished with a small cup, *a*. Below the cylinder C hangs a cup in the shape of a double cone, *dmne* and *fgmn*, the latter being open downwards, *mn* serving as a common bottom to both vessels: here, or in the lower cup, are a few small holes to allow the air to escape, which would collect below *mn* on the immersion of the instrument. The whole apparatus must be of less specific gravity than water; so that a certain weight, x, must be put in the cup *a* to sink the hydrometer to *b*, the water-mark.

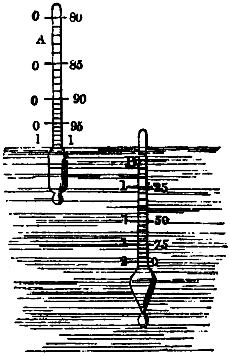
Fig. 101.



This instrument is used thus:—Suppose it is required to find the specific gravity of a body, B, whose weight is less than x, place the body in the cup *a*, instead of x, and add weights till the level of the water coincides with *b*; x minus the sum of these weights will be the weight of the body B. Now take the body out of the cup *a*, and put it in the upper cone *demn*, *i. e.* below the water. It will now weigh as much less as equals the weight lost in water; weights must be added to *a* till the water-mark *b* is again brought to the level. The weights last placed in the cup will be the weight lost by the body on immersion, whence the specific gravity can be found by § 172. If the substance be specifically lighter than water, it must be put in the lower compartment *fgmn*, and the upward pressure of the fluid will retain it in its place. If it be required to find the specific gravity of a fluid, we must first learn what weight the instrument itself loses on being immersed in water and sunk to the mark *b*. This loss is equal to the weight of the instrument w' plus the weight x, which was laid in the cup to make the hydrometer sink to *b*. If it be plunged in a fluid heavier than water, as many weights, x' must be added as are necessary to depress the instrument to the normal point *b*; on the contrary, if it be lighter than water, so many weights, x' must be taken out of the cup as will allow *b* to rise to the level of the fluid. In the former case, the weight displaced of the fluid under examination is $w' + x + x'$; in the latter, $w' + x - x'$; whence, by § 174., the specific gravity of the fluid is equal to $\frac{w + x + x'}{w' + x}$, or $\frac{w' + x - x'}{w' + x}$.

The *Scale-Areometers* consist generally of glass tubes, at the end of which is blown a globe (or pear-shaped body) loaded with some heavy substance, usually quicksilver, in such a manner that the instrument floats upright. The *figs.* A and B, *fig.* 102. represent areometers of this description. The application and use of the scales graduated on the tubes depend on this, that a body floating on different fluids sinks deeper as their densities approach that of the body, and conversely (§ 167.). These hydrometers are mostly used for fluids lighter than water; whilst the latter, B, are for such as are heavier. The former, *fig.* 102., A, sink if immersed in pure water to such a depth that the glass tube with the scale stands above the surface; whilst the latter, as B, sink even to the upper end of the graduated part below the level. In both species the water-line is marked with 0, or more correctly with 1, and the register on the scale gives the specific gravity answering to the depth of the immersion.

Fig. 102.



Besides these common areometers, applicable to determining the specific gravities of all fluids, there are others made for particular and mixed liquors, in which case the scale is graduated so as to express the component parts per cent. either by weight or volume, they are hence called *per cent. hydrometers.* They are named after the fluid they are used to test; as spirit of wine hydrometers, beer hydrometers, &c. To this class also belongs the powder areometer, lately invented by Becker, a lieutenant of artillery, whose scale is so marked, that in a solution of 400 grains of powder in 1 lb. of distilled or rain water, at a temperature of 14° R.$=63\frac{1}{2}$° F., each degree represents $\frac{1}{4}$ per cent. of nitre in the powder; as 400 grains have been dissolved, it is necessary to multiply the number indicated on the scale by 20, and the product will be the proportion per cent. of saltpetre in the powder.

The most useful hydrometer for general purposes is that of Beaumé. It contains for fluids heavier than water a scale, on which 0 represents the depth to which it sinks in distilled water; this is its highest point, and 15 that to which it descends when immersed in a solution of 3 parts bay-salt, to 17 of water. About 50 or 60 degrees are marked between these two extreme points. In the instrument used for fluids lighter than water, 0 is marked at that point to which it sinks, in a mixture of 1 part bay-salt and 9 water, and 10 at that where it stands in pure water. There are generally 50 degrees ascending from the lower of these two points to the upper.

Richter and Tralle's areometers give the proportions of water and alcohol per cent., the former by weight, and the latter by volume.

§ 176.

The knowledge of the specific gravities or densities of different substances is of great importance, not only for scientific purposes, but also for its application to many of the useful arts. By means of this knowledge the following problems may be resolved.

1. The weight of any substance may be calculated, if the volume and specific gravity be known; and
2. The volume of any body may be deduced from its weight, if its specific gravity be given.

To resolve either of these problems we must first find the weight of one solid unit of the substance, according to which its volume is calculated; this is found by multiplying the weight of the same cubic unit of water into the specific gravity of the substance.

If the weight of a body be required, multiply its solid content into the weight of a cubic unit of the body; but if its volume be sought, divide the weight of the whole by the weight of one solid unit.

Let w express generally the weight of one cubic unit of water, s the specific gravity of the body, W the weight of a body containing V cubic units.

$$W = Vs.w,$$

and the solid content V of a body whose weight is W,

$$V = \frac{W}{sw}.$$

Examples:

1. Required the weight in pounds of a dry block of fir containing 50 cubic feet, one cubic foot of water weighing 62½ lb., the specific gravity of the wood being 0·47.

$62{\cdot}5 \times 0{\cdot}47 = 29{\cdot}375$ weight of 1 cubic foot of fir,

$29{\cdot}375 \times 50 = 1468{\cdot}75$ „ 50 „

2. How many cubic inches are there in a 12-pound cannon ball, 1 cubic inch of water weighing ·5787 oz., and the mean specific gravity of cast-iron being assumed at 7·25?

$\frac{{\cdot}5787}{16} . 7{\cdot}25 = 0{\cdot}2622$ lb. weight of 1 cub. in. of cast-iron.

$12 \div 0{\cdot}2622 = 45{\cdot}555$ cubic inches in the ball.

The densities of the celestial bodies calculated from their masses and volumes — Cubic content of empty spaces, whose dimensions do not admit of measurement, obtained by finding the weight of that bulk of water which exactly fills them.

SECTION V.

OF ELASTIC FLUIDS OR GASES.

I. PROPERTIES OF ELASTIC FLUIDS OR AËRIFORM BODIES.

§ 177.

Elastic fluids, *aëriform bodies*, or *gases*, constitute the third aggregate form under which matter presents itself to our notice. They resemble the non-elastic fluids in *the readiness with which their particles move among each other*, a property they possess in a far higher degree than the latter, whilst they are distinguished from them mainly by their *expansibility*, *i. e.* by the tendency they manifest to diffuse themselves in all directions, whereas the non-elastic fluids extend themselves only laterally (§ 145.), and by their high degree of *compressibility*, that of the latter being almost *nil.* (§ 142.)

These characteristic properties of gases are obviously owing to the different degree of intensity that obtains between their molecular forces, repulsion in this form of aggregation greatly preponderating.

Besides these essential peculiarities, there are also other characteristics by which aëriform bodies are distinguished from the other two forms of aggregation. Between solids and liquids, a gradation is observable (§ 14.), and in the degree of fluidity a strongly marked variety obtains (oil, water, ether, § 143.), but between elastic and non-elastic fluids no similar connecting links are to be found. Again, as a general rule, gases are highly transparent, for the most part colourless, and consequently invisible, and are distinguished from all other bodies by their small degree of density.

Aëriform bodies are divided into *permanent gases*, *compressible gases*, and *vapours*. To the first belong such as retain unchanged their aggregate form, and resume their original volume when the force by which they were acted on ceases to operate, whatever may have been the change of temperature or the degree of compression to which they were subjected. They are the only *perfectly elastic substances* (§ 7.) in existence; they are oxygen gas, nitrogen gas, and hydrogen gas, for modern research has shown that several others which had previously been reckoned as belonging to this class, are not really permanent gases; as, *ex. gr.*, carbonic-acid gas, sulphuric-acid gas, ammoniacal gas, &c., which may be reduced to a fluid, nay, even to a solid, form. The compressible gases, of which the last-named are examples, are such as, through the action of cold and compression, lose their gaseous form. It is uncertain whether or no the atmosphere is of the number, for it has been said by means of considerable pressure to have been reduced to a liquid. All those gases may be classed as vapours, which, at an ordinary atmospheric pressure and at a mean temperature, lose this aggregate form. These vapours exhibit many peculiarities distinguishing them from the gases, the fuller account of which belongs to a treatise on heat.

§ 178.

The different properties which we observe as belonging to aëriform bodies are, as is in fact the case with both solids and liquids, partly fundamental peculiarities, owing to their form of aggregation, and partly such as are caused by external attractive forces, and especially by terrestrial gravitation.

In order to simplify these investigations, we will first consider what consequences must follow if an elastic fluid be subjected only to the operation of external pressure and its internal expansive force, and will reserve till afterwards all discussion on the modifications effected by the attraction of the earth.

§ 179.

Every fluid whose atoms, by virtue of their predominating force of repulsion, have a tendency to fly further from each other, must necessarily expand so as to occupy a continually larger space, until it meets with some obstacle that limits its further extension, or till some counteracting force cancels its tendency to expand.

Equilibrium, or rest, can therefore be established in aëriform bodies only when their further extension is checked on every side by some boundary, or when a force of compression or attraction acts upon them equal to their own force of expansion.

As every liquid exerts a pressure on the sides of any vessel containing it, so elastic fluids exert a similar pressure in all directions upon the surfaces by which they are bounded; this pressure is equal in all directions, and proportional to the force of expansion. If the resisting medium offer an equal resistance on all sides, the gas, by virtue of this eccentric expansibility, will assume a spherical form.

Soap-bubbles, and the small bubbles which rise in water, have invariably a globular figure, and show the eccentric expansibility of atmospheric air.

If the boundary of the solid which limits the gas does not enlarge with it, or if the force of the latter exceed the solid's cohesion, then disruption takes place, and the gas escapes in that direction where the rupture happens. If, on the contrary, this boundary be subjected to an external pressure equal to and contemporary with that which it suffers from within, these two forces counteracting each other, no escape will be allowed to the gas, however thin and weak the solid itself may be.

§ 180.

All these fluids are in like manner susceptible of *compression* (§ 177.); *i. e.* by means of some external force the repulsion subsisting between their particles may be overcome, and their mass reduced to a smaller volume. The component atoms will consequently be brought closer together, and their mutual repulsion will be increased; *i. e.* the expansive force and the density of the fluid increase in the same proportion as the compression. When the external force ceases to act, the tendency of the fluid to expand developes itself uncontrolled, and this impulse is obeyed till it again fills its former space, and its expansive force recovers its equilibrium.

§ 181.

If an elastic fluid, in addition to the action of its molecular forces, be subjected to an attractive force from some other body without it,—as from our earth,—certain limits will be set by this external force to the fluid's expansion. It will not fly altogether from the attracting body, but recede from it merely till the two forces counteract each other, when the elastic fluid will surround the body as a shell or *atmosphere* of limited height.

The atmosphere will not be of uniform density throughout, as the strata nearer to the attracting body must necessarily be compressed by those which lie upon them; they will therefore be denser, and this density will diminish as the distance from the attracting body increases.

Hence it follows that the denser fluid has the greater expansive force, and that this force is always proportional to the pressure of the strata above, since through the attraction of the body the above-named pressure and the force of expansion must always be *in equilibrio* the one to the other.

§ 182.

Every change in the expansive force, or in the density, that may be brought about by some external cause, in any one of the strata, will disturb the equilibrium subsisting between these antagonistic forces, and produce motion in such an atmosphere as we have described; this motion will last till the repulsion of each individual particle is restored to equilibrium with all the others that surround and press upon it.

II. PHENOMENA OF ATMOSPHERIC AIR, RESULTING FROM ITS COMPRESSIBILITY AND EXPANSIBILITY.

§ 183.

The *atmospheric air*, as being the most important of the aëriform bodies, may be taken as the representative of the whole class. It is to this form of bodies what water is among liquids. As it exists all over the world, we can most readily observe in it those properties noticed in the preceding sections, and experimentally test the laws of its equilibrium and motion.

§ 184.

First of all, the fact of its material nature must be established. In consequence of its transparency, its freedom from colour, and especially its extreme tenuity, the ordinary proofs of materiality, viz. the limits of space occupied by a body and its impenetrability, are not appreciable by the sight. The existence of these properties must therefore be proved experimentally. The experiment of the diving-bell, already described (§ 3.), confirms our belief in the impenetrability of the air. Its materiality is demonstrated in many ways by its motions, inasmuch as when agitated by storms and hurricanes it makes complete havoc of solid bodies of such strength as often set at defiance the most powerful mechanical means.

The materiality of our air is of service in communicating motion to windmills, ships, &c.; also in firing air-guns.

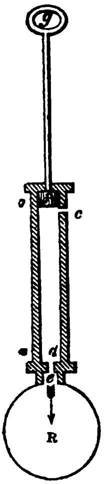
§ 185.

Our atmosphere is *compressible* and *elastic*. By means of an external force it may be condensed so as to occupy a smaller space. Any further condensation it resists with a force increasing in proportion to the condensation already effected; if the force by which it is compressed ceases to operate, it again resumes the space it originally filled. This expansion takes place with a force that is in inverse proportion to the space occupied; and if the surrounding bodies are not able to withstand the expansion, they will be put in motion with a greater or less velocity.

Our daily experience proves that these properties are possessed by the air; as, *ex. gr.*, the compression and expansion of an inflated bladder, so also the air-cushions and pillows, with the use of which we are familiar.

To increase the density of air in a given space, we make use of a *condensing syringe* or *pump*. It consists of a small

Fig. 103.

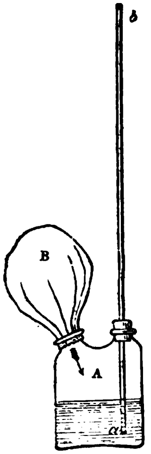


hollow metal cylinder, *a b*, *fig.* 103., in which a metal piston-rod having an air-tight piston, *f*, works. At the lower end of the tube is a small aperture, *d*, on which a valve of leather or bladder, *e*, closes; this part of the apparatus screws into a receiver, R, in which the air is to be condensed. If *f* be depressed, by means of the piston-rod *gf*, to *d*, the air being forced down opens the valve *e*, and rushes into R. Now draw up the piston, and the pressure of the air against the valve *e*, since it opens downwards, will prevent the escape of any air from R; when *f* has again arrived at *b*, a fresh charge of air rushes in at an opening, *c*, made for this purpose in the cylinder. By continually repeating the process already described, the air is more and more condensed in R. Condensing syringes of this kind are used in charging air-guns.

The following may serve as instances of motion generated by the compression of the air.

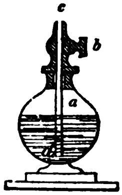
First Exp. A liquid may be forced to a considerable height up a tube by means of the air contained in a bladder. A, *fig.* 104., is a two-necked bottle containing the liquid to be experimented on. B is an inflated bladder attached by the neck to one of the mouths of the glass bottle. A glass tube, *a b*, is fitted air-tight in the other mouth, its lower end, *a*, reaching almost to the bottom of the vessel. On compressing the air in the bladder, the liquid will be seen to mount up the tube.

Fig. 104.



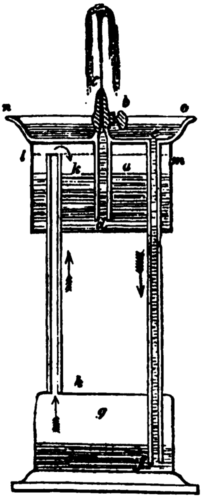
Second Exp. *Hero's Ball.*—By means of this apparatus a fluid, usually water, is forced up through a fine tube in a jet by the compression of the air. Its construction is, for the most part, as described below:—A hollow globe, *a*, *fig.* 105., from which the external air can be excluded by turning a cock, *b*, contains a tube, *c d*, that reaches nearly to the bottom of the globe, and fits in the neck with a screw. Fill the vessel about half way up with water, screw in the tube *c d*, and breathe through *c*, the cock *b* being turned; the breath of the operator will ascend through the water, and mingling itself with the air already there in the space *a*, will press on the surface of the fluid with a greater expansive force. So long as the cock *b*, through which the breath is driven, remains closed, the water cannot escape; but on its being turned the fluid finds vent in a jet through *c*. A still greater compression of the air may be produced, and consequently a higher fountain, if the condensing syringe, *fig.* 103., be screwed on, and the air in *a* condensed by it.

Fig. 105.



On this principle depend the operations of the air chamber in fire-engines and similar machines.

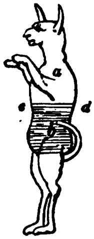
Fig. 106.



Third Exp. Hero's Fountain.—In this apparatus, also, the compression of the air is used to produce a jet of water. It consists of two vessels, commonly of a cylindrical shape, united by means of tubes (*fig.* 106.), the upper vessel, *a*, being similar in construction to the Hero's Ball described above; excepting that it has two other tubes in addition to the tube *c d*, and the cock *b*, which belong to that instrument. One of these pipes, *e f*, goes through the upper cistern, and empties itself at *f*, nearly at the bottom of the lower one, *g*; the second, *h k*, reaches from the top of the lower vessel, and terminates just below the lid of the upper one. Care must be taken that the air is not admitted at those parts of the cisterns through which the pipes pass. The upper vessel is first filled with water through *c d*, the stop-cock having been previously taken off; when the cistern is full the cock must be replaced, and the external air excluded. Now pour water into the basin *n o*, that fits on the upper cistern, the fluid will find its way into the lower vessel at *f*, through the tube *e f*. By this means there is exerted on the air in *g*, above the fluid, a pressure proportioned to the height, *f e*, of the column of water (§ 156.). The air thus compressed rises through *h k* into the upper vessel, transmitting the pressure, as in the Hero's Ball, to the water in it, which is forced out at *c* in a column whose height is proportional to the height of *e f*. The fluid discharged by the jet is received in the basin *n o*, whence it again descends the pipe *e f* to *g*, thus maintaining an uninterrupted jet-d'eau so long as there is water in *a*.

Fourth Exp. The Cartesian Diver.—This is a well-known glass figure, constructed, as *fig.* 107. shows, so that it may float in a vessel of water above whose level a small portion of air is confined, in such a manner that if this air be compressed, the figure will immediately descend, and rise again to the surface when this force ceases to be exerted. This is thus contrived: in the middle of the figure *a* is a capillary tube *b*, through which so much water is admitted into the interior of the body that its specific gravity is little less than that of the water it is to float in. Having thus adjusted it, the figure is to be immersed in a wide-mouthed glass over which a piece of bladder has been strained, in order to confine the air on the surface of the fluid. Now if the finger be pressed upon the covering the air will be slightly compressed, this pressure will be transmitted by the water to the air within the figure *a*, and a small additional quantity of water admitted in consequence through *b*: the mass becomes specifically heavier than water and sinks. On removing the finger the air expands at the surface of the water, and the expansive force of that within the image discharges the extra water through the opening *b*, so that the figure again rises. It is easy so to arrange the discharge of the water that the figure shall move up and down as it becomes alternately lighter and heavier than water (§ 166.) If the opening *b* be in a tube that is curved sideways, the efflux of the fluid generates a rotatory motion, according to the principle of Segner's machine. (§ 163.)

Fig. 107.



On the compressibility of our air, and its consequently greater expansive force, depend the action and construction of the various bellows used in the arts to accelerate the combustion of bodies and so generate an intense heat, mainly by means of a blast of condensed air. There are numerous applications of this apparatus: it will suffice to mention the single and double bellows, from the latter of which a blast is obtained by the alternate discharge of the chambers,—the hydrostatic blow-pipe, &c.

§ 186.

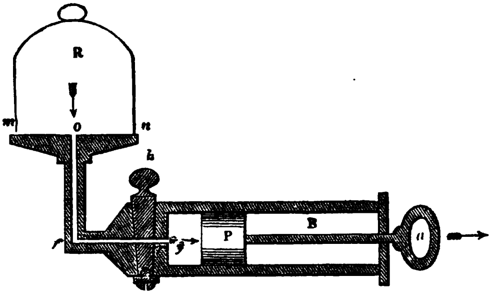
Our atmospheric air has a constant tendency to expand or occupy a larger space. This expansion actually takes place when the limits within which the air is confined enlarge or yield to the pressure. The same mass of air with its atoms uniformly distributes itself over a larger space; a necessary consequence of which is that the *rarefied* air has a less density and expansive force than it had in its original state: that portion of the atmosphere which has not undergone this process may, as compared with it, be considered as condensed.

In order conveniently to rarefy the atmospheric air and bring it to any required degree of tenuity, and at the same time exhibit the circumstances that follow this change of condition as they are displayed in other bodies, we make use of *the air-pump* or *exhausting syringe*, one of the most important pieces of apparatus used by the natural philosopher.

The mode of working it will be best understood by describing first an extremely simple machine of the kind. It consists of three principal parts.

1. The *receiver* R, *Fig.* 108., or chamber from which the atmosphere is excluded, that the air within may be rarefied. This is commonly a bell-shaped glass, with ground edge, over which a small quantity of grease is smeared that no air may be admitted between any remaining inequalities on its surface, and the polished metal-plate *mn*, on which it stands.

Fig. 108.

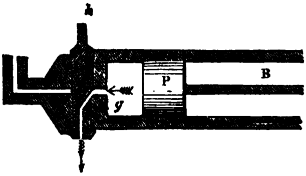


2. The *barrel* B, or chamber where the air contained in the receiver is to expand itself. It is a hollow metal cylinder, connected with the receiver by the communication *ofg;* in the interior of the barrel an air-tight piston, P, plays.

3. The *stop-cock* *h*, by means of which the communication between the two above-named parts of the pump is opened and shut at pleasure, and the rarefaction of the air regulated accordingly. This cock is a conical piece of metal fitting into an aperture just at the lower end of the barrel; it is air-tight, but

pierced in two directions; one of these perforations runs transversely through, as is shown above in *fig.* 108., and when in this position, the communication is perfect between the receiver and the barrel; the second passes in the direction of the axis from the small end, and then inclines sideways so as to stand almost at an angle of 90° with the other opening (see *fig.* 109.). In this position of the cock the communication of the cylinder with the receiver is cut off, whilst that with the external air is opened at the same time.

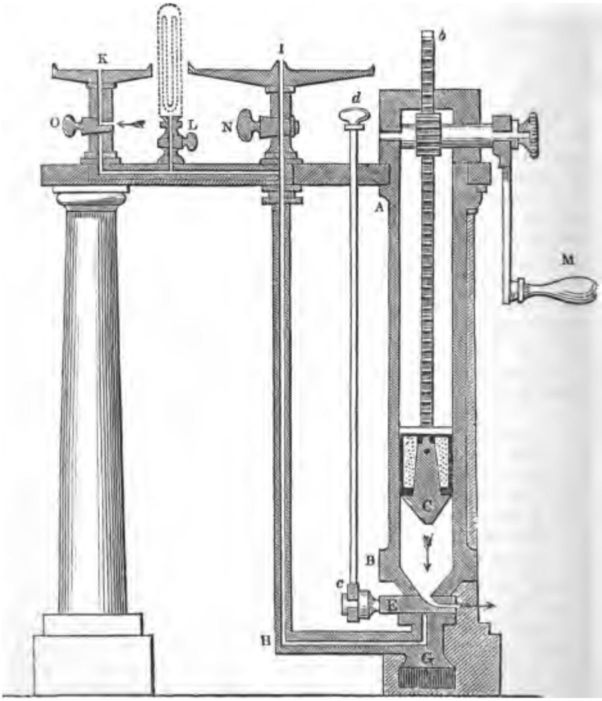
Fig. 109.



Suppose now that the piston P is at the bottom of the barrel, let it be drawn up by the piston-rod *a* P, and the air contained in the receiver R will at once rush out in the direction indicated by the arrow, through the communication *ofg*, into the vacant space within the barrel, the cock being as represented in *fig.* 108. It is clear that the air occupying the receiver and barrel must be less dense than if it filled the receiver only. By a quarter turn of the cock, as in *fig.* 109., the communication between the receiver and the barrel is closed, whilst the latter is opened to the atmosphere, which therefore enters below the piston, and can be forced out of the barrel only by thrusting the piston quite down to the bottom of the barrel. Again reverse the position of the cock; and as often as the operation just described is repeated, the air still in the receiver will be continually rarefied, the mass remaining, as also that discharged at each stroke of the piston constantly diminishing.

Repeated attempts have been made to bring the air-pump to still higher degrees of perfection since the time of Otto von Guericke, burgomaster of Magdeburg, who first invented this machine in 1560, when he exhibited before the emperor Ferdinand III. his famous experiments with the Magdeburg hemispheres. Self-acting valves, opening and shutting by the elastic force of the air, have been used instead of cocks, the manipulation of which is tedious. Two barrels have been given to the air-pump instead of one, so that an uninterrupted and more rapid rarefaction of the air is brought about, the piston in one barrel being made to ascend as that of the other descends. The most serious defect in the air-pump was, that the atmospheric air could not be entirely got rid of from the barrel, but that it remained between the piston and the barrel in the space *g r*, *fig.* 108. This intervening space is filled with air of the ordinary density at each descent of the piston, as contact between the latter and the bottom of the cylinder does not take place; when the cock is turned, and the communication re-established with the receiver, this portion of air forces its way in and diminishes the degree of rarefaction already attained. If the air in the receiver is so far rarefied, that one stroke of the piston will raise only such a quantity as equals the air obtained in this space, it is plain that no further exhaustion can be effected by continuing to pump. This limit to rarefaction will be arrived at the sooner, in proportion as the space below the piston is larger; whence one chief point in the improvements has been to diminish this space as much as possible. In *fig.* 110. A B is a highly-polished cylinder of brass which serves as the barrel of the pump; within it the piston C works perfectly air-tight.

Fig. 110.



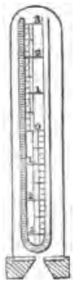
The interior of the piston consists of washers of leather soaked in oil, or of cork, covered with a leather cap, and tied together by means of two metal plates connected by a common axis. The piston-rod *ab*, which is toothed, is elevated and depressed by means of a cog-wheel that is turned by the handle M. If a thin film of oil be poured on the upper surface of the piston the friction will be lessened, and the whole will be rendered more perfectly air-tight. To diminish to the utmost the space between the bottom of the barrel and the piston-rod, the form of a truncated cone is given to the latter, so that its extremity may be brought as nearly as possible into absolute contact with the cock E; this space is therefore rendered indefinitely small, the oozing of the oil down the barrel contributing still further to lessen it. The exchange-cock E has the double bore already described, and is turned by a short lever, to which motion is communicated by the rod *cd*. The communication G H is carried to the two plates I and K, on one or both of which receivers may be placed: the two cocks N and O below the plates serve to cut off the rarefied air within the receivers when it is desired to leave them for any length of time; the cock O is also an exchange-cock, so that the external air can be readmitted through it into the receivers.

Pumps thus constructed have one advantage over such as work with valves, that they last longer, rarefy the air more completely, as sufficient elasticity is not left in the air after exhaustion has been carried beyond certain limits to raise the valves in the latter, and they allow of the greatest reduction in the space below the piston. Again, these pumps may be used for condensing the air, if suitable receivers are fixed on them, by merely inverting the manipulation; this is impracticable with valve-pumps, which are, however, more convenient for performing experiments with, as neither labour nor time is expended in reversing the cocks.

From the description just given of this apparatus it is evident that an absolute vacuum cannot be obtained; the exhaustion, however, progresses in a good pump, the under surface of whose piston is brought in contact with the bottoms of the barrel at each stroke of the piston. The degree of rarefaction arrived at may be found approximately from the relative dimensions of the receiver and barrel (1). It is indicated in a more simple and convenient manner by means of either the *barometer* or *syphon-gauge*, which show at every instant on a scale the degree of rarefaction already accomplished; this instrument also serves as a perpetual register, showing immediately when any portion of the external air finds admission below the piston (2). The mode in which this apparatus gives the rarefaction of the air will be better understood when the use and action of the barometer itself shall have been explained.

1. To illustrate the mode of calculating the degree of exhaustion arrived at after any given number of strokes, we will suppose the receiver with its communication to contain 900 in., and that portion of the barrel in which the piston works 100 in. After the first stroke the 900 in. of air in the receiver, &c. will have spread themselves over a space of 1000 in., whence the density of the fluid will be $\frac{900}{1000}$ or $\frac{9}{10}$ of what it was at first. The portion of the barrel filled with this rarefied air by our hypothesis equals $\frac{1}{10}$th of the entire volume, consequently with every stroke of the piston $\frac{1}{10}$th part of the mass of air in the receiver and communication will be drawn out, so $\frac{9}{10}$ths must remain. At the second stroke there will be $\frac{9}{10} \times \frac{1}{10} = \frac{9}{100}$ pumped out, whence there remains $\frac{9}{10} \times \frac{9}{10} = \frac{9^2}{10^2} = \frac{81}{100}$. In like manner at the third stroke there will be $\frac{81}{100} \times \frac{1}{10} = \frac{81}{1000}$ of the air drawn out, and there will remain $\frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = \frac{9^3}{10^3} = \frac{729}{1000}$. After n strokes the exhaustion will be $\left(\frac{9}{10}\right)^n$; or, in general terms, $\left(\frac{R}{R+B}\right)^n$, R being put for the content of the receiver and its communication, B for that of the barrel. The inconveniences attending such a mode of calculation are obvious, as also why the actual rarefaction will not coincide with the results obtained by this theorem. By the aid of the best pumps air may be rarefied from four to six hundred times.

Fig. 111.



2. In the air-pump described above (*fig.* 110.) is a communication that may be opened and closed at pleasure by means of the cock L; it is intended that the gauge, *fig.* 111., should be screwed on when the pump is used.

§ 187.

The following are some of the most interesting experiments that can be performed with the aid of an air-pump, showing the expansive force of the atmosphere, and also the relations between air of ordinary density and that which is highly rarefied.

First Exp. Under a receiver place a bladder tied tightly about the neck, and partly filled with air: exhaust the air in the receiver, and that confined within the bladder will gradually distend, proving experimentally the expansive force of our atmosphere. When the air is readmitted into the receiver the bladder will resume its former dimensions.

An analogous appearance will be exhibited if a jar over which some Indian rubber has been tied be placed beneath a receiver and the air be then exhausted.

Second Exp. The expansive force of our atmosphere is further shown if a long-necked flask, or retort, be inverted so that its mouth shall be below the surface of some water, contained in a vessel, and the whole be placed under the receiver of an air-pump; when the air within the receiver is rarefied, that which was contained in the bulb, expanding, escapes through the water; and on re-admitting the atmosphere the water will rise and occupy the space vacated by the air. The apparatus required for this experiment is represented in *fig.* 112.

Third Exp. The transfer of a fluid from one flask to another. Let there be a fluid in flask A, *fig.* 113. The neck of this flask contains a glass tube fitted air-tight into it, reaching almost to the bottom of the fluid; the other end of the tube passes freely through the neck of a second bottle, B. Place this compound apparatus under a receiver, exhaust the air, and the fluid will mount out of A into B in the direction of the arrow *a*. Re-admit the air, and the fluid returns to A in the direction *b*

Fig. 112.

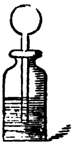
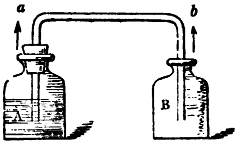
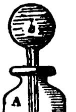


Fig. 113.



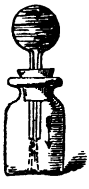
Fourth Exp. Vessels may be filled with liquid, if they have very small openings, by rarefying the air within them. *a b*, *fig.* 114., is a capillary tube, at the end of which a bulb is blown. To fill this with any fluid, invert the tube *a* in the flask A which contains it; place the whole under a receiver, exhaust the air from the bulb, and, on re-admitting it, the fluid will rise from A to *b*.

Fig. 114.



Fifth Exp. Similar vessels, having small apertures, may be emptied by rarefying the air. In this case, the glass tube, as *fig.* 115. shows, is to be stuck through a cork, and so placed in the bottle that its end may be at some distance from the bottom. As the air is rarefied in the receiver under which the apparatus is put, the liquid descends into the glass.

Fig. 115.



Sixth Exp. The jet from Hero's Ball in rarefied air, or Fountain by expansion. Place a small Hero's Ball beneath a tall receiver, exhaust the air, and the jet will quickly begin.

§ 188.

Atmospheric air, when rarefied, displays many other properties besides those already described. The most important are detailed in the following paragraphs.

In treating of the descent of heavy bodies, and the laws by which this kind of motion is regulated (§ 33.), it was stated that the resistance of the air greatly modified the actual results, since experience proves that all bodies do not fall with an equal velocity, and that this difference is greatest in the case of those whose specific gravities are smallest. The following experiment will show that the resistance of the atmosphere is the sole impediment, and that *in vacuo* the lightest bodies will descend as rapidly as the heaviest.

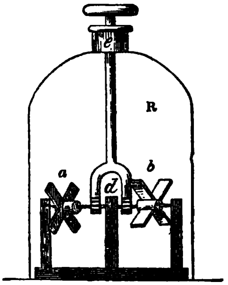
First Exp. In a highly rarefied medium a feather will fall as quickly as a coin.

To exhibit this experiment, familiarly known as *the guinea and feather* experiment, a glass receiver, open at both ends, about 3 feet high and 3 inches wide, is required; stand this on one of the plates of the air-pump and exhaust the air, having previously covered the receiver with a metal plate furnished with a stage, on which the feather and coin are laid: on turning the stage by means of a handle that runs through the plate, the two substances will be let fall together, and if the exhaustion be tolerably complete they will arrive simultaneously at the bottom of the receiver.

The resistance offered by the air to bodies in motion is illustrated by the following experiment.

Second Exp. Let *a* and *b* (*fig.* 116.) be two wheels resembling the arms of a windmill, with this difference only, that the vanes of *b* shall cut the air with their broad faces, whilst those of *a* shall meet it edgewise; each has a separate axis on which it revolves. By means of a mechanical contrivance a rapid rotatory motion is communicated to them. In order that this may act under a receiver, the rod must be made to pass through an air-tight leather stuffing-box, *e*; at the end of the rod is a curved arm, *d* which drives the wheels. If the rotation take place *in vacuo*, the two wheels *a* and *b* will cease to revolve simultaneously;

Fig. 116.

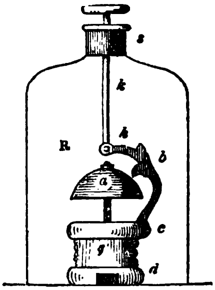


whereas if in the ordinary atmosphere, the resistance will bring *b* to stand long before *a* ceases to turn.

Our atmosphere is the ordinary medium through which sound is transmitted to the ear. If the mass of air be rarefied, the sound becomes more feeble; and if the rarefaction be carried far enough, sound ceases to be heard at all. The causes which produce this will be more fully explained in the section which treats of acoustics. The following experiment will confirm the truth of the assertion just made.

Third Exp. Under a receiver furnished with a leather stuffing-box let a bell be placed, whose clapper can be struck at pleasure by means of a rod passing through the stuffing-box; care must be taken that the bell shall rest on some body that is a bad conductor of sound to separate it from the metallic plate of the air-pump. *Fig.* 117. represents such an arrangement of the apparatus, which may however be considerably varied; *a* is the bell, *b* the clapper attached by a spring to *c*; *c* is a thin cylinder of wood, into which the support of the bell is screwed; *g* is a leather drum stuffed with horse-hair, fitting into the upper wooden cylinder, *c*, and into the lower, *d*, by which the whole apparatus is fastened down to the plate of the pump: lastly, *h* is the lever by which the clapper is agitated. — After about ten strokes of the piston the sound becomes sensibly more feeble, and if the exhaustion be continued long enough it ceases altogether.

Fig. 117.



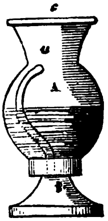
The process of respiration is soon interrupted by rarefying the air. The experiments given below will illustrate this.

Fourth Exp. Introduce a bird beneath the receiver, a few strokes of the piston will cause it to make convulsive struggles, and death would soon ensue unless air be re-admitted.

Warm-blooded animals, as birds, die if rarefaction be carried to but a small degree — a plain proof that in those regions of the atmosphere to which they ascend the density of the air exceeds that of the air within the receiver; cold-blooded animals, on the contrary, endure a high degree of rarefaction before the vital principle fails.

Fifth Exp. To perform the experiment on fishes, the apparatus represented in *fig.* 118. will be found convenient

Fig. 118.



(it will be useful in performing other experiments, to be detailed presently). A is a globular glass vessel, having pretty wide openings at each end. The lower may be closed by means of the metal foot *b*, by which the apparatus is securely fixed on the plate of the pump, the tube *a* connecting the interior of the glass with the communication of the pump. Into the glass pour the water in which it is intended to put the fish, &c. and close it at the top by the air-tight metal plate *c*. — The apparatus is now prepared for the experiment.

Sixth Exp. Introduce a small taper beneath a bell-shaped receiver full of ordinary atmospheric air, and observe the time it will continue to burn. Light the taper again, place it beneath the receiver, and exhaust quickly, after it has been replenished with fresh air. — The flame will expire much sooner than before.

To the same cause is owing the fact, that *in vacuo* no light is produced by striking a flint and steel together.

III. WEIGHT AND PRESSURE OF THE ATMOSPHERE, AND THE CONSEQUENCES THENCE ARISING.

§ 189.

Our atmosphere further possesses *Weight*, *i. e.* like every other material substance, it is subject to the attraction of the earth: this is evident from the fact, that it encases the earth; for if, destitute of weight, it were to obey no other impulse than its expansive force, it would recede farther and farther from the surface of the earth, and extend itself throughout space.

In consequence of this property, it follows that air must exert a *pressure*, and possess some definite *weight:* this may be proved experimentally.

First Exp. Procure a hollow globe of glass or metal, about two or three inches in diameter, provided with an air-tight stop-cock. Suspend it to one arm of a good pair of scales, and find its weight when filled with air of ordinary density. By means of the air-pump exhaust the air it contains, and it will be found to be lighter than before; re-admit the atmosphere by turning the cock, and equilibrium will be re-established between the two arms of the balance.

Second Exp. Force into the globe an additional quantity of air by means of the condensing-pump (§ 185.), and the weight will be found to be increased.

§ 190.

In order to determine the weight of any particular space, as 1 foot or 1 inch of atmospheric air, it is necessary to learn its density or specific gravity. This may be obtained from the first of the experiments in the preceding §, if we know the cubic content of the vessel, and compare the weight of air that it holds with the weight of an equal volume of water. There are two ways of determining the cubic content of the air pumped out of the flask, viz. from the degree of rarefaction attained, the content of the flask being known; or by plunging the flask in water, and opening the cock, when a quantity of the fluid will enter equal in volume to the air that has been extracted. From the weight of this quantity of water the content of air exhausted may be reckoned.

Suppose the globe to contain 100 cubic inches, and that the air is rarefied 100 times, there will consequently remain 1 inch in the flask, the other 99 inches having been pumped out. The globe on being weighed after exhaustion will be found to have lost about 31·32 gr. troy. As 99 inches of water weigh 25056 gr., the density of the air extracted must be about $\frac{31 \cdot 32}{25056}$ gr. $= \frac{1}{800}$. By the second experiment it will be found that 25056 gr. of water would exactly occupy the space left vacant by the air if the vessel were plunged into water; so that by either process a like result is obtained.

The air is of *unequal* density, and this variety is occasioned partly by its own pressure, partly by the proportion of aqueous vapour it contains, but chiefly by variations in its temperature. Regard must be had to all these circumstances in determining the specific gravity of the atmosphere. By careful experiment and calculation it has been ascertained that at a mean temperature, and with a mean pressure, the air is 800 times lighter than water, whilst at the freezing point it is only 770 times lighter. These proportions being known, the absolute weight of any volume of air may readily be estimated.

From the above remarks, it is clear why the exact calculation of the specific gravity of air is a matter attended with some little difficulty. Biot and Arago have determined, by accurate experiment, that with a mean pressure, such as obtains at the level of the sea (this level being ascertained, as will be hereafter shown, with the aid of the barometer), and at the temperature of freezing water, it $=$ ·0012955, or $\frac{1}{77}$th nearly, the weight of water; and at a temperature of 59° F. its density is nearly $\frac{1}{800}$th that of water.

Since 1 cubic foot of pure water = 1000 oz. avoirdupois,
= 911·5 oz. troy - - - - - = 437520 gr.
1 cubic foot of air = $\frac{437520}{770}$ - - = 568·207 gr.

1 cubic in. of air - - - - - = ·328 gr.
or at a temperature of 59° F.
1 cubic ft. of air = $\frac{437520}{800}$ - - - = 546·9 gr.
1 „ in. „ - - - - - = ·316 gr.

Heat, as has been laid down for a universal law, § 4., is the principal agent in producing a variation in the density of the air. It has been found experimentally that the addition of each degree of heat in Fahrenheit's scale produces in the atmosphere an expansion = ·00208, or about $\frac{1}{480}$th of its volume. Call $V = 1$ the volume of the air at 32°, then at t°,

$$V = (1 + \cdot 00208\, t)$$

Next let S, s, represent the specific gravities or densities of similar masses, which, of course, are inversely to their volumes (§§ 5. and 31.), whence

$$S : s = v : V,$$

and consequently the specific gravity of air at any temperature whatever, is

$$s = S\frac{V}{v}$$

But at the normal temperature 32°, $S = \frac{1}{770}$ and $V = 1$, whence

$$s = \frac{1}{770\, v}$$

For t assume 132°, then

$$s = \frac{1}{770\,(1 + \cdot 00208 \times 100)} = \frac{1}{930 \cdot 16}.$$

In like manner the specific gravities of other gases may be found by admitting them into a flask from which the common air has been entirely exhausted, and then ascertaining the weight. In such cases the specific gravity of dry atmospheric air at its greatest natural density is assumed as unity; the temperature being that of the freezing point and the barometer standing at 30 inches.

Suppose now that the quantity of dry atmospheric air pumped out of the flask was 20 gr., that a gas was then introduced weighing 30 gr., its specific gravity will be thus expressed: $\frac{30}{20} = 1 \cdot 5$.

§ 191.

Since our atmosphere, as has been shown already, is a ponderous elastic fluid, it must display the same peculiarities as we became acquainted with in liquids, modified, however, by its compressibility and expansibility.

By virtue of its weight the atmosphere exerts a pressure not merely on all bodies below it, but also on its own subjacent particles. The absolute amount of this atmospheric pressure was first discovered by Torricelli (1643), a pupil of Galileo.

First Exp. Fill a tube about 32 inches long, and from 2 to 3 lines wide, with mercury, the other end being either secured by an iron cock, or else by its being hermetically sealed. Close the open end by pressing against it with the finger, invert the tube and plunge the open end into a cistern of mercury; remove the finger, and the metal will not escape, but remain at a height nearly 30 inches above the level of the quicksilver in the cistern.

This is known as the *Torricellian experiment*, the tube is called a *Torricellian tube*, the vacuum in the tube above the surface of the mercury is the *Torricellian vacuum*, to distinguish it from that obtained by the air-pump, sometimes called the *Guerickian* or *Boylean vacuum.*

If the circumstances attending the experiment just described be carefully considered, it will be seen that the tube containing the mercury, forms with the cistern of the same fluid a system of communicating tubes, in which the mercury might have been expected to stand at an equal height in both. (§ 154.) But observe that the atmospheric air rests on the surface of the quicksilver in the cistern, whilst there is a vacuum above that which fills the tube. To this circumstance we must attribute the rise of the mercury in the tube to a height varying from 28 to 31 inches above the level of that in the cistern. — That this is an accurate explanation the following experiment proves.

Second Exp. Open the tube at top, the air will rush in upon the quicksilver, which will sink to the same level within the tube as without it.

The atmospheric air consequently presses on the quicksilver, and indeed upon all other bodies, with a force sufficient to maintain the quicksilver in the tube at a height of about 29½ or 30 inches, whence *this pressure must be equal to the weight of a column of mercury whose base is equal to the surface it presses on, and its height about* 30 *inches.*— The force thus exerted on any surface whatever, is called *the Atmospheric Pressure.*

Throughout Germany the mean pressure is assumed at 29·84 English inches; in France it is estimated at 76 centimetres = 29·9218 English inches; in England at 30 inches; all which amounts differ but little one from another.

The body of a man of ordinary stature has a surface of about 2000 square inches, whence the pressure at 15 lb. per square inch will be 30,000 lb.— Required the entire pressure on the surface of the earth. — Each change of one line in the height of the mercury causes a difference of $\frac{1}{360}$th in the pressure of one inch $= \frac{1}{30}$th.

If the height of the column of mercury in the *Torricellian* tube be 30 inches as we have assumed, the pressure of the air will support a column of water 34 feet high, mercury being 13·6 times heavier than water.

Hausen and Sturm have actually performed the experiment at Leipzig with columns of water, and have fully verified what is stated above.

Third Exp. Procure a glass tube 6 feet in length, one end being provided with a cock; apply the mouth to this end and exhaust the air; then turn the cock, and if the opposite extremity of the tube has been immersed in a vessel of water the fluid will fill the tube.

§ 192.

The atmosphere can exert this pressure only if no other force counteract it. If the counter-force be exactly equal to the weight of the air, no effect is produced; if it be less, the atmosphere presses with a force equal to the difference of the forces. The pressure is perceptible when it acts only in one direction, *i. e.* when there is a vacuum on the opposite side of the body to that on which the air presses.

The following experiments illustrate this fact.

First Exp. Lay the hand on a cylindrical receiver open at top, exhaust the air, and it will be firmly held down by the superincumbent air. — For a similar reason, concave bodies hang to the lips when the air is withdrawn by means of the mouth. — This furnishes us with one means for testing the density of air in an imperfect vacuum.

Second Exp. The receivers are firmly fixed to the plate of the air-pump, entirely by the external pressure which is not counteracted by the rarefied air within them.

Third Exp. The force of the atmospheric pressure is most strikingly shown in the *Magdeburg hemispheres.* They are two hollow hemispheres of brass or copper, whose edges fit accurately, each hemisphere being furnished with a strong ring or handle, one of them also having a tube, that fits into the screw-hole of the plate, and a cock to shut off the communication. — Place the hemispheres one on the other, exhaust the air, turn off the cock, unscrew it from the pump, put on the handle, and it will be found that a great force is requisite to separate the apparatus. If now the area of the circle of contact were 100 sq. in., and the height of the mercury were 30 inches, then the pressure would be exactly

$100 \times 14{\cdot}75 = 1475$ lb. = 13 cwt. 0 qr. 19 lb. If now the greatest force that a horse can exert for a short time be 700 lb., it would require 2 horses to pull the hemispheres asunder. — In Guericke's experiment, referred to above, there were successively from 14 to 30 horses harnessed to the hemispheres, which were 2 feet in diameter, without effecting a separation; when more horses were added, the hemispheres parted with a loud report.

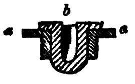
Fourth Exp. Tie a piece of bladder tightly over a low glass cylinder about 4 inches across ; extract the air quickly, and if the bladder be well dried it will be torn asunder by the pressure of the air with a loud noise.

If the top of the glass be covered with a thin plate of glass, a similar consequence will ensue.

But if a thin piece of Indian rubber be substituted for either of the above substances, in consequence of its elasticity it will be forced into the interior of the vessel.

Fifth Exp. The pressure of the air will force fluids through such bodies as are porous. The apparatus described in the fifth experiment, § 187., will serve for this purpose. Cover the upper opening of the metal plate *a a*, *fig.* 119., in the middle of which is a circular hole 1¼ in. across, to admit the wooden cylinder *b*, its pores being in the direction of its axis. Now fill the cavity with water or quicksilver, exhaust the air, and the fluid will escape into the vessel A in a fine shower.

Fig. 119.

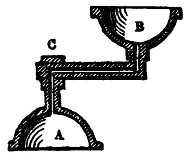


§ 193.

The atmosphere presses not downwards only, but also upwards and laterally, in all directions, with equal force. This is most clearly shown by the following experiments :—

First Exp. The two hemispheres, A and B, *fig.* 120., are connected by means of a communicating tube, in such a manner that one of them, B, will turn about the cubical part C of the tube at pleasure. — Place the hemisphere A on the plate of the air-pump, and over the other, B, lay a flat plate of metal or glass, which will firmly adhere to the rim of B, in whatever position it may stand.

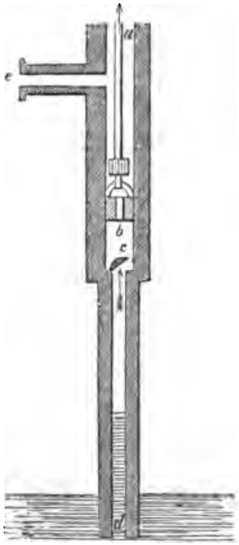
Fig. 120.



This pressure of the atmosphere, exerted in all directions equally, produces motions in liquids, in the direction in which it is unchecked by any antagonist force, resembling those motions already noticed as resulting from the unequal rarefaction or condensation of the air, but of far more general application, and of more extensive utility. The following are practical illustrations:—

1. *Pumps.* These are well-known contrivances by means of which water is elevated from various depths to the surface of the earth. They are either *suction* or *force-pumps*. The first come more immediately under our notice, as in them the water is raised solely by atmospheric pressure. — A *suction-pump* consists of the *suction-pipe c d*, *fig.* 121., which descends below the level of the water in the well, and of the *barrel*, *a c*, in which an air-tight *piston*, *b*, works up and down as far as the *spout*, *e*, by which the water is discharged. There are, besides the parts named, two valves, the lower valve *c* at the top of the suction-pipe, and the other *b* in the piston, both opening upwards. After the piston has been elevated and depressed a few times, the water gushes out from the spout. This may be accounted for thus; — let the piston be at its lowest point, let it then be drawn up, and a vacuum will be formed in the barrel between the two valves; the air in the suction-tube forces open the lower valve, enters the barrel, and thus creates a vacuum in the suction-pipe. The external air will then force a column of water up into the suction-pump, such that its upward pressure shall condense the rarefied air in the pipe, so that its density shall equal that of the air in the well. The piston-valve continues closed; the external air being denser than that in the barrel, the pressure downwards exceeds the upward pressure. Now let the piston be thrust down, by which means the air in the barrel is more and more condensed, and acquires a corresponding force of expansion which closes the lower valve whilst it opens the piston-valve through which it passes. Each time the piston is raised, the water ascends in the suction-pipe, until at length it rises above the lower valve, and when the piston is sufficiently depressed above its valve also, the weight of the water prevents its return into the well; so it has no escape but by the spout.

Fig. 121.

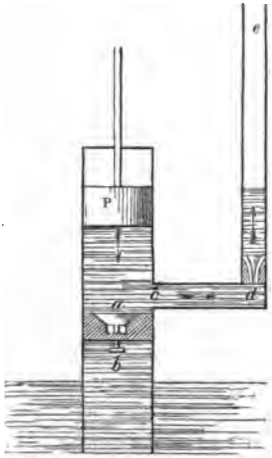


A glass model of a pump will greatly facilitate the comprehension of this written description.

Pure water can be raised at the utmost 35¼ feet, the column of mercury standing at 31 inches, supposing all parts of the pump to fit with the greatest nicety (§ 190.); salt water cannot be elevated to quite so high a point. Why? Compare this with the Torricellian experiment. — The atmospheric pressure forces the water above the piston; it is lifted through the barrel up to the spout by mechanical forces, independent of the pressure of the air.

In the *force-pump*, comparatively little use is made of atmospheric pressure; the suction-pipe, *a b* (*fig.* 122.), is short, the piston P has no valve; there is, however, the suction-valve *a*, and also a second valve *d*, opening into the *chamber*, *e d*, into which the water is forced: this part of the machine is connected with the suction-pipe by means of a tube *c d*. After a few strokes of the piston the barrel becomes filled with water; when the piston is depressed the fluid is driven into the chamber *e d*, up which it rises to a height proportioned to the mechanical force exerted.

Fig. 122.



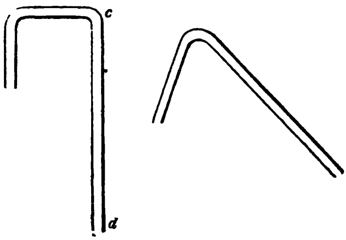
The force-pump of a fire-engine drives the water into the air-vessel, and thus condenses the air it contains. (Compare with this § 185., and the second experiment in this §.)

2. The common *squirt*, or *hand-syringe*, consists of a cylinder, or barrel, with a pretty accurate piston; at the barrel is a short suction-pipe: there is no valve. The action of the atmospheric pressure is the same as in the suction-pump. The water that is raised into the barrel is driven thence into the suction-pipe, and projected into the air by mechanical force.

The act of respiration, suction with the mouth, drinking, smoking, &c. may all be explained by reference to the same principle.

3. The *Syphon* is a tube, usually of two unequal legs, bent either at one or two angles, as in *fig.* 123., and open at each end.

Fig. 123.



Plunge the shorter leg into a vessel filled with some fluid, in such a manner that the end of the longer leg shall be lower than the level of the liquid: fill the tube with the liquid; this may generally be done most readily by applying the mouth to the longer end, and drawing in the breath; the fluid will discharge itself at the open end, and will continue to do so, so long as the extremity of the shorter leg is below the surface of the liquid.

This uninterrupted efflux is owing to the unequal pressure of the columns of liquid in the two legs.

In both legs the upward pressure of the atmosphere is equally exerted on the fluid they contain; to the shorter leg this force is transmitted by the liquid in the vessel, whilst the air acts immediately at the aperture of the longer one. The antagonist force to the atmospheric pressure is the weight of the columns of liquid in the two legs, but as these are of unequal lengths their forces must also be unequal, and the resistance offered to the upward pressure of the air by the column in the shorter leg must be less than that made by the other. In obedience to this greater force the liquid will flow from the longer leg so long as the atmospheric pressure continues to send a supply up the shorter leg, *i. e.* so long as its opening lies below the level of the liquid in the vessel.

Let $a\,b\,c\,d$, *fig.* 123., represent a syphon filled with liquid, the whole atmospheric pressure acts on the aperture a with a force = a column of water 34 feet high (§ 191.). Against this is exerted the pressure of the column $b\,a$. In like manner the pressure of the air at $d = 34 - c\,d$. But $c\,d > a\,b$, whence the remaining pressure on $a >$ that on d, so the fluid must pour from d.

As the efflux does not depend on that portion of the shorter leg which is below the level of the liquid, the two legs may be of equal length.

Use of the syphon: to empty vessels by inserting a syphon at the bunghole. — If fluids of different specific gravities be put into a vessel, the lower may be drawn off, by means of a syphon, without the necessity of previously drawing off the others. — Water conducted over elevated ground by the aid of syphons. — What limits are there to the height it may be thus raised?

Fig. 124. *a.*

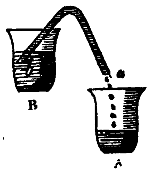
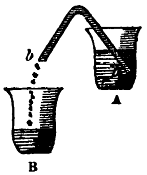


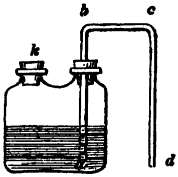
Fig. 124. *b.*



Second Exp. The efflux from both the apertures of a syphon may be shown thus: — Introduce into the glass B, *fig.* 124. *a*, the full syphon *a b*, and the water will flow from *a* into the glass A. When A has become partially full, lift it and the syphon, *fig.* 124. *b*, above B, and the water will run from the other end, *b*; which operation may be repeated by merely lifting the syphon with the other glass.

Third Exp. That the atmospheric pressure is concerned in the efflux from a syphon may be thus proved: —

Fig. 125.



With the help of a cork insert the syphon *a b c d*, *fig.* 125., air-tight into a double-mouthed bottle. If the syphon be filled in the manner shown above, the liquid will issue at *d* so long as the other mouth of the bottle stands open; but if it be closed with a cork, *k*, the water ceases to escape.

Introduce a small syphon under the receiver of an air-pump, and let it discharge some quicksilver from a small vessel, its action will be found to cease when rarefaction has been proceeded with far enough.

Fourth Exp. If a syphon be concealed in the side of a cup, one of its openings being within the cup and near to its bottom, whilst the other, which is lower, discharges externally, and liquor be poured into such a cup, it will be found, on the fluid rising to a height equal to the length of the syphon's leg, that the contents of the cup will entirely run out; this is owing to the fact, that the syphon, obeying the laws which regulate vessels that communicate, continues full of the fluid: if the fluid does not rise as high as the syphon, the cup will not be drained. On this principle the *Cup of Tantalus*, or *Diabetes Heronis*, is constructed. — The canal of Languedoc, or *Canal du Midi.*

It is probably owing to a similar cause that some springs are intermittent, and that standing water and pools occasionally become drained when the water rises to a particular height.

Fifth Exp. By the aid of this force we may make a *Syphon-jet*, or *Perpetual Syphon.* Its construction is as follows: A, *fig.* 126., is a glass vessel closed at top, and fitting closely on a

Fig. 126.

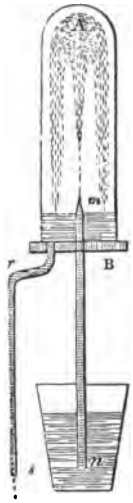


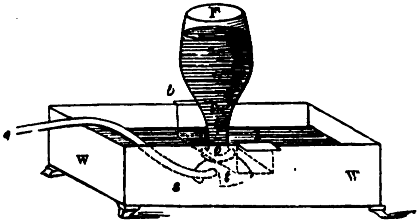
plate of brass, through which two tubes pass. The shorter, *m n*, rises but little above the plate, and terminates in a point *m*; the longer opens into the vessel A. Invert A, and pour in water enough through *s r*, at least to fill *s r* on the apparatus being reversed. Before reversing it close the aperture *s*, and plunge *m n* into a vessel of water, removing at the same time the finger from *s*. The consequence is that the water will rise through the shorter tube into the glass A, whilst it escapes at the same time through the longer. By the discharge of the water through *r s* a vacuum is created in A; the pressure of the air within is less than that of the atmosphere resting on the fluid in the glass, whence it rises up the tube *n m*.

The pressure of the atmosphere, transmitted as it is in all directions, prevents the efflux of liquor from a small aperture in a cask, unless the pressure be admitted to the surface of the fluid. This is a fact abundantly confirmed by experience, and in accordance with which several useful pieces of apparatus have been constructed. The following experiments may still further illustrate these observations.

The *vent-peg* is raised in order to admit the air to the surface of any liquor which it is wished to draw off from a cask.

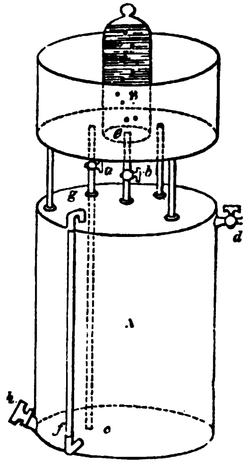
By means of the atmospheric pressure water (and other liquids) may be kept in full vessels, whether the aperture in the vessel be large or small, care only being taken that the opening shall be under water: other gases, besides common air, are thus introduced into receivers.

Fig. 127.



The *pneumatic trough* is a vessel of wood or lead, of either an oval or oblong form (*fig.*.127.): it has a moveable stage or *bridge*, *b b*, above which the water is to be poured to the height of about an inch; in the middle of the bridge is a round hole *o*, communicating with an inverted funnel *t*, under which the tube *g s* is brought, that conveys the gas it is wished to collect. Bell-shaped glasses filled with water are placed above the hole in the bridge. The gas passing through the tube at *s* into the water, ascends the receiver F in the form of air bubbles, and expels the water thence. In order to preserve the gas, it is advisable not to fill the receiver quite full; a small quantity of water left at the bottom of the receiver, will cut off the gas from communication with the external air; or if narrow-mouthed flasks be used for collecting gas, they may be corked under water, and then placed in an inverted position; if wide jars are employed, slip them off the bridge into a deep vessel, like a soup-plate, which will contain water enough to prevent contact between the gases.

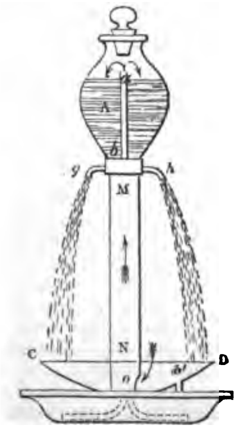
Fig. 128.



The portable *gasometer* consists of two cylindrical vessels, A and B, *fig.* 128.: the best are made of copper. The lower vessel A, usually about 2 feet high and 1 foot wide, is air-tight at top and bottom, and is intended to hold the gas; the upper, of equal width, but only one-third the height of the lower, is open to contain water. Both are connected by two tubes, one of which, *a c*, reaches from the bottom of the upper vessel to the bottom of the lower, but the other, *e b*, goes no lower than the lid. The communication can be opened and closed by means of the cocks *a* and *b*. The weight of the upper cylinder is supported on three columns. Just below the top of the vessel A is a cock *d*, through which the gas, when collected, may be drawn off, whilst the aperture *h*, about 1 inch across, near the bottom of the vessel, is for introducing the pipe by which the gas is conducted to the gas holder. A stout glass tube, *fg*, about ½ an inch wide, placed outside the lower cylinder and reaching from end to end of it, serves as a gauge to show the proportions of water and gas at any moment in the vessel (§ 154.).— To transfer a gas into this apparatus, A is first filled with water; this is done by opening the three cocks, *a*, *b*, *d*, and pouring water into the upper vessel B. The fluid descends through *a* and *b* into the lower vessel, expelling the air previously in it through *d*. When the water rises to *d* the cock is turned, and the apparatus is ready for use. *a*, *b*, and *d* are to be closed, and the covering unscrewed from *h*, the atmospheric pressure preventing the escape of the water at that point. Insert the tube proceeding from the retort, or other vessel whence the gas is obtained, into the gasometer at this opening, and as the bubbles of gas rise to the top of the water in B, they will expel the latter at *h*. When the gauge shows that a full supply has been obtained, take out the tube and secure the aperture *h*. The gas may now be taken off either at *d* or *e*; if at the latter point, in jars, as B, previously charged with water. To cause the gas to ascend, water must again be poured into the upper cylinder, and the communication *a* be re-opened, if you intend to take it off at *d*; if you prefer taking it in a receiver at B, open the cock *e*.

Sixth Exp. Fill a glass to the brim with water; cover it with a sheet of sufficiently stiff paper, so as to exclude the air from entering into the glass; press the paper firmly against the rim; invert the glass quickly; remove the support, and both paper and water will be supported by the upward pressure of the atmosphere.

Fig. 129.



Seventh Exp. Kirchner's Intermittent, or *Magic Fountain*. It consists of a hollow glass globe (A, *fig.* 129.), having a neck into which a ground-glass stopper fits accurately; at bottom it has a rim of brass, in which are several holes. This rim is surrounded by a second outer one, having two, or at most four, short fine tubes, *g* and *h*, so placed that by turning the globe round they may be connected with the corresponding holes in the inner rim, or the communication be interrupted at pleasure.

Through the inner rim a tube, *ab*, passes, open at both ends, reaching almost to the stopple, whilst it empties itself below into the pipe M N, whose foot is fixed in the dish C D; in this pipe is a small lateral opening *o*; about the middle of the dish is the little tube *o'*. To make this apparatus play, the globe is turned so that the communication is cut off with the tubes *g* and *h*, the glass is filled with water, and the stopple put in. Now move the globe so that the openings in the inner and outer rims shall correspond: the water will immediately begin to issue from *g* and *h*, and accumulate in the dish. In a short time the side-opening, *o*, will be covered with the water's rising above it, when the external air will cease to be admitted to the water in the globe: the efflux through *g* and *h* will consequently be discontinued. As the water in the dish escapes through the tube *o'*, the aperture *o* is soon above the level. The admission of the external air causes the water again to issue through the tubes as at first; this process will be repeated until all the contents of the globe have run out.

On the same principle *Mariotte's flask* acts.

§ 194.

The existence of many liquids, as such, is owing to the atmospheric pressure, as, but for it, they would pass off into gases; this same force likewise prevents the gases which exist in fluid and solid bodies from disengaging themselves.

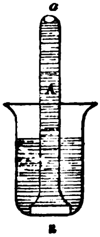
If, by rarefying the air, its pressure on these bodies be diminished, they either assume the form of vapours, or else the gas detaches itself altogether from the other body.

First Exp. Lukewarm water, milk, spirits of wine, and several other liquids, if placed under an exhausted receiver, either pass off as vapour, or else go through a process of ebullition.

The next experiment will most clearly show the change from the liquid to the vapoury form.

Second Exp. Fill with water a glass tube, closed at one end, about 10 in. long and 1 in. wide, invert it in a glass partially full of water (the pneumatic trough, described in the preceding §, will be found still more convenient). The atmospheric pressure will retain the water in the tube. Introduce a few drops of sulphuric æther, which by virtue of their small specific gravity will ascend to the top of the tube, and appear as represented in *fig.* 130.: A being the water in the tube, *a* the æther floating on the top, and B the glass which serves as a substitute for the pneumatic trough. Place the apparatus beneath the receiver of the air-pump,

Fig. 130.



exhaust the air, and the small quantity of æther will soon be seen to assume the gaseous form, expanding in proportion to the rarefaction of the air under the receiver, so that it gradually expels the water from the tube, and fills up the entire space itself. On re-admitting the air, the æther becomes condensed and the tube is once more filled with water.

Third Exp. The gases held in combination with beer, mineral waters, and other similar fluids, may be disengaged if the atmospheric pressure be withdrawn. Even common spring water contains so large a quantity of air, that it may be seen to make its escape in bubbles if a tumbler containing some be placed beneath a receiver and the air be exhausted.

Fourth Exp. To a piece of box-wood, which is but little lighter than water, attach a bit of lead sufficient to sink it; place the whole in a glass of water, and put it under the receiver of an air-pump. As the exhaustion proceeds, bubbles of air escape from the entire surface of the wood, whose pores had been previously filled with this air. Again, admit the air into the receiver, the air bubbles will disappear, and the pores become filled with water to such an extent that the specific gravity will be so increased that the wood will not float again.

If a piece of coal be used instead of wood, numerous bubbles of air will ascend from it, proving how great the absorptive power of this substance (§ 57.) is for gases.

Place an egg in water; on exhausting the air its shell will be covered with an infinite number of air bubbles. The egg therefore contains a quantity of atmospheric air for the support of the chick; this air exudes through the pores of the shell, when rarefaction has been carried to some extent in the receiver.

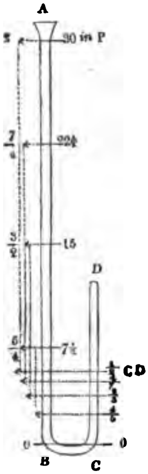
IV. THE LAWS OF ATMOSPHERIC DENSITY AND ELASTICITY, THE CHANGES OCCURRING IN THESE PROPERTIES, THE ACTION OF THE BAROMETER AS DEPENDING ON THEM, AND THE VARIOUS USES TO WHICH THIS INSTRUMENT IS APPLIED.

§ 195.

It has been already shown in the preceding sections, and demonstrated experimentally, that our atmosphere is capable of compression, as also that its elasticity or expansive force is hereby increased. Boyle (1660) and Mariotte (1668) experimentally investigated the ratio subsisting between the density of air, its elasticity, and the space it occupies. The following law was detected:—That *the density and elasticity of atmospheric air are directly, but the space it occupies is inversely, as the force of compression.*

First Exp. To perform *Mariotte's Fundamental Experiment*, as it is generally called, a syphon-shaped tube, A B C D, *fig.* 131., is used, open at the longer leg A, but hermetically sealed at D. Between the legs is a scale, marked off in inches.

Fig. 131.



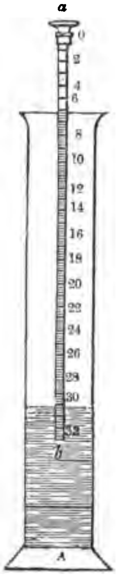
Pour in at the open end, A, as much quicksilver as will just fill the horizontal part of the tube, and stand at zero in the scale; a quantity of atmospheric air, of ordinary density, will therefore be inclosed in the shorter leg, which, at a mean temperature, is equivalent to the pressure of a column of mercury 30 inches high. Now, if there be poured into the longer leg any additional quantity of quicksilver, the fluid will also rise in the shorter leg, and compress the air above it into a smaller space, but the levels at which it will stand in the two legs must be unequal. The difference between these two heights marks the increased pressure or force with which the air is compressed into a smaller space.—For instance, if the mercury in the long open part of the tube be 15 inches higher than in the shorter, the pressure of the mercury will equal half an atmosphere, and the contained air is compressed with $1\frac{1}{2}$ or $\frac{3}{2}$ atmospheres. It will be seen that the air in the shorter leg occupies only $\frac{2}{3}$ of its original dimensions; and, as $1 : \frac{3}{2} :: \frac{2}{3} : 1$, the left-hand terms representing the pressure, and the right-hand their inverse, or the space occupied by the air, thus confirming the law just laid down. Now, if so much more mercury were to be added as would raise the column 30 in. higher in the one leg than in the other, then the pressure of the longer column would equal one atmosphere, and, consequently, the included air is acted on with a force of two atmospheres. The space occupied by the air in this case is only half that filled by it at first, so again the spaces $1 : \frac{1}{2}$ are inversely as the forces of compression. Corresponding results are obtained if the compressive power be equal to $\frac{5}{4}$ or $\frac{7}{4}$ atmospheres, in which case the spaces into which the air will be compressed will be $\frac{4}{5}$ and $\frac{4}{7}$ of its original volume, as is shown in *fig.* 131. In this experiment it is necessary to wait a short time after the mercury is poured in to allow the heat evolved by condensation, which causes an expansion of the air, to go off.

That this law holds good, not only in condensing the air, but likewise when it is rarefied, is proved in the next experiment.

Second Exp. — For this there will be required a glass tube, *ab*, *fig.* 132., about 33 inches long, one end, *a*, being fitted with an air-tight cock; the entire length of the tube must be graduated in inches. Turn the screw *a*, immerse the tube with its open end downwards into the vessel A, previously half filled with mercury: which will of necessity stand at an equal height within and without the tube. Now close the open end of the smaller tube, and so shut a portion of air at its ordinary density within the glass above the surface of the mercury.

Elevate the tube any distance whatever above the quicksilver, taking care that its open end shall be below the surface, and the air will expand, and fill a larger portion of the tube, though a column of mercury will still stand at an elevation above the outer level, so that this column, with the elastic force of the inclosed air, counterbalances the natural pressure of the atmosphere. The pressure therefore which the included air sustains is equal to the weight of a column of mercury 30 inches high, minus the column supported in the tube.—Let the space full of air above the mercury in the closed tube be 3 inches, lift up the tube so that this space shall be $\frac{1}{3}$, $\frac{3}{3}$, &c. greater, *i. e.* so that it shall be 4 and 6 inches, these lengths will then be to the original space as $\frac{4}{3}$ and 2, and the columns of quicksilver supported $\frac{1}{3}$ and $\frac{1}{2}$ the height of that in the Torricellian tube, about 10 and 15 inches respectively, whence the pressure borne by the included air equals that of a column of mercury of $30-10=20$ in., or $\frac{2}{3}$, and of $30-15=15$ in., or $\frac{1}{2}$ the atmosphere. Hence it appears that the truth of this law is confirmed as respects rarefied air, or such as exerts a pressure less than that of our atmosphere.

Fig. 132.



By experiments lately made at Paris, it has been found that this law obtains when air is condensed 27 times, and when rarefied 112 times. Other gases obey it also, until the pressure becomes a few atmospheres less than that at which they assume a liquid form.

§ 196.

Our atmosphere at the same time that it is an elastic fluid being also heavy and compressible, is condensed by its own weight; its density therefore cannot be uniform, but must necessarily diminish as its altitude increases, *i. e.* the atmosphere becomes rarer as you approach its limits (*a*), (§ 181.).—The ratio of its density according to Mariotte's law, being proportional to the compressive forces, *i. e.* in this case to the height and weight of the superincumbent strata.

a. If a vessel be closed air-tight at the foot of a high mountain, and the stop-cock be opened on arriving at the summit of the mountain, the air will rush out of the vessel. —Explain this phenomenon.—If the vessel be closed in like manner at the top of the mountain, and opened on reaching its foot, the external air will rush into the vessel.—Why?

§ 197.

Experience teaches us that the height of the mercury in a Torricellian tube varies at the same place. Since the column of mercury in such a tube measures the intensity of atmospheric pressure (§ 191.), it follows that these variations in the altitude of the column denote corresponding changes in the air's density and elasticity. When the density and elasticity of the atmosphere are increased, the quicksilver must rise; when they are diminished, it must sink.

In order to measure accurately the height of the column and the variations in it, some unit of length is applied as a measure of the tube. A Torricellian tube, provided with such a scale, is called a *barometer*, or weather glass; the *height of the barometer* is the height of the column of mercury accurately measured according to the scale; by the *mean height of the barometer* at any particular place is understood the arithmetical mean of a great number of different altitudes observed at that place.

In Germany the old French standard, or Paris measure, is almost universally adopted; throughout France the decimal scale, and in England the English measure, is used for the barometric scale.

For different reasons a variety of construction has been used in barometers; the most important are described below.

1. The *Cistern-Barometer* (*fig.* 133.) is a Torricellian tube, whose open end is immersed in a cistern of mercury. The zero point is on a level with the surface of the mercury. This is the oldest form of the instrument.

2. The *Common Barometer* (*fig.* 134.) is a Torricellian tube, whose open end being turned up, has a spherical form. The globe performs the office of the cistern described in the other barometer; the scale is graduated from the same point in both these instruments.

Fig. 133.

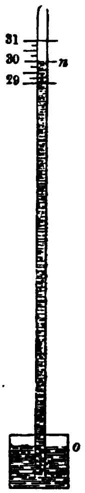
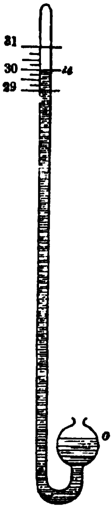


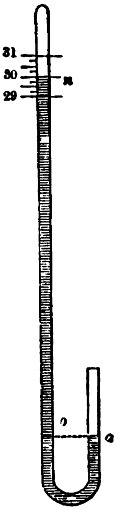
Fig. 134.



As in barometers of this kind the distance *o n* of the two surfaces of the mercury is the height of the barometer, it follows that the scale must begin at *o*, the level of the quicksilver in the cistern or bulb. If it do so at any particular height of the mercury, yet it will no longer coincide when the column rises or falls in the tube, as a rise or fall must also take place in that which is contained in the cistern. By making the cistern very wide as compared with the tube, these changes in the level of the cistern are rendered almost imperceptible under ordinary circumstances, so that for common purposes the variation in the level may be disregarded; if accurate measurements are required, the level of the mercury in the cistern must be regulated by some such mechanical contrivance as Fortin and Horner have invented. In the common barometer these differences are still greater, and as correction is impossible, this instrument is not adapted for exactly measuring the altitude of the column.

3. *De Luc's Syphon Barometer, fig.* 135., consists of a glass tube bent upwards so as to form two parallel legs: the longer is hermetically sealed, and constitutes the Torricellian tube; the shorter is open, and the pressure of the atmosphere is exerted on the surface *a* of the quicksilver. Both legs must be of equal width, at least as far as the changes in the height of the mercury extend. The difference between the levels in the longer and shorter legs is the barometric height. — As the open leg is of equal width with the closed one, the changes in height are equal in both, whence in these barometers it is indispensably necessary in observing the altitude of the mercury to note the change of level in the shorter leg. The most convenient and practicable way of doing this is to adjust a moveable scale between the two legs prior to each observation, so that its zero may coincide with the level of the mercury in the shorter leg.

Fig. 135.



Different contrivances have been adopted to render the minute variations in the atmospheric pressure, and consequently in the height of the barometer, more readily perceptible by enlarging the spaces, all which devices must impair the exact measurement of the length of the column. Of these we may name Morland's Diagonal, and Hook's Wheel-barometer, but especially Huygen's Double-barometer.

Essential properties of a good barometer: width of tube; purity of the mercury, which is effected by distilling it; it must be deprived of air by boiling it in the tube; accurate graduation of the scale, to which a *vernier* should also be added.

Correction of the barometer for the temperature. Heat, as has been already observed § 4., affects the density of bodies. If their temperature be raised, they expand and become lighter, bulk for bulk; if it be lowered, the opposite takes place. The quicksilver in the barometer is subject to the operation of this universal law. If the heat be increased, a similar atmospheric pressure will counterbalance a higher column than at a low temperature. The different heights of the barometer, if compared to obtain their true magnitudes, must consequently be reduced to one standard temperature. For this purpose, it will be most accurate to assume 32°. The most exact experiments of Dulong and Petit have fixed the expansion of the mercury in the barometric tube at $\frac{1}{9990}$ of its volume with every additional degree of heat, and a corresponding contraction for each degree of heat that may be deducted: thus the increased altitude of the column caused by the heat for T degrees above the freezing point must be deducted from the height as shown by the barometer, and for T degrees below that point it must be added to it. Hence we have the following formula by which to correct the barometer:—

$$b = B\left(1 \pm \frac{T}{9990}\right) = B\,(1 \pm 0{\cdot}0001001\ T)$$

B being the altitude observed, T the temperature of the mercury, and b the correct altitude.

§ 198.

The barometer may be used not only to measure the pressure of the free or natural atmosphere, but also to determine the density and expansive force of any other gas.

For this purpose, it is the most convenient and exact gauge for showing the degree of rarefaction attained in the receiver of an air-pump; applied to this purpose, it is called a *Barometer-gauge.* — The object proposed can be attained in two ways; either the external pressure of the atmosphere can be made to drive the mercury up the tube till the desired degree of exhaustion has been reached, or the column in the gauge may be made to sink as the exhaustion proceeds. In the former case, the difference between the altitude of the barometer and that of the gauge shows the density of the rarefied air; and in the latter the gauge itself shows it. The degree of rarefaction is found by dividing the altitude of the barometer by that of the gauge: thus, if the barometer stood at 30 inches, whilst the gauge stood at 1 inch, the air would have been rarefied thirty times; if it stood at $\frac{1}{12}$th of an inch, then the air has been rarefied 360 times.

A pressure gauge of the first-named sort consists of a glass tube about 33 inches long, its upper end communicating with the receiver that is to be exhausted, whilst the lower is plunged in a cistern of mercury. The scale commences from the level of the mercury in the cistern. — These gauges are liable to the same source of error that was noticed in the cistern-barometer. (§ 197.)

The other, generally known as the *Syphon gauge, fig.* 111.

is a shorter syphon-shaped barometer, about six inches in length. The closed leg is filled with quicksilver, which rises in the open leg no higher than the end of the scale whose zero stands in the middle of the tube. The quicksilver in the closed leg will begin to fall as soon as the rarefied air fails by its pressure to counterbalance the weight of the column. The mercury would stand at an equal height in both the legs, if the pressure of the air were altogether annihilated; but whilst it exerts any force at all less than that of the column in the gauge, the difference between the levels in the two legs will express the degree of rarefaction attained. Such a gauge is either placed beneath the receiver of the pump, or, as is more convenient, it is put under a receiver of its own, as in *fig.* 111., which, during the operation of the air-pump, *fig.* 110., is connected by means of the cock L with the receiver and with the communicating pipe of the machine.

In order to determine the pressure or expansive force of gases by the aid of the barometer, they must be inclosed in a tube, and a column of mercury made to act upon them in the place of the atmosphere itself. The quicksilver which is contained in the two legs, as being in vessels that communicate, would stand at an equal height in both; but as the gas or vapour under examination is made to press on the free surface of the mercury in the open leg, it will drive the mercury upwards until its column holds the gas *in equilibrio*. — The *steam-barometer* or *manometer* is used chiefly to measure the force of the steam in boilers.

Fig. 136.

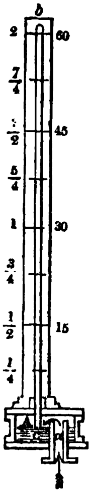
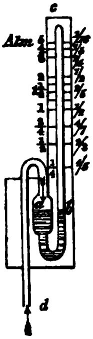


Fig. 136. is a representation of such a barometer. A is a steam-tight vessel, containing quicksilver; *a* is a tube communicating with the boiler, through which the steam flows and presses on the quicksilver. The barometer tube *b c*, open at top, reaches nearly to the bottom of A, having attached to it a scale whose zero point coincides with the level of the mercury in A.

If a very high pressure be exerted, one of several atmospheres for instance, an apparatus thus constructed would require a tube of great length (for each atmosphere 30 inches). In such a case it is more convenient, instead of an open tube, to use a closed one of moderate length, or a Marriotte's tube (§ 195.), which being graduated according to Marriotte's law, will show the amount of pressure.

A *Mariotte's Manometer* is shown in *fig.* 137. The surfaces of the mercury *a* and *b* will stand at the same level so long as no steam is admitted. In the space *bc* above the quicksilver is some air of the ordinary density, the tube at *c* being hermetically sealed. The steam is admitted at *d*, and so pressing on the surface *a* of the mercury forces it above its natural level *b*, and compresses the air in *bc*. The scale from *b* to *c* marks the force of compression in atmospheres. The greater width of the tube at *a* is therefore necessary, in order that the level at *a* may not be materially affected by the ascent of the mercury above *b*, *a* being assumed as the zero of the scale. Instead of this construction, that represented in *fig.* 136. may be adopted if the tube *cb* be hermetically sealed at *b*, and the scale be graduated according to Marriotte's law.

Fig. 137.



§ 199.

The changes in the altitude of the barometer at the same place are in part *regular*, and in part extremely *irregular*. The regular variations occur almost always at the same time of day, attaining daily two *minima* and two *maxima*. They show an ebb and flow to obtain in the atmosphere, similar to what we observe in the sea. Further, they are very small, the oscillation seldom amounting to more than a line, and for the most part observable only in the torrid zone. The irregular changes in its altitude are much greater in degree than the regular, and also are more frequent and observable as we approach the poles; for which reason less notice is paid by us to the regular alterations in the height of the mercury.

The greatest depression of the barometer occurs daily about 4 A. M. and P. M., and its highest elevation about 10 A. M. and P. M. In summer these extreme points are reached from 1 to 2 hours earlier in the morning, and as much later in the afternoon.—The greatest changes at Peru extend only to a few tenths of an inch, whilst in our latitude they amount to from 2 to 3 inches, and further north they exceed even this last measure.

The cause of the irregular changes in the barometer's altitude in the temperate and frigid zones, appears to be principally the frequent variation in the wind, which is produced by the alteration both in the temperature and form of aggregation of the aqueous vapour combined with the atmosphere: these changes are more considerable during the winter than during the summer months. Hence we may perceive why the altitude of the barometer corresponds so frequently with the prevailing state of the weather, and particularly with the predominant wind.

The following remarks may perhaps illustrate this alleged coincidence between the weather and the height of the barometer. The fall of the mercury on the approach of bad weather seems owing principally to a reduction in the elasticity of the aqueous vapour in the atmosphere, which gradually contracts till it becomes condensed into the liquid form, and descends to the earth as rain. The cause of this change of aggregation is to be found in the alteration of temperature and the winds generated by this alteration. Throughout the greater part of western Europe it may be assumed as a general rule, that south winds, which are mostly warm and moist, produce the lowest, and north winds, which usually are cold and dry, produce the highest elevation of the mercury, whilst its mean altitude shows that the wind blows from an intermediate quarter between N. and S. A change in the wind is attested by sudden oscillations in the barometric column; stormy weather sometimes causes an unusual ascent of the mercury, but more commonly a considerable depression. Successive rising and falling of the mercury frequently precedes showery weather. A considerable elevation of the column, however, is not invariably a proof of a dry, nor is a low state of the mercury an indubitable evidence of a moist state of the atmosphere, as both the pressure of the atmosphere and the nature of the weather are greatly affected by currents of air; so although the barometer is used as a *weather-glass*, it cannot in this character be implicitly relied on.

§ 200.

Since the altitude of the barometer depends on the pressure of the atmosphere, and this force again is affected by the height of the column of air resting upon it, it is clear that a shorter column of air will exert a less pressure and sustain a less column of mercury than a longer one. What theory would have led us to expect experience confirms. The quicksilver in the barometer falls as the instrument is carried from the foot to the summit of a mountain, and rises again as it is brought down from this elevation to the surface of the ground; if taken down the shaft of a mine, the barometer will stand at a greater height than at the surface of the earth. At the foot of the mountain the entire column of the atmosphere from its utmost limit presses with its entire weight on the mercury; on the top of the mountain, however, this force must be diminished by the weight of all the intervening strata of air between the summit and foot of the mountain, so that only a shorter column of mercury will be sustained by it. Analogous conclusions may be drawn if the process be reversed; the height of the barometer will be increased by descending from a greater elevation to a less.

It is well known that the surface of our earth is not uniform, consequently it does not sustain an equal column of air at all points, whence the mean altitude of the barometer (§ 197.) will vary at different places; for similar reasons this altitude gradually increases at the level of the sea from the equator towards the poles. (a.)

As has been remarked above, *a very important application of the barometer is measuring the heights of mountains.*

Since the length of the mercurial column invariably depends on and is proportional to the atmospheric pressure, if we know the law by which the altitudes increase, we can determine the difference between the elevation of two places, their mean barometric altitudes being given; or, this difference may be found by observing simultaneously the heights at which the mercury stands at the two stations. Now it has been demonstrated mathematically that *the pressure of the atmosphere decreases in a geometrical progression, as the height of the place of observation increases in an arithmetical progression.* (b.)

In order to measure the height of any place by means of the barometer, the altitude of the mercury must be accurately observed simultaneously at both the lower and upper stations, the logarithms of these two barometric altitudes found, and the less be subtracted from the greater. The difference will be proportional to the difference of the elevations of the two points; and in order to express this last difference in some known measure of length, the logarithmic difference must be multiplied by a constant number previously found both by practice and theory for the various measures of length. — The same process must be gone through if the mean barometric altitudes of the two stations are given.

If great accuracy be required, the calculation becomes more complex, as allowances must be made for the influence of temperature both upon the mercury and the atmosphere; beside which, regard must be had to the degree of moisture in the air, as well as to the irregular variations in the height of the barometer at the places of observation. The improved construction of instruments, and the exact formulæ deduced from mathematical investigation have rendered this method easy of application, and comparatively free from error.

a. The following heights of the barometer taken at the level of the sea for the various degrees of latitude, the temperature being 32° Fahr., may be considered as tolerably exact approximations, according to Munke's experimental and theoretical observations; many anomalies may still be detected in them, owing partly to local peculiarities, and partly, as *von Humboldt* proves, to the imperfect character of the instrument used, as well as to the want of perfect exactness in making the observations.

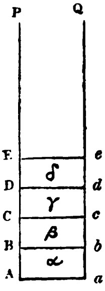
Geog. Latitude.	Mean Altitude of Barometer in Millimètres.
0°	760·214
10	760·341
20	760·706
30	761·266
40	761·951
50	762·682
60	763·367
70	763·927
80	764·290
90	764·414

b. If the atmosphere were an incompressible fluid, like water, it is obvious that on ascending, its pressure would diminish by equal decrements; and that by ascending equal heights the altitude of the mercury which measures this force would sink equal portions for each successive elevation attained. This proportion, however, is not found to obtain, but wherever the barometer stands at a considerable altitude, the atmosphere is of greater density, being compressed by a greater force; a column of air of any definite height at the place where the atmosphere is more dense, will be heavier than a similar column at another station where the compressive force is less, *i. e.* where the barometer stands at a less altitude.

It may be demonstrated that the pressure of the atmosphere and the consequent height of the barometer decrease in geometrical progression as the heights increase in an arithmetical series.

Let A P Q *a*, *fig.* 138., be a column of air extending from the surface of the earth to the limits of the atmosphere. Suppose this column to be divided into pretty equal portions A B, B C, C D, D E, &c., beginning at the base, so that the horizontal lines B*b*, C*c*, D*d*, E*e*, &c. form the boundaries of equal strata. The size of these portions A B, B C, C D, &c. is altogether optional; let them, however, be so small that the density of the air in each stratum may be assumed to be equal throughout. Again, let the density of the different strata be represented by α, β, γ, δ, &c., and the altitudes of the barometer, *i. e.* the atmospheric pressures, by *a*, *b*, *c*, *d*, &c.

Fig. 138.



Having these data, the proportion of the density $\alpha : \beta$ may be expressed in two different ways:

Regarding merely the density, which, with equal volumes, varies as the weight. Now, the weight of the entire column of air from the limit of the atmosphere to A a is equal to the weight of a column of mercury a; the weight from the top B b equals the weight of a column of mercury b. Whence the weight of the lowest stratum A a B $b = a - b$, that of B b C $c = b - c$, and so on. Thus we have

$$\alpha : \beta = a - b : b - c$$

2. According to Mariotte's law the density is as the pressure, so

$$\alpha : \beta = b : c$$

whence it appears that

$$a - b : b - c = b : c$$

$$\text{or } a - b : b \quad = b - c : c$$

$$\text{and} (a - b + b) : b = (b - c + c) : c$$

whence

$$a : b = b : c$$

In like manner it may be proved that

$$b : c = c : d$$

$$c : d = d : e \ \&c.$$

whence it is evident that the magnitudes a, b, c, d, e, &c., or *the altitudes of the barometer, and the densities of the atmosphere at the elevations* A, B, C, D, E, *&c., form a decreasing geometrical series.*

If there were no other causes in operation than the gravity of the atmosphere, the method already explained would be sufficient to compute the difference of altitudes of places by with sufficient accuracy; but since the mercury in the barometer is affected by change of temperature—that the temperature of the air is found to diminish the higher we ascend into it—and also since the mercury in the barometer does not immediately assimilate with the temperature of the external air of the place to which it is carried—it becomes necessary to correct the observations with a view of clearing them from the effect of these several causes. It therefore becomes necessary to employ two thermometers, one to show the temperature of the external air, and the other to show that of the mercury in the barometer. When the observations have been made at both stations, those made at the upper station must be reduced to what they would have been if made under the same circumstances as those at the lower station.

Various methods have been given to effect this object both by English and foreign mathematicians; but the one most general in its character, and embracing in the investigation all the circumstances affecting the results, is given by Laplace in the fourth volume of the *Méchanique Céleste,* which is generally designated Ramond's formula. Several modifications of this formula have been made by different authors; among whom may be mentioned Gauss, Baily, and Littrow.

The author of this treatise has in the text given a method compounded of those of several authors, which being too long for the space that could be afforded for it in the translation, the translator has taken the liberty to substitute the method given by the late Francis Baily, F.R.S., President of the Astronomical Society, in a small work entitled *Tables and Formulæ,* published by him for private circulation; and also that given by Professor Littrow, in the *First Vol.* (*part* 2.) *of the Astronomical Society's Transactions,* pp. 299—307.

BAILY'S METHOD.

Let c = a constant determined by observation, and which is here assumed equal to 60158·53 English feet.
g = increase of gravity from equator to the poles = ·00539.
ϕ = the latitude of the place.

$$\left.\begin{array}{l}\beta = \text{the height of the barometer}\\ \tau = \text{the temperature (Fahr.) of mercury}\\ t = \text{the temperature (Fahr.) of the air}\end{array}\right\}\text{at the lower station.}$$

$$\left.\begin{array}{l}\beta' = \text{the height of the barometer}\\ \tau' = \text{the temperature (Fahr.) of mercury}\\ t' = \text{the temperature (Fahr.) of the air}\end{array}\right\}\text{at the upper station.}$$

a = the expansion of air for 1° of Fahr. = ·0020833.
m = the expansion of mercury for 1° of Fahr. = ·0001001.
r = radius of the earth at ϕ; assumed = 20898240 feet.
h = the height of lower station above the level of the sea; assumed = 4000 feet.
M = the modulus of the common logarithms = ·434294.
χ = the difference of altitude required in English feet.

Then by Ramond's formula—

$$\chi = c\left\{1+\tfrac{1}{2}g\cos.2\phi\right\}\times\left\{1+\frac{2h}{r}\right)\times\left(1+\tfrac{1}{2}(t+t'-64^\circ)\,a\right\}$$
$$\times\left\{\left(1+\frac{\chi}{r+h}\log.\frac{\beta}{\beta'\{1+(\tau-\tau')m\}}+\frac{\chi}{r+h}\times 2\mathrm{M}\right\}$$

"By expanding the last term agreeably to the method adopted by M. Biot, and assuming the numerical values for the several quantities as above, we have

$$\chi = 60345{\cdot}51\left\{1+{\cdot}001111\ (t+t'-64^\circ\right\}\times\log.\text{ of}$$
$$\left\{\frac{\beta}{\beta'}\times\frac{1}{1+{\cdot}0001(\tau-\tau')}\right\}\times(1+{\cdot}002695\cos.2\phi);$$

putting the logarithm of the first term in English feet = A; the logarithm of the last = C; and the logarithm of $1 + \cdot 0001 (\tau - \tau')$ = B, then the formula will be reduced to

$\chi = \text{A} + \text{C} + \log. \left\{ \log. \beta - (\log. \beta' + \text{B} \right\}$; and making $\log. \beta - (\log. \beta' + \text{B}) = \text{D}$, it is reduced to $\text{A} + \text{C} + \log. \text{D} = \chi$.

By assuming the expansion of air for one degree of Fahrenheit = ·00222222, Table G has been computed, in employing which the heights of the barometers may be taken in any measure whatever, whether English, French, German, or any other scale.

Example. M. Humboldt made the following observations on the mountain of Guanaxuato in Mexico, in latitude 21°:—

	Upper Station.	Lower Station.
Thermometer in open air -	$t' = 70{\cdot}4$	$t = 77{\cdot}6$
Thermometer to barometer -	$\tau' = 70{\cdot}4$	$\tau = 77{\cdot}6$
Barometer - - -	$\beta' = 23{\cdot}66$	$\beta = 30{\cdot}05$

What was the difference in the height of the two stations?

By referring to the formula, $\chi = \log. \text{D} + \text{A} + \text{C}$, the operation will stand thus:

$$
\begin{array}{rlrl}
\text{B} & = 0{\cdot}00031 & & \\
\log. \beta' & = \underline{1{\cdot}37401} & & \\
 & \underline{1{\cdot}37432} & & \\
\log. \beta & = \underline{1{\cdot}47784} & & \\
\text{D} & = 0{\cdot}10352 & \log. & = 9{\cdot}01502 \\
 & & \text{A} & = 4{\cdot}81940 \\
 & & \text{C} & = \underline{0{\cdot}00087} \\
 & & 6843{\cdot}7 \log. & = 3{\cdot}83529
\end{array}
$$

Consequently the difference in the altitudes of the two stations was 6843·7 English feet.

The foregoing method, though very simple and elegant, is, notwithstanding, objectionable on account of its requiring the aid of logarithms. To obviate this, Professor *Littrow* has given the following method, which, as it is short, I here give entire.

The Professor states his object to be to give an *exact* method of determining altitudes by barometrical observations more simple than any hitherto known, and such as would enable the traveller to obtain the result of his observations on the spot by a simple calculation, involving only the first three rules of arithmetic.

LITTROW'S METHOD.

"I take for the basis of the whole research the well-known formula of Ramond, which is generally acknowledged to be the best.

"If we call b, t, T, the barometer, the external and interior thermometers (where consequently T represents the temperature of the mercury and t that of the outward air) for the upper station; and b', t', T', the same things for the lower; ϕ the geographical latitude; and lastly, H, the difference of altitudes of the two stations in toises, then Ramond's formula is—

$$\left.\begin{aligned} H = \log. \big[9436{,}966\,(1 + 0{\cdot}00284\cos. 2\phi)\,(1 + 0{\cdot}0025\,(t + t')\big] \\ \times \left[\frac{b'}{[1 + 0{\cdot}00023\,(T' - T)\,b}\right] \end{aligned}\right\}\ \mathrm{I}$$

"We must now endeavour to express this somewhat complicated formula by a small table, so constructed as to require only the three first rules of arithmetic without the need of any knowledge of proportions of logarithms, &c.

Thermometer in open Air.				Thermometer attached to Barometer.		Latitude of the Place.	
$t+t'$	A	$t+t'$	A	$\tau-\tau'$	B	ϕ	C
°		°		°		°	
40	4·76891	102	4·79860	0	0·00000	0	0·00117
42	4·76989	104	4·79953	1	0·00004	5	0·00115
44	4·77089	106	4·80045	2	0·00009	10	0·00110
46	4·77187	108	4·80137	3	0·00013	15	0·00100
48	4·77286	110	4·80229	4	0·00017	20	0·00090
50	4·77383	112	4·80321	5	0·00022	25	0·00075
52	4·77482	114	4·80412	6	0·00026	30	0·00058
54	4·77579	116	4·80504	7	0·00030	35	0·00040
56	4·77677	118	4·80595	8	0·00035	40	0·00020
58	4·77774	120	4·80687	9	0·00039	45	0·00000
60	4·77871	122	4·80777	10	0·00043	50	9·99980
62	4·77986	124	4·80869	11	0·00048	55	9·99960
64	4·78065	126	4·80958	12	0·00052	60	9·99942
66	4·78161	128	4·81048	13	0·00056	65	9·99925
68	4·78257	130	4·81138	14	0·00061	70	9·99910
70	4·78353	132	4·81228	15	0·00065	75	9·99900
72	4·78449	134	4·81317	16	0·00069	80	9·99890
74	4·78544	136	4·81407	17	0·00074	85	9·99885
76	4·78640	138	4·81496	18	0·00078	90	9·99883
78	4·78735	140	4·81585	19	0·00083		
80	4·78830	142	4·81675	20	0·00087		
82	4·78925	144	4·81763	21	0·00091		
84	4·79019	146	4·81851	22	0·00096		
86	4·79113	148	4·81940	23	0·00100		
88	4·79207	150	4·82027	24	0·00104		
90	4·79301	152	4·82116	25	0·00109		
92	4·79395	154	4·82204	26	0·00113		
94	4·79488	156	4·82291	27	0·00117		
96	4·79582	158	4·82379	28	0·00122		
98	4·79675	160	4·82466	29	0·00126		
100	4·79768	162	4·82553	30	0·00130		

"Besides which, the computation must be short and easy, and the result *perfectly exact.*

" To attain this end, let us take the two magnitudes M m, so as to have

$$M = 9436{\cdot}966\ (1 + 0{\cdot}0025\ (t+t')$$
$$m = 1 + 0{\cdot}00023\ (T'—T)$$

If, then, for a while we leave out the consideration of the factor dependent on ϕ, Ramond's formula gives

$$H = M \log. \frac{b'}{m\ b}$$

Suppose, then, we assume

$$H = M \log. \frac{b'}{b} + x$$

Equating the two values of H, and observing that $m-1$ is always a very small quantity compared with unity, we find

$$x = -\ 0{\cdot}4342945\ M\ (m-1)$$

Or restoring the values of M and m,

$$x = -\ {\cdot}094264\ (T' - T) - 0{\cdot}0023565\ (T' - T)\ (t' + t)$$

" After this introduction it is easy to give the formula (I) the following form, where $\theta = \frac{t'+t}{4}$

$$H = [M \log. \frac{28{\cdot}16666}{b} + (0{\cdot}943 + 0{\cdot}0096\ \theta)\ T]$$

$$-\ [M \log. \frac{28{\cdot}16666}{b'} + (0{\cdot}943 + 0{\cdot}0096\ \theta){\cdot}T']\ \text{II.}$$

an expression which it is very easy to reduce into a table having the above-mentioned properties. I assume, for instance, that the tables shall give for every value of b the quantity

$$A = 9436{\cdot}966 \log. \frac{28{\cdot}16666}{b}$$

and then proceed as follows :—

" We first seek A for the upper station, entering the table with b; then place the decimal point in A two places forward, and multiply this new number by θ. We lastly seek $B = 0{\cdot}943 + 0{\cdot}0096\theta$, and multiply this number by T. The sum of these three quantities is an approximate value of the height h of the first station above a certain point.

" In other words

$$h = A + \frac{A\ \theta}{100} + B\ T$$

" In like manner we seek the quantity

$$h' = A' + \frac{A'\ \theta}{100} + B\ T'$$

and the difference of altitudes required will be in toises

$$H = h - h'$$

We see that the quantities θ and B are the same for both the two stations, and need therefore only be computed once. The thermometer here is supposed to be Reaumer's. If we use Fahrenheit's or any other scale, it must be reduced to Reaumur's. The barometrical altitudes, $B\,b'$ may, however, be taken in any scale, English, French, and German, &c., and require no reduction.

"If we would avoid computing B, we may use the following little table:—

θ	B	θ	B
—10°	0·85	+0°	0·94
9	0·86	1	0·95
8	0·87	2	0·96
7	0·88	3	0·97
6	0·89	4	0·98
5	0.90	5	0·99
4	0·91	6	1·00
3	0·92	7	1·01
2	0·93	8	1·02
1	0·94	9	1·03
0	0·95	10	1·04
		11	1·05
		12	1·06
		13	1·07
		14	1·08
		15	1·09

"For the factor in ϕ I have provided by a peculiar and very convenient table, which follows from the equation

$$\text{Corrected } H = H' \, (1 + 0{\cdot}00284 \cos. 2\,\phi).$$

"To find the errors to which these interesting observations are usually liable, the equation (I) gives, making H, t, and t' variable,

$$\frac{d\,H}{H} = 0{\cdot}0025 \frac{dt + dt'}{a}$$

putting $a = 1 + 0{\cdot}0025\,(t + t')$; whence it follows that a degree of error in the construction or reading off of the two exterior thermometers renders H erroneous by 0·0025 part of its value, and thus produces errors as follows:—

	Toises.		Toises.
In an altitude of	400	an error of	1
„	800	„	2
„	1200	„	3
	&c.		&c.

We must therefore take care to be provided with good thermometers when the altitudes are great, if we would obtain correct results.

"If we make H T T′ variable, we have

$$\frac{d\,\mathrm{H}}{\mathrm{H}} = \frac{0{\cdot}943}{\mathrm{H}} \,.\, a \,.\, (d\,\mathrm{T} - d\,\mathrm{T}')$$

Whence it follows that the interior thermometers, or those attached to the mounting of the barometer, exert a less hurtful influence in producing error the greater the altitude is.

"For the levelling of rivers, streets, &c., the exterior thermometer requires to be particularly good.

"Finally, if we make H$b\,b'$ variable,

$$\frac{d\mathrm{H}}{\mathrm{H}} = \frac{4098{\cdot}4224}{\mathrm{H}}\, a \left(\frac{db'}{b'} - \frac{db}{b}\right)$$

"Whence it follows that an error in the barometer is the most important of all, and that this relative error $\frac{d\mathrm{H}}{\mathrm{H}}$ is so much greater, the smaller is the altitude H. Hence, for the purposes last enumerated, the barometer should be used with great caution. It remains still to show how one may conclude the altitude above the sea from one insulated observation. It is well known that this process does not give the altitude so exactly as the method of corresponding observations. We require first the height of the barometer b' on the level of the sea at the time of observation.

"By a great number of observations I find that we may take

In. Lines. In.
$b' = 28$ „ $2 = 28{\cdot}16666$ Paris.

"It is more difficult to estimate the temperature t' on the level of the sea at the time of observation. Lindenau has employed a number of observations of Saussure (in the 11th volume of the *Monatlichen Correspondenz* of Baron Von Zach, and more lately in his *Tables Barométriques*, Gotha, 1809), and grounded thereon a table. I find that this table represented very well such observations as I could collect on this point, and that the number in it might be represented with sufficient accuracy for our purpose, by the following simple equation,

$$t' \text{ or } \mathrm{T}' = 53 + t - 2\,b;$$

when t and b represent the heights of the exterior thermometer and the barometer at the upper station. It is right to elucidate the use of the tables by an example, by which their convenience will appear."

Example 1.

Upper Station.
In. Lin.
$b = 22 \quad 5{\cdot}6$
$t = 8^\circ{\cdot}4$ Réaum.
$T = 9{\cdot}3$

Lower Station.
In. Lin.
$b' = 27 \quad 3{\cdot}2$
$t' = 15{\cdot}2$
$T' = 164$

$\phi = 58^\circ{\cdot}0'$

This gives $\theta = 5{\cdot}9$ . . . . $B = 1{\cdot}0$
935·9 . . . . 135·6
—9·2 . . . . —2·5
$A = 926{\cdot}7$ . . . . 133·1
$\frac{A\,\theta}{100} = 54{\cdot}7$ . . . . 7·9
$B\,T = 9{\cdot}3$ . . . . 16·4
$h = 990{\cdot}7$ . . . . $h' = 157{\cdot}4$
$h' = 157{\cdot}4$
$H = 833{\cdot}3$
Cor. for ϕ —1·0
H (cor.) = 832·3 toises.

Example 2.

Upper Station.
In. Lin.
$b = 19 \quad 8{\cdot}7$
$t = -\ 12{\cdot}3$
$T = -\ 10{\cdot}5$
$\phi = 50^\circ \; 0' \quad \theta = -2{\cdot}5$

Lower Station.
In. Lin.
$b' = 26 \quad 4{\cdot}9$
$t' = 2{\cdot}3$
$T' = 3{\cdot}5$
$B = 0{\cdot}919$

Consequently,

1472·3 . . . . 275·9
−12·1 . . . . −6·4
1460·2 . . . . 269·5
−36·5 . . . . −6·7
—9·6 . . . . +3·2
$h = 1414{\cdot}1$ . . . . $h' = 266{\cdot}0$
$h' = 266{\cdot}0$
$H = 1148{\cdot}1$
−0·5
$H' = 1147{\cdot}6$ toises.

Example 3.

Let

$b = 19$ in. 10·14 lines

$t = 3·2$

$T = 7·6$

required the height above the sea.

This gives $t' = T = 16·5$

$\theta = 4·92$

$B = 0·99$

Consequently,

$$\begin{array}{rr} & 1437·7 \\ & -2·4 \\ \hline A = & 1435·3 \\ \frac{A\,\theta}{100} = & 70·7 \\ B\,T = & 7·5 \\ \hline & 1513·5 \\ & 16·3 \\ \hline \end{array}$$

1497·2 toises = the height required.

TABLE I.

Barom.		A.	Diff. for 1-10 line.	Barom.		A.	Diff. for 1-10 line.	Barom.		A.	Diff. for 1-10 line.
In.	Lin.			In.	Lin.			In.	Lin.		
29.	6	−189·5	1·16	27	8	73·5	1·23	25	10	354·5	1·31
	5	−177·9	1·16		7	85·8	1·24		9	367·6	1·34
	4	−166·3	1·16		6	98·2	1·25		8	381·0	1·33
	3	−154·7	1·17		5	110·7	1·24		7	394·3	1·33
	2	−143·0	1·18		4	123·1	1.25		6	407·6	1·36
	1	−131·2	1·17		3	135·6	1·25		5	421·2	1·33
29	0	−119·5	1·19		2	148·1	1·27		4	434·2	1·35
28	11	−107·6	1·18		1	160·8	1·26		3	448·0	1·35
	10	− 95·8	1·18	27	0	173·4	1·27		2	461·5	1·36
	9	− 84·0	1·20	26	11	186·1	1·27		1	475·1	1·37
	8	− 72·0	1·19		10	198·8	1·27	25	0	488·8	1·37
	7	− 60·1	1·19		9	211·5	1·29	24	11	502·5	1·38
	6	− 48·2	1·21		8	224·4	1·28		10	516·3	1·38
	5	− 36·1	1·20		7	237·2	1·28		9	530·1	1·38
	4	− 24·1	1·20		6	250·0	1·30		8	543·9	1·39
	3	− 12·1	1·21		5	263·0	1·29		7	557·8	1·39
	2	0·0	1·22		4	275·9	1·29		6	571·7	1·39
	1	12·2	1·21		3	288·8	1·30		5	585·7	1·39
28	0	24·3	1·23		2	301·8	1·32		4	599·6	1·40
27	11	36·6	1·22		1	315·0	1·30		3	613·6	1·41
	10	48·8	1·23	26	0	328·0	1·33		2	627·7	1·42
	9	61·1	1·24	25	11	341·3	1·32		1	641·9	1·42

TABLE I.—*continued.*

Barom.		A.	Diff. for 1-10 line.	Barom.		A.	Diff. for 1-10 line.	Barom.		A.	Diff for 1-10 line.
In.	Lin.			In.	Lin.			In.	Lin.		
24	0	656·1	1·43	20	11	1219.7	1.64	17	10	1873·3	1·93
23	11	670·4	1·43		10	1230 1	1.65		9	1892·6	1·93
	10	684·7	1·44		9	1252.6	1·65		8	1911·9	1·93
	9	699·1	1·44		8	1269·1	1·65		7	1931·2	1·94
	8	713·5	1·44		7	1285·6	1·65		6	1950·6	1·96
	7	727·9	1·45		6	1302·1	1·68		5	1970·2	1·97
	6	742·4	1·46		5	1318·9	1·67		4	1989·9	1·97
	5	757·0	1·46		4	1335·6	1·68		3	2009·6	1·97
	4	771·6	1·46		3	1352·4	1·68		2	2029·3	2·00
	3	786·2	1·47		2	1369·2	1·71		1	2049·3	2·01
	2	800·9	1·48		1	1386·3	1·70	17	0	2069·4	2·02
	1	815·7	1·48	20	0	1403·3	1·72	16	11	2089·6	2·02
23	0	830·5	1·49	19	11	1420·5	1·72		10	2109·8	2·03
22	11	845·4	1·50		10	1437·7	1·73		9	2130·1	2·04
	10	860·4	1·50		9	1455·0	1·73		8	2150·5	2·06
	9	875·4	1·51		8	1472·3	1·74		7	2171·1	2·06
	8	890·5	1·50		7	1489·7	1·74		6	2191·7	2·07
	7	905·5	1·51		6	1507·1	1·76		5	2212·4	2·09
	6	920·6	1·53		5	1524·7	1·77		4	2233·3	2·10
	5	935·9	1·52		4	1542·4	1·76		3	2254·3	2·10
	4	951·1	1·53		3	1560·0	1·77		2	2275·3	2·13
	3	966·4	1·53		2	1577·7	1·79		1	2296·6	2·13
	2	981·7	1·55		1	1595·6	1·80	16	0	2317·9	2·15
	1	997·2	1·55	19	0	1613·6	1·80	15	11	2339·4	2·15
22	0	1012·7	1·56	18	11	1631·6	1·81		10	2360·9	2·17
21	11	1028·3	1·56		10	1649·7	1·82		9	2382·6	2·17
	10	1043·9	1·58		9	1667·9	1·83		8	2404·3	2·18
	9	1059·7	1·57		8	1686·2	1·83		7	2426·1	2·19
	8	1075·4	1·58		7	1704·5	1·83		6	2448·0	2·22
	7	1091·2	1·57		6	1722·8	1·86		5	2470·2	2·22
	6	1106·9	1·60		5	1741·4	1·86		4	2492·4	2·23
	5	1122·9	1·60		4	1760·0	1·86		3	2514·7	2·24
	4	1138·9	1·60		3	1778·6	1·87		2	2537·1	2·26
	3	1154·9	1·60		2	1797·3	1·89		1	2559·7	2·27
	2	1170 9	1·62		1	1816·2	1·89	15	0	2582·4	
	1	1187·1	1·63	18	0	1835·1	1·91				
21	0	1203·4	1·63	17	11	1854·2	1·91				

TABLE II. — LATITUDE.

H	+ 44° − 46°	+ 42° − 48°	+ 40° − 50°	+ 38° − 52°	+ 36° − 54°	+ 34° − 56°	+ 32° − 58°	+ 30° − 60°
Toises.								
50	0·0	0·0	0·0	0·0	0·0	0·0	0·1	0·1
100	0·0	0·0	0·0	0·0	0·1	0·1	0·1	0·2
200	0·0	0·1	0·1	0·1	0·2	0·2	0·2	0·3
300	0·0	0·1	0·1	0·2	0·3	0·3	0·4	0·4
400	0·0	0·1	0·2	0·2	0·3	0·4	0·5	0·6
500	0·0	0·1	0·2	0·3	0·4	0·5	0·6	0·7
600	0·1	0·2	0·3	0·4	0·5	0·6	0·7	0·8
700	0·1	0·2	0·3	0·4	0·6	0·7	0·9	1·0
800	0·1	0·2	0·4	0·5	0·7	0·8	1·0	1·1
900	0·1	0·3	0·4	0·6	0·8	0·9	1·1	1·2
1000	0·1	0·3	0·5	0·7	0·9	1·0	1·2	1·4
1100	0·1	0·4	0·5	0·7	1·0	1·2	1·4	1·6
1200	0·1	0·4	0·6	0·8	1·0	1·2	1·5	1·8
1300	0·1	0·4	0·6	0·8	1·1	1·3	1·6	1·9
1400	0·1	0·5	0·7	0·9	1·2	1·4	1·7	2·0
1500	0·2	0·5	0·7	1·0	1·3	1·6	1·9	2·2
1600	0·2	0·5	0·8	1·1	1·4	1·7	2·0	2·3
1700	0·2	0·5	0·8	1·1	1·5	1·8	2·1	2·4
1800	0·2	0·6	0·9	1·2	1·6	1·9	2·2	2·6
1900	0·2	0·6	0·9	1·3	1·1	2·0	2·4	2·8
2000	0·2	0·6	1·0	1·3	1·7	2·1	2·5	2·9
2100	0·2	0·7	1·1	1·4	1·8	2·2	2·6	3·1
2200	0·2	0·7	1·1	1·5	1·9	2·3	2·7	3·2
2300	0·2	0·7	1·2	1·6	2·0	2·4	2·9	3·4
2400	0·2	0·7	1·2	1·6	2·1	2·5	3·0	3·5
2500	0·3	0·8	1·3	1·7	2·2	2·6	3·1	3·6
2600	0·3	0·8	1·3	1·8	2·3	2·7	3·2	3·7
2700	0·3	0·8	1·4	1·9	2·4	2·9	3·4	3·9
2800	0·3	0·9	1·4	1·9	2·5	3·0	3·5	4·0
2900	0·3	0·9	1·5	2·0	2.5	3·1	3·6	4·1
3000	0·3	0·9	1·5	2·0	2·6	3·2	3·7	4·2

V. RELATIONS SUBSISTING BETWEEN SOLID AND GASEOUS BODIES IN THE ATMOSPHERE, AS REGARDS THEIR DIFFERENT DENSITIES, AND THE PRINCIPAL CAUSES OF ATMOSPHERIC DISTURBANCES.

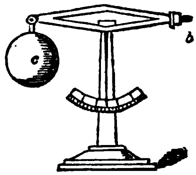
§ 201.

It has been already remarked that if a solid be completely immersed in a liquid, the former loses such a portion of its weight as is equal to the weight of a similar volume of the fluid (§ 165.). Now since our atmosphere is a ponderable fluid, it follows that every body contained in it, being subjected to this law, must lose a portion of its weight equal to that of the air it displaces.

This affects the accuracy of our results in determining the specific gravity of bodies, as the weight lost by any particular substance in air is greater in proportion to its volume, *i. e.* as the density of its mass is diminished. In ordinary cases this difference may be disregarded, as a body of the same density with water would lose only the $\frac{1}{800}$th part of its weight, air being at a mean temperature 800 times lighter than water. That bodies do actually sustain such a loss in weight when suspended in air, or, in other words, that they become heavier when weighed *in vacuo*, or in a highly rarefied medium, may be shown by the following experiment: —

Exp. If to the arm of a small balance, *fig.* 139., there be suspended a light but bulky body *c*, as, *ex. gr.*, a hollow sphere of thin glass or copper, and on the opposite arm *b*, a counterpoise be hung of some substance of great specific gravity, and consequently of small volume, on placing the apparatus beneath a receiver and exhausting the air, the equilibrium of the balance will be disturbed, and that arm which carries the body whose volume is greater descends, whilst the other rises. On readmitting the air the beam recovers its horizontal position. — The absence of the air is obviously the cause of the circumstances just described; in the rarefied medium the body which had the greater volume was heavier than the other, which had counterbalanced it when hung in air, consequently it was heavier than it had itself been then, and this excess of weight disappeared on the readmission of the air. If the volumes of the two bodies had been equal, they would have lost equal weights in air, and would therefore have been *in equilibrio* also when placed in the exhausted receiver.

Fig. 139.



The instrument described above was invented by Otho von Guericke, and is called a *Dasymeter*; sometimes also it is spoken of as a *Manometer*, being used instead of a barometer to test the density of the air.

§ 202.

The observations made, § 164., about bodies immersed in liquid are equally true as respects those existing in air. Thus every substance whose density exactly corresponds with that of the atmosphere, when immersed in it loses all its weight; it will float in the air, and be carried about like an integral part of it. On the other hand, if the body be denser than the air, it descends to the earth with a force equal to the excess of its weight above that of an equal volume of air; lastly, if it be lighter than this fluid, it is borne upwards by the pressure of the particles of air with a force equal to the difference of its weight and that of an equal volume of air (§ 169.). Now if the atmosphere were of uniform density throughout, such a body would be elevated to its highest regions, where it would float as a piece of wood floats on the surface of water. As, however, the density of the atmosphere decreases as the altitude increases (§§ 181. and 196.), the body in the course of its ascent is constantly entering into a rarer and still rarer stratum, until at length it arrives at one of equal density, *i. e.* of equal specific gravity with itself, where it will float, and above which it will be unable to rise.

§ 203.

The science of *Aerostation* depends on the statical operations of our atmosphere. In order that a body may ascend in the air, it needs only that it be connected with some other body which, with an inconsiderable weight, has such a volume that the air displaced by both the bodies weighs more than the sum of their weights.

For this purpose hollow spheres are well adapted, composed of flexible and air-tight materials, and filled with some elastic fluid whose specific gravity is much less than that of atmospheric air. The bag for such *air-balloons*, or *aerostats*, is sometimes of paper, and the air which it contains is rarefied by means of a fire kindled beneath a pretty large opening in the under part of the balloon; sometimes a spheroid of fine silk, well varnished, is substituted for this, and inflated with hydrogen gas, which is from 7 to 15 times lighter than the atmosphere. The first of these two kinds of balloons is called a *Montgolfier*, from the name of its inventor.

The force with which a balloon rises, or is borne upwards by the air, is found by subtracting its weight and that of the car together, with the weight of the gas with which it is filled, from that of an equal bulk of common air. (a)

> Cavallo and Lichtenberg, in 1782, performed the first experiments with soap bubbles inflated with hydrogen gas. The following year the brothers Montgolfier constructed the first Montgolfier balloon, at Annonay, in France. The same year Professor Charles performed his first experiments at Paris with those balloons which have sometimes been called, after him, Charliers. Accounts have been published of the aerial trips of Pilatre de Rozier, Romains, Blanchard, Count Zambeccari, Garnerin, and Professor Reichard. Sacharow and Robertson, on the 30th of June, 1804, undertook an ascent from Petersburg, for purely scientific objects. A similar trip was made by Biot and Gay-Lussac, who ascended from Paris on the 24th of August in the same year. The latter rose to an elevation of 21,474 Paris, or about 23,000

English feet. To diminish the great expense of preparing a supply of hydrogen, an English aeronaut, Green, has substituted the carburetted hydrogen that is used for the purpose of illumination. In November, 1836, he, with two companions, travelled in such a balloon from London to the neighbourhood of Coblenz in about 19 hours.

First Exp. The ascent of a Montgolfier by heating the inclosed air.—The balloon should be made of fine tissue-paper, and should contain at least 4 or 5 cubic feet. At the lower part is an aperture 10 or 12 inches in diameter, within the circumference of which opening two wires cross each other; from the point at which they intersect a wire hook is suspended, carrying a wisp of cotton saturated with spirits of wine, which, being lighted, quickly heats and rarefies the air inclosed in the balloon.

Smaller Montgolfiers may be made to ascend by the aid of such mechanical means as will overcome their weight. Tie a thread to the balloon, and carry it over a pulley in the ceiling of a room, and hang to the other end of it a small weight which will exactly counterbalance the balloon. If the flame of a spirit-lamp be held beneath the opening, the apparatus will immediately rise.

Second Exp. The ascent of soap-bubbles and small balloons filled with hydrogen.

To fill soap-bubbles with hydrogen, attach to a bladder containing this gas a common tobacco-pipe, or a metallic tube, with a stop-cock; plunge the end of the pipe below the surface of the soapy water, turn the cock, and compress the bladder; bubbles will be formed which, as they disengage themselves from the end of the tube, will mount upwards.

The small balloons that are usually sold in the shops are made of turkeys' craws.—To fill them, insert a curved tube into the neck, which must be well secured, by tying with thread, that there may be no escape of gas, the other end of the tube being connected with the discharging cock *d* of the gasometer (*fig.* 126.), which contains the hydrogen gas. Open the cock, and the balloon is quickly filled; disengage its neck from the tube, tie it securely with a thread, and it will at once rise in the air. Observe that the common air must be expelled from the balloon before it is fastened to the tube.

(a.) Supposing a balloon to be perfectly spherical, its content in cubic feet will be $0{\cdot}5236\ d^3$. Next allow that it is but three-fourths full of gas; this would amount to $0{\cdot}3927\ d^3$ solid feet. Now if one cubic foot of gas weigh x lb., the weight of that which the balloon contains will be $0{\cdot}3927\ d^3\ x$; add to this the weight of the balloon itself with its car $=$ B then the whole weight which it is required to raise in the air$=0{\cdot}3927\ d^3\,x+\mathrm{B}$. Let a lb. be the weight of a cubic foot of air of mean density and heat ($=\frac{1}{800}$th the weight of 1 cubic foot of water), then the weight of that mass of air which the apparatus will displace will be $0{\cdot}3927\ d^3\,a$, whence the upward force of the balloon

$$\begin{aligned} f &= 0{\cdot}3927\ d^3\,a-[0{\cdot}3927\ d^3\,x+\mathrm{B}] \\ &= 0{\cdot}3927\ d^3\,[a-x]-\mathrm{B}. \end{aligned}$$

If the air contained in a Montgolfier be heated from 32° to 212° F. it will be expanded 0·375 of its volume (the freezing point being assumed as unity), and will therefore be just that much lighter: substitute for this decimal the vulgar fraction $\frac{1}{3}$, and call the entire weight of the included air F, then from $\frac{\mathrm{F}}{3}$ deduct B, and the remainder is the force with which it will ascend. Now, as hydrogen gas, when prepared in large quantities, is never more than nine times lighter than the atmosphere, the ascending force of a balloon inflated with this gas will be $\frac{8}{9}$ F, whence the upward forces of these two kinds of balloons are as $\frac{1}{3} : \frac{8}{9} = 3 : 8$, or nearly $1 : 3$, whence a balloon filled with gas may be one third the size of a Montgolfier in order that it may mount with an equal force. Gas obtained from coal is on an average half the weight of atmospheric air, so the upward force of a balloon filled with it $=\frac{1}{2}$ F., and the forces of three balloons of equal dimensions, filled in these three different modes, will be as $1 : 1\frac{1}{2} : 2\frac{2}{3}$.—Reason why balloons are not completely filled with gas?—Use of valve at the top of the balloon.—Use of ballast.—Mode in which the *Parachute* acts.

§ 204.

If the air at the same distance from the earth's surface were of uniform density and elasticity throughout, no motion could occur in the atmosphere; but such uniformity does not obtain at different places, nor does it remain undisturbed even at any one place. Consequently, complete repose does not prevail in the atmosphere, for, from various and frequently from small causes, the equilibrium of a fluid so easily set in motion is frequently disturbed, and the movements generated, according to the variety in the exciting cause, are transmitted with a greater or less force and velocity, and extend through a wider or more limited space. These currents of air are known under the appellations of *Winds*, *Storms*, or *Hurricanes*, according to their respective velocities; sometimes they are named from the regions of the earth in which they prevail, and at others from their peculiar characteristics.

If air thus set in motion meet with any resistance, it will exert upon the obstacle an impact similar in its effects to that produced by water impinging on any body, modified in some degree by the great compressibility of the particles of the former fluid among each other. That currents of air are employed as motive powers, the reader will observe in the case of ships and windmills, whilst their increased power is displayed as resulting from a high degree of velocity in the destructive effects of hurricanes and tornados.

The most general cause of currents of air is the unequal degree in which the atmosphere is heated. Heat, by expanding the air, increases its elasticity, and at the same time lessens its density. The hotter, and specifically lighter air, agreeably to the laws above explained, must ascend into the denser strata (just as the Montgolfier rises), and thereby occasion a flowing together of cool air to fill the place left vacant by the ascent of the hot air.

On this principle the following circumstances may be explained; in heated rooms the upper strata of air are invariably found to be warmer than the lower. Open the door of a hot room a little way, and the heated air will escape at the top of the door, whilst the cold draught sets in at the bottom. This current is most perceptible near the stove and chimney, at which the heated and lighter air escapes, and in which direction the draught rushes to replace it. Winds and local breezes have in part a similar origin. Thus at night the land-breeze, or wind from the islands, sets towards the sea, whilst, during the day the sea-breeze blows towards the shore off the sea, the latter being sooner heated by the sun. On hot days local currents may be observed near the boundaries of the shadows cast by the clouds. Again, to this cause is owing the motion of ventilators in the windows of rooms, and the revolution of little paper fly-wheels, or spiral serpents, balanced on an upright stem, and placed on the hob of a stove.

Another cause generating a perpetual current of air, is the diurnal motion of the earth, which, in consequence of the inertia of the atmosphere, produces a motion from east to west, being in an exactly opposite direction to that in which the earth itself turns.

Instruments which serve to measure the velocity of the wind are called *anemometers.*

§ 204. That the agitation of the atmosphere by winds is of the last importance to the well-being of both the animal and vegetable world, will appear from the fact that "The proper, constant, and inexhaustible sources of oxygen gas are the tropics and warm climates, where a sky, seldom clouded, permits the glowing rays of the sun to shine upon an immeasurably luxuriant vegetation. The temperate and cold zones, where artificial warmth must replace deficient heat of the sun, produce, on the contrary, carbonic acid in superabundance, which is expended in the nutrition of the tropical plants. The

§ 205.

It has been demonstrated above, that as regards the strata of a homogeneous atmosphere, they will arrange themselves according to their specific gravities, the lighter rising through and floating above such as are more dense; such an arrangement, however, does not obtain when the atmosphere contains various vapours and gases, not even if they are those which constitute it. On the contrary, it has been found that all gaseous bodies, without chemically combining, extend and mix one with another so uniformly that gases of most unequal densities are found in juxta-position.

Even gases confined in closed vessels display this tendency to an equilibrium of mixture, if separated from the atmosphere, or from some other gas, by a porous lamina. Thence arises a universal and reciprocal mixing of the gases, carried on through the capillary tubes of the containing vessel. This process is spoken of as the *diffusion* of gases, and closely corresponds with the *Endosmosis* which obtains under similar circumstances between fluids. (See *Exp.* 11. § 54.)

It is a well-known fact that our atmosphere consists of oxygen and nitrogen. Though the latter is specifically lighter than oxygen, and does not enter into a chemical combination with it, yet at every possible elevation these two gases have been found distributed in one uniform and constant proportion.

First Exp. If carbonic-acid gas, which is about half as heavy again as common air, be put into an open vessel, in a short time a stratum of the gas will be found only at the bottom of the glass; and in no very long period this will be dissipated, and the glass will contain only the atmospheric air with the same quantity of carbonic-acid gas as the rest of the air in the room.

Second Exp. Fill a flask with any gas, close the mouth with a piece of moist bladder tied pretty firmly; if the gas be heavier than the common air, let the mouth be upwards;

---

same stream of air, which moves by the revolution of the earth from the equator to the poles, brings to us, in its passage from the equator, the oxygen generated there, and carries away the carbonic acid formed during our winter.

" The experiments of De Saussure have proved that the upper strata of the air contain more carbonic acid than the lower, which are in contact with plants; and that the quanity is greater by night than by day, when it undergoes decomposition.

" Plants thus improve the air, by the removal of carbonic acid, and by the renewal of oxygen, which is immediately applied to the use of man and animals. The horizontal currents of the atmosphere bring with them as much as they carry away, and the interchange of air between the upper and lower strata, which their difference of temperature causes, is extremely trifling when compared with the horizontal movements of the winds." —*Liebig's Organic Chemistry of Agriculture and Physiology*, p. 23.

but if the contrary, invert the flask, and in either case it will soon be found that the gas has disappeared, and that the flask has become filled with the ordinary atmospheric air.

Hydrogen gas, in consequence of this diffusion, escapes after some considerable time from glass-vessels inverted in mercury, to cut off communication with the external air.

§ 206.

Besides the currents already noticed as arising in the open air, there are likewise motions, viz. an influx and efflux between such portions of the atmosphere as may be contained in two enclosed spaces that communicate with each other, or between the main body of the atmosphere and such an enclosed space, provided only that there be a communication between them. In both these cases draughts will set in if the aeriform fluids be of unequal densities, as the denser air will flow into that which is more rare.

Thus we see the origin of the draughts which are experienced in chimneys, in shafts and air-passages of tunnels and mines, as well as in all kinds of apparatus for warming rooms, where, as has already (§ 204.) been shown, the difference of the temperature of the air in the apartment, as compared with that of the external atmosphere, is the cause of its rarefaction; the application of the various sorts of draughts or blasts for furnaces, the action of steam-engines, of air-guns, and of fire-arms, all depend on the escape of the air which they severally contain, into the rarer atmosphere.

The laws of motion which the elastic fluids obey, are in the main analogous to those to which the non-elastic fluids are subject; they are, however, variously modified, and are therefore more complex than the latter in consequence of the great compressibility of aeriform bodies, and the changes in their expansibility arising from an alteration in their temperature.

If a gas of any density and expansive force whatever issue from an aperture in the bottom of a vessel into a vacuum, its velocity in the first second of time will be

$$v = \sqrt{2gb.\frac{q}{d}}$$

Suppose, for instance, that in the space from which the air had been exhausted a second vessel were placed, containing to a height *h*, a certain non-elastic fluid, whose density equalled that of the air at the bottom of the first-named vessel; in this case it is obvious that the bases of both the vessels sustain equal pressures; the air will therefore issue from its orifice with the same velocity as the non-elastic fluid, whose velocity in the first second of time is $\sqrt{2gh}$ (see § 158.). Now we have only to determine the altitude h. Let the density of the air at the orifice be represented by d, the pressure of that point on the base of the vessel = the height b of a column of mercury, and lastly the specific gravity of the mercury q; then, according to § 154.,

$$d : q = b : h$$

$\therefore\ h = b.\frac{q}{d}$, whence is deduced the above formula

$$v = \sqrt{2gb.\frac{q}{d}}$$

But since d varies as the temperature of the air varies, it follows that v must do so likewise; *ex. gr.*, when the thermometer stands at the freezing point, and the altitude of the barometer is 30 inches $= 2\frac{1}{2}$ *ft.*, $d = \frac{1}{770}$ (§ 190.), $q = 13{\cdot}59$; whence h or $b.\frac{q}{d} = 2{\cdot}5.\frac{13{\cdot}59}{\frac{1}{770}} = 26160$ *ft.*; allowing $g = 32{\cdot}2$ ft., we obtain

$$v = \sqrt{2 \times 32{\cdot}2 \times 26160} = 1297 \text{ ft.}$$

The atmosphere will consequently rush into any space entirely, or almost, void of air with the velocity just found above, supposing the density and pressure to be what we have assumed. The remark made concerning liquids is true also in the case of elastic fluids, that the actual velocity falls short of that found by theory. Schmidt says, when the side of the containing vessel is thin and the aperture is small, the actual velocity is only ·52 of the theoretical, but that if funnel-shaped tubes be inserted in the orifices, the velocity may be raised to ·6; this, it will be remembered, corresponds with what was observed in liquids.—After these deductions have been made, such high velocities still remain as will clearly account for the loud report which accompanies the discharge of fire-arms, as well as the combustion of explosive mixtures, such as detonating gas and the different detonating salts. At the moment when combustion takes place, the combined gases expand with prodigious force into a larger space, thus creating instantaneously an almost absolute vacuum in that space which they had previously occupied, the surrounding air rushes in to fill up this room with such violence as to cause a loud noise.

For any other gas whose density is d',

$$v' = \sqrt{2gb.\frac{q}{d'}}, \text{ whence}$$

$$v : v' = \sqrt{2gb.\frac{q}{d}} : \sqrt{2gb.\frac{q}{d'}}$$

$$= \sqrt{\frac{1}{d}} : \sqrt{\frac{1}{d'}}$$

$$\text{or } v : v' = \sqrt{d'} : \sqrt{d}$$

i. e. the velocities with which any two gases issue from an aperture are in the inverse ratio of the square roots of their densities. In fact, the lighter any gas is, the greater is its velocity; and that of hydrogen is the greatest.

It is more difficult to determine the velocity when the air flows into a space already occupied with some other gas. If the solution of the problem depended solely on the respective expansive forces, $b - \beta$, of the two gases, the velocity would be $V = \sqrt{2g(b-\beta).\frac{q}{d}}$; but this result is affected by the temperature, density, and chemical composition of the gases. It would be extremely difficult to resolve such cases by theoretical calculation, as indeed problems would arise which could not be accurately solved even with the aid of the higher branches of the mathematics.

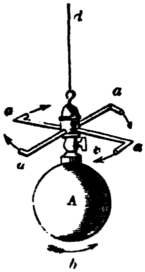
An impulse or pressure is produced by the elastic fluid against that side of the vessel containing it that stands opposite to the aperture whence the efflux takes place; this pressure is equal to the motive power with which the fluid issues: compare with this § 163. If this force be sufficient to incline the vessel in this direction, motion ensues, which will last as long as the efflux continues.

The following experiments will be confirmatory of the above statements.

First Exp. Condense the air in the metallic vessel A, *fig.* 140., by means of the condensing syringe (§ 185. *fig.* 103.), turn the cock *c*, unscrew the condensing apparatus and fix on the air-tight exit-canal, whose apertures *a* are placed at tangents to the revolving axis. Now suspend it by a thread *d*, open the cock *c*, and the air will issue in the direction shown by the arrow *a*, whilst the rotation of the globe takes place in that indicated by the arrow *b*.

Second Exp. The same may be shown still more simply thus. Let A, *fig.* 140., be a bladder provided with a stopcock, with which two hollow tubes, *a a*, are connected so as to

Fig. 140.



turn easily about the axis of the cock, whilst the gas contained in the bladder makes its escape through them. The four orifices stand, as shown by the arrows, tangentially to the periphery of rotation. Press the gas out of the bladder, and the cross-tubes will revolve in an opposite direction to that in which the current of air escapes. In both experiments a motion is obtained similar to that in Segner's machine, § 163.

The recoil of cannons and fire-arms, the ascent of rockets, the revolution of catherine-wheels, &c., are all owing to this reaction of gases against the sides of those vessels from which they issue.

SECTION VI.

UNDULATIONS.

§ 207.

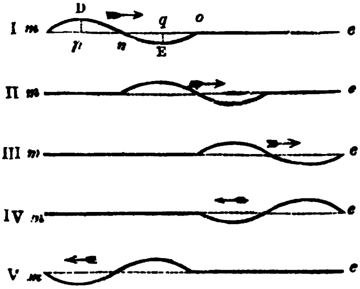
THE different parts of all bodies are ordinarily held, through the operation of certain internal and external forces, in a condition of equilibrium. Now, if the constituent particles of a body have been disturbed by any cause whatever (provided they have not been carried beyond the limits within which the stability of their equilibrium is confined), they will return again to a state of repose. This is effected by the particles alternately approaching the position of rest and receding from it, until, after some time, equilibrium is restored. The movements thus generated are termed *undulations*, *vibrations*, or *oscillations*.

§ 208.

These movements are of two kinds. In the first, the undulation successively traverses the different parts of the body; those particles which have been immediately excited by the disturbing cause communicating motion to such as are next them, and so on. In this case the movement of the particles is successive, so that the position they assume at any particular moment during the continuance of their motion appears to advance from one place to another. This constitutes a *progressive undulation.*

This kind of undulation may be observed in a stretched cord; for strain tight with the hand a rope made fast at one end, and then move the end you hold sharply up and down; that portion of the cord which is nearest you will assume the form represented in *fig.* 141. I. The curve *m* D *n* E *o*, produced by thus disturbing the equilibrium of the cord, is called a *wave*. The portion *m* D *n*, which rises above its original position, is the *elevation* of the wave, D the *summit* or point of greatest elevation; *n* E *o*, which is below the level, is called the *depression* of the wave, E being its *lowest* point. Again, D *p* is the *height*, *m n*, the *length* of the *elevation*; *q* E the *depth*, *n o* the *length* of the *depression*, and *m o* the *length of the entire wave.*

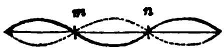
Fig. 141.



Such a wave does not continue stationary, but advances, as *fig.* 141. II. represents, towards the other extremity of the cord; on arriving at which, *fig.* 141. III., an inverted curve, *fig.* 141. IV., is produced; and the wave returns, *fig.* 141. V., to the point *m*, from which it started; its position being the same as at first, only the relative situation of its elevation and depression is reversed; this movement may be repeated several times.

In the second class of these movements all the particles of a body begin their undulations simultaneously, and complete them at the same instant. Even when the body is divided into several vibrating portions, each of them ceases to move independently of the rest, the different parts of the body being separated by points which remain throughout in a state of rest. This kind of vibration is called a *stationary undulation*; it may be produced by means of a tense cord, which must be agitated, in the manner described above, at equal intervals of time, until it has assumed throughout its entire length the form of elevations and depressions impinging on each other, when this form will pass into its converse, and *vice versâ*, as is represented in *fig.* 142. by the continuous and dotted lines. The points *m* and *n*, which bound an elevation and a depression, continuing at rest during the undulations, are termed *Nodal Points.*

Fig. 142.



§ 209.

Bodies of all three forms of aggregation admit of these vibrations being induced in them; the movements, however, are modified both by the form of the body itself, and by the nature of the exciting force. Such forces are either purely internal, as elasticity, which is the principal agent in solid and aeriform bodies; or external, as gravity, which causes this kind of motion in fluids.

An acquaintance with the laws of undulations is on many accounts of great importance in the pursuit of physical science. Our perception of sounds by the organs of hearing depends entirely on the vibrations of sonorous bodies communicating an undulatory motion to the air by which we are surrounded. The phenomena of light and radiant heat may also be most satisfactorily accounted for on the hypothesis that there exists a certain imponderable medium which is put into a state of undulation; reasoning from analogy, too, it seems probable that other effects which are commonly attributed to the motions of different imponderable bodies will admit of a similar explanation. We shall now proceed with greater exactness to consider the most important laws by which these movements are regulated in solid, fluid, and aeriform bodies.

I. UNDULATIONS OF SOLID BODIES.

§ 210.

Such solids as possess a sufficient degree of elasticity can alone be made to vibrate for any length of time. Many bodies of this class, as cords and membranes, acquire the requisite elastic force by means of tension; in others it is an innate property of their mass, as in rods and plates of metal, glass, and even wood.

Bodies of a *lineal* form, as, *ex. gr.*, tense strings, and fine wires, are susceptible of three distinct kinds of vibrations, that is to say, such bodies may incline from a position of rest on both sides in the direction of their length, or they may alternately extend and contract, or, lastly, their particles may move about the axis, alternately advancing and receding. The first and most common movement is called a *transverse* vibration, the second a *longitudinal*, and the last a vibration *of torsion.*

Exp. All the three varieties of undulations may be very plainly illustrated by a simple piece of apparatus invented by Professor August; it consists, as *figs.* 143. and 144. show, of a spiral-shaped piece of brass wire tightly twisted, having one end attached to a frame from which it is suspended, and at the other a weight by which it is strained tight.

Fig. 143.

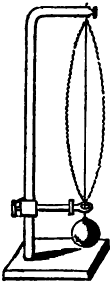
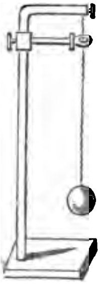


Fig. 144.



The *transverse* vibrations are produced by securing a portion of the wire by means of the moveable clamp, as represented in *fig.* 143.; then pulling the wire in the middle from its position of rest, and suddenly letting it go.

The *longitudinal* vibrations may be excited by raising and then letting go the weight by which the wire is strained tight.

The *rotatory* vibrations are induced by twisting the wire by means of the weight attached to it, and then allowing the weight to fall suddenly.

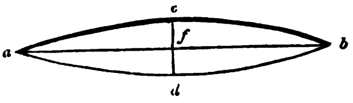
Bodies having a large *plane* surface, such as elastic discs and tense membranes, are susceptible of transverse vibrations only, whilst bodies of pretty nearly equal dimensions in all directions may be made to vibrate in each of the modes above described.

§ 211.

In all these movements elasticity is the force which, like gravity in the case of a pendulum, tends to restore the particles of the vibrating body to their former state of equilibrium.

The oscillations of a pendulum, as to external appearance, most nearly resemble the transverse vibrations of a body; this will be seen in the apparatus described above, or even in a tense piece of wire or catgut. Let any such body, *afb*, *fig.* 145., be drawn in the middle out of a straight line into the curve *acb*, on being let go it will contract by virtue of its elastic force, and return to its former position. This movement is effected with an accelerated velocity, and like the oscillation of a pendulum (§ 97.), is at a *maximum* when the body has regained the position *afb*. Conse-

Fig. 145.



quently it cannot retain this position, but moves on with a constantly decreasing velocity till it arrives at *adb*, where its velocity is *nil.* The force of tension again brings the cord back to *afb*: it recedes again till, as has been just shown, its velocity is annihilated, and so on. — One complete movement from *acb* to *adb*, and back to *acb*, is termed an *oscillation* or *vibration*, and the time occupied in performing it is called *the time of an oscillation.*

The principal laws which have been deduced from both theory and experiment respecting these vibrations of tense bodies are the following:—

1) The vibrations are perfectly *isochronous*, or occupy each an equal duration of time; in this point resembling the oscillations of the pendulum. Thus no difference is made in the pitch of the note, that is in the time of vibration, by pulling a tense cord further from its position of equilibrium.

2) The number of vibrations in equal successive portions of time increases, or, in other words, the vibrations become more rapid (a), if the force of tension, or the body's elasticity, be increased, and that in the ratio of the square root of the force (b); if the string be shorter, in which case the number of vibrations will be inversely as the lengths; and (c) inversely as the diameter of the cord is greater.

Let S, *s*, represent the elastic force of two strings; L, *l*, their lengths; D, *d*, their diameters; and N, *n*, the number of vibrations in any particular unit of time; then the above laws may be generally expressed in the subjoined formulæ:

$$N : n = \frac{\sqrt{S}}{LD} : \frac{\sqrt{s}}{ld}$$

and if the times of an oscillation be represented by T and *t*, the times being inversely as the number of oscillations, we have

$$T : t = \frac{\sqrt{s}}{ld} : \frac{\sqrt{S}}{LD}$$

§ 212.

If a body in a state of tension acquire a stationary undulation, such that it becomes divided into several vibrating parts, then these parts will all be of equal size, they shall all vibrate with equal velocities, but those which are contiguous in opposite directions, and the number of the vibrations will be inversely as the size of the parts.

Exp. An undulation of this kind may be very plainly shown by means of the tense wire (*fig.* 144.), thus: Press on the uppermost quarter of the wire with two fingers of the left hand, whilst with two fingers of the right hand you press on the lowest quarter, draw the wire thus held in opposite directions from its position of rest, and let go with both hands simultaneously. The wire will divide itself into two stationary undulations, between which you will find the nodal points.

§ 213.

Elastic rods in like manner vibrate transversely in an isochronous manner, if brought from their natural position either by a blow, impulse, or bending. For this purpose they may be free at one or both ends. Theory and experiment both prove that in equal times the number of complete vibrations in a perfectly uniform rod is inversely as the squares of the lengths of the vibrating parts.

If, for example, a steel wire fastened at one end makes two vibrations in a second, with half the length it would make four times as many, or eight; with one-fourth the length, sixteen times as many, or thirty-two vibrations, and so forth. Supposing the rod to be perfectly uniform, we may hence calculate how many vibrations it would perform at any particular length.

The particles of a rod thus set in motion describe paths which do not always lie in the same plane, but return in variously curved lines. This is clearly seen if a polished knob be mounted on the free extremity of the rod.—On this depends *Wheatstone's Kaleidophon.*

Exp. Fasten some fine steel wires—knitting-needles ten or twelve inches long will do very well—vertically by one end into a board; on the other end fix one or more polished metal knobs. If these wires be made to vibrate, the bright speck on each of the knobs will be seen to describe different curves.

Elastic rods are equally capable with strings of being divided by stationary undulations into several vibrating parts: the part at the detached end of the wire is half the length of a part contained between two nodal points, but it vibrates with equal rapidity, and the number of vibrations increases uniformly with the number of parts into which the rod divides itself. *Fig.* 146. represents such a rod, *a* being the fixed and *b* the free end, *c* each nodal point.

§ 214.

In the movements of elastic planes complete rows of points, *nodal lines*, will be formed which do not participate in the movement of the plane, but are in a state of rest. These nodal lines are produced by the continuous succession of nodal points in the strings of which you must conceive the plane to consist. They run in various directions across the plane, dividing it into numerous vibrating portions, the contiguous ones invariably moving in contrary directions. Conceive such a plane to be intersected by a second plane at right angles to the nodal lines; then the intersecting plane will form an *unda* with stationary waves, just like a rod vibrating with both its extremities free. See *fig.* 146. The bounding areas are always half the size of the interior. The number of vibrations increases with that of the nodal lines.

Fig. 146.

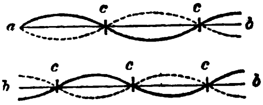
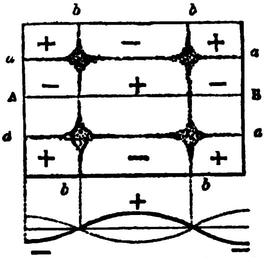


Fig. 147. will render the above easy to be understood. *a a*, *b b* are the nodal lines, the areas bounded by them and marked + are elevated, those marked − are depressed. The figure beneath shows the undulation, supposing the plane to be intercepted by another plane at right angles to it in A B.

How the existence of these nodal lines may be rendered visible will be shown presently in treating of sound.

Fig. 147.



II. UNDULATIONS OF NON-ELASTIC FLUIDS.

§ 215.

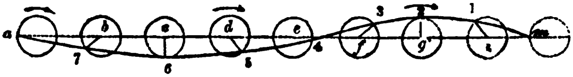
If a depression be made at any point in the surface of a fluid in a state of rest, either by dropping a solid into the fluid or by immersing and then withdrawing the solid, an *undulation* will be produced. Around the point of depression there first arises a circular elevation above the level of the water when *in equilibrio*, and immediately beyond this a corresponding depression; this constitutes a *circular undulation*. The wave is progressive, forming continually enlarging concentric circles further and further from the point at which it originated. A succession of waves will arise and traverse the same course from the place where the first arose, and this process will continue so long as the particles of the fluid are in motion. Thus the movement of alternate elevations and depressions in concentric circles goes on, each new wave decreasing in height and depth, until at length equilibrium is restored.

Conceive now that a vertical plane passes through the point whence the waves originated, and it is clear that on it will arise a *progressive lineal undulation* resembling that of the cord in the experiment described in § 208. On this line the same nomenclature is used as in the former: thus the length of the wave is the length of both its elevated and depressed portions, or the distance between the summits of two successive elevations.

In a movement of this kind the fluid appears as if its entire mass advanced progressively from the point of excitation. Such is, however, by no means really the case, but each particle of the fluid describes a vertical circle or ellipse, about the spot where it may happen to be, in the direction in which the wave is advancing; the particle returns again to its former position in the same curve, one half being above and the other half below the level of the fluid. Every particle of the fluid thus put in motion necessarily imparts a similar motion to its contiguous particle, this again to the next, and so the movement is propagated. Each successive particle must therefore begin to move at a point of time later than the preceding; and consequently at any particular moment the movement of each individual particle in its ascent or descent will differ from that of the others in height, hence the surface of the fluid derives its undulating form. The fluid, as a whole, occupies the same place, and the moving form of its undulation is merely the figure of the curve in which the individual particles of the fluid lie. This is obvious if you observe how a body floating on the waves of the sea is almost stationary in one spot, just rising and falling with the elevations and depressions of the water.

Fig. 148. is intended to illustrate this remark. For the sake of simplicity, we will consider only eight out of the very many particles of fluid which we may conceive as occupying the horizontal surface between the points *a* and *m*: imagine, then, these eight particles to be contiguous and successively set moving, so that each describes its own circle, as is represented by the row of circles in the figure. As the particles begin to revolve at successive points of time, it follows that every particle shall be at a new point in its orbit each in-

Fig. 148.



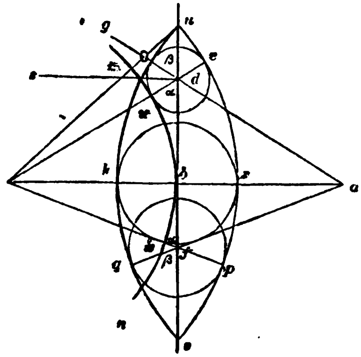
stant. Assume, now, that each particle of the fluid begins its movement in the direction of the arrows $\frac{1}{8}$th of a revolution later than its neighbour next it on the left hand, then, at the instant when *a* has completed one entire revolution, the second will be $\frac{1}{8}$th of a revolution behind it, viz. at 7; the third $\frac{2}{8}$ths behind, viz. at 6; and the fourth, fifth, sixth, seventh, and eighth, at the points 5, 4, 3, 2, 1, respectively; whilst the ninth particle is but just beginning to move. Connect the points *a*, 7, 6, 5, 4, 3, 2, 1, *m*. and the line will represent the form of the fluid's surface at that precise moment of its undulation. From this it will be perceived at once that the radius of the circle, described by each of these particles, is equal to the height of the elevated or the depth of the depressed portion of the wave, and also that each wave advances a distance equal to its own length in the same time as each particle accomplishes its revolution. Again, those particles which lie below the surface of the wave have their equilibrium disturbed simultaneously with the superficial particles, so that, to a certain depth at least, the successive strata assume a position parallel to that taken by the surface. The brothers Weber in their experiments found that in water this sympathy extended to a depth below the surface equal to the altitudes of 350 waves.

§ 216.

If progressive waves impinge on solid objects which check their further advance, a backward movement of the waves will ensue. This *reflexion of the waves* is owing, not to the elasticity of the particles of the fluid, but is occasioned by the action of gravity; it obeys precisely the laws which regulate the impact of perfectly elastic bodies, whence it follows that *all the points in a wave will be reflected from the surface of a solid under the same angle at which they struck it.*

Lines drawn from the point at which the impulse is communicated to a circular wave to different points in the circumference of the circle will express the directions in which these points are respectively advancing. Such radiating lines are appropriately termed *rays of undulation.* Those drawn from one centre of impulsion are diverging, those drawn from two such central points at an infinite distance asunder are parallel. Rays moving from their centre of impulsion against any solid body, as *ex. gr.*, *c x f*, *c b*, *c w d*, *fig.* 149., are said to be *incident;* those which, after impact, recoil from the resisting object, are *reflected* rays, and their direction is determined by the law given above; that is to say, *the angle of incidence* which the ray makes with a *perpendicular,* drawn through the point of impact, is equal to *the* *angle of reflexion.* Thus, let the ray *c d* impinge on the solid *u v*; from *d*, the point of impact, draw the perpendicular *ds*, then the angle α or *cds* is equal to the angle β or *gds*, and these two angles are on opposite sides of the perpendicular. Hence it follows that parallel rays falling on a plane surface will be parallel when reflected from it, diverging rays diverging, converging converging, and such as are at right angles to it return on their own track.

Fig. 149.



When the diverging rays of a circular wave impinge on a plane surface, their reflexion ensues exactly as if the undulation originated at a point *a*, *fig.* 149., as far beyond the line of resistance as the centre *c* of the original wave *m b n* is before it, hence the reflected wave forms a circle, *v k u*.

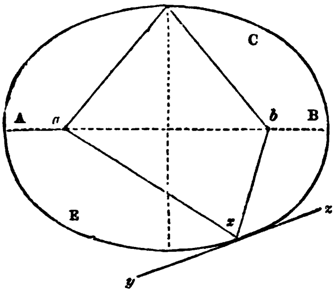
If a circular wave advance progressively towards a plane surface, it is evident that all its points will not impinge simultaneously on the solid. The perpendicular *c b* of the wave *m b n*, referring still to the same diagram, is the first to reach the plane at the point *b*; all the other points in the circumference of the wave will arrive at the plane after it, just in the order of their respective distances from *b*. Thus, *ex. gr.*, the reflected ray at *f* is later by all the time the wave *m b n* requires to traverse the space *x f*, the ray at *d* by the time needed to go from *w* to *d*.

As each point of a wave impinges on a solid it excites instantaneously a fresh undulation similar to what would be produced were the equilibrium of the fluid disturbed at the point of impact. So new circular waves originate from these points; advancing, however, from the solid merely as semi-circles, apparently, and succeeding each other as the centres whence they start are more or less remote from the perpendicular *cb*.

§ 217.

If the solid from which the wave is reflected have not a plane but a curved surface, then the rays will be thrown off from such a surface in directions answering to the curves, and the path of the returning wave may be deduced from the laws laid down in the preceding section. On account of the results arising from these reflected waves, as they are met with in the case of sound, light, and heat, the circle, the ellipse, and the parabola possess peculiar interest.

Fig. 150.

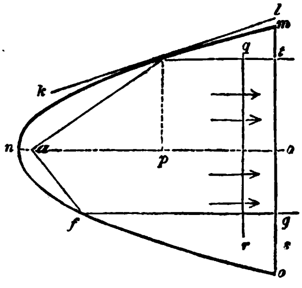


All rays of undulation proceeding from the centre of a circular vessel containing a fluid will impinge perpendicularly, and at the same instant, on the boundary; from it they will be reflected, and arrive again together at the centre.

If an undulation be excited at one focus, *a*, *fig.* 150., of an ellipse, all the rays diverging from this point will be so reflected from the bounding surface as to fall simultaneously on the other focus. Hence just such an effect follows as if the disturbance had originated in this latter focus.

The ellipse has this property, that at every point *x* in the periphery the angle $axy = bxz$, and the sum of the focal distances is always equal, *i. e.* $ax + bx = av + bv$.

Fig. 151.

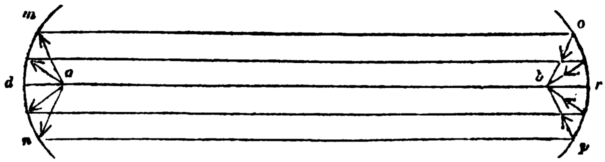


First Exp. Pour some quicksilver into a vessel of elliptical form (the vessel may be made of wood, or even of pasteboard) so as completely to cover the bottom. Then let a few drops of the metal fall through a fine glass funnel on the fluid surface at the focus *a*, *fig.* 154., and corresponding undulations will be excited at *b*.

If the surface have a parabolic form, *fig.* 151., and an undulation travel from its centre *a*, then the rays impinging on the solid boundary will be reflected in such a manner that they will move on in a direction parallel to the axis *nb* of the parabola (see the arrows in the figure), and combining so as to form one entire wave, they will strike at right angles against the ordinate *mo* which bounds the figure. On the contrary, if there be progressive waves, as *ts*, *qr*, moving in a direction perpendicular to the axis *nb* of the parabola, then will all the rays of such a wave be reflected from the interior of the curve so as to impinge simultaneously on the focus *a*.

Hence it follows, that if there be two parabolas having a common axis *dr*, and an impulse be excited at the focus *a*, *fig.* 152., of one parabola, and the waves of undulation reflected from its interior *mdn* impinge on the second parabola *orp*, then being reflected from *orp* they will all meet together at the focus *b* of this second parabola.

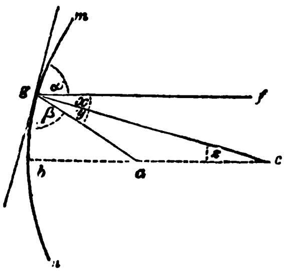
Fig. 152.



The motion of the waves described above is owing to these two properties of the parabola,—that, at any point *c* of the curve the angle *ack* will be equal to the angle *tcl*, if *tc* be parallel to the axis: and secondly that the sum of the lines *ac* and *ct* is equal to that of *af* and *fg*.

Second Exp. These properties in the movement of undulations, propagated in a parabolic surface, may be experimentally shown, by filling a vessel of the required form with mercury, and proceeding as in the preceding experiment. Care must be taken that the curves have a common axis, and that the sides fit when put together.

Fig. 153.



Rays reflected from a very small circular arc will in their direction approximate closely to those reflected from a parabolic boundary. The focus of such an arc lies at the distance of half the radius from the centre of the circle.

Let mbn, *fig.* 153., be an arc of a circle, bc its radius, and a a point about half-way between b and c, then $ca=ab$, and if the arc bg be assumed very small, then ab may be considered $=ag$; cag will therefore be an isosceles triangle, and angle y will be $=$ angle z. Draw gf parallel to bc, then angle $x=$ angle z, and consequently angle $x=$ angle y, and angle $\alpha=$ angle β, as in the parabola.

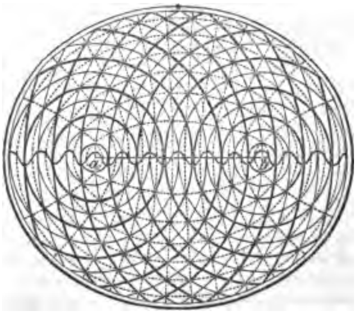
§ 218.

If an impulse be communicated at two points in the surface of a fluid, then the waves thus generated will intercept each other, and at the places where this occurs there will be a change in the form of the waves. If two elevations meet in exactly opposite directions, an elevation of double the height will be produced; if two depressions meet, a depression of double depth will be produced; but if an elevation and a depression meet, the surface of the fluid will remain undisturbed, each counteracting the other. This mutual annihilation is termed the *interference* of waves.

After the undulations have thus crossed each other's path, they continue their progress unaffected by this interception.

Exp. The crossing of the waves and the formation of the lines of interference in a fluid may be readily illustrated by the first experiment described in the preceding paragraph. In consequence of the reflection a similar undulation is described by both the foci of the ellipse, and the lines of interference arrange themselves regularly in the form of hyperbolas and ellipses about these foci. *Fig.* 154. repre-

Fig. 154.



sents this arrangement. The strongly marked curves are the elevations, the faint curves the depressions, and the dotted lines the lines of interference.

§ 219.

If two similar systems of undulations intersect each other, their points of impulse being distant *an odd number of times the length of a half wave*, a *stationary undulation* will be the consequence. The space intervening between the two points of impulse is divided into several distinct sections, each half a wave in length, separated by lines of interference in a state of rest; these portions alternately rise above and fall below the ordinary level of the fluid in such a manner that every depression has an elevation on each side of it, and *vice versâ*.

Fig. 155. represents a vertical section of the line of undulation between the points of impulse, *a* and *b*, which are 7 half-waves' lengths asunder. The arrows mark the directions of the waves from their respective points of impulse; 1 and 6 are the points of interference.

Fig. 155.

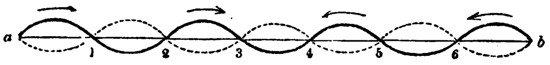
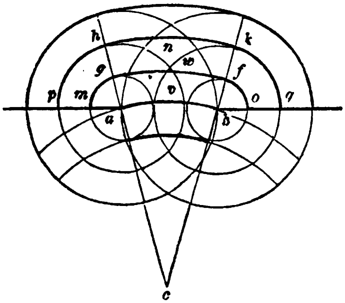


Fig. 156.



§ 219. "Stationary oscillations or waves are produced in a straight channel by exciting a succession of waves at regular intervals of time, which are then reflected at the opposite extremity of the channel; or, in a circular vessel, by exciting waves from its centre by impulses repeated at regular intervals. The MM. Weber also observed the stationary waves in vessels filled with fluid, and placed upon a drum or a cane chair, when impulses were given to the elastic surface at regular intervals."— *Müller's Elements of Physiology, translated by Dr. Baly.* Pt. V. 1219.

§ 220.

If a circular wave impinge on a solid in which there is an aperture, that part of the wave which comes against the opening will pass freely through, but the other portion which strikes against the solid will be curved about the edges of the aperture, so that one semicircle will be in front of the aperture, and the other beyond it. This is the *inflexion* of waves. The new circular waves formed about the opening will intersect the original wave and each other (§ 218.). so that interference points and lines will be produced.

In *fig.* 156. let *c* be the point of impulse of a circular wave, *ab* the aperture through which the portion *avb* passing freely expands into the arcs *gwf*, *hnk*, &c. At the points *a* and *b*, two new similar systems of undulation will be produced, having *a* and *b* for their centres; these circular waves, combining with that portion of the original wave which has already passed through the opening, acquire the form *mgwfo*, *phnkq*, and so on. In this case the series of waves advancing from *a* and *b* must necessarily intersect each other, and cause those results which have already been noticed in § 218.

§ 221.

Undulations can be excited in incompressible fluids only when their surface is free; if they are enclosed on all sides no undulation will ensue, but an impulse, however produced, will be transmitted with great velocity.

According to accurate experiments performed by Colladon and Sturm (1826) on the Lake of Geneva, it was found that in water vibrations are transmitted with a velocity of 1435 French metres, or 4708 English feet, in a second.

The rapidity with which vibrations may be transmitted through other fluids cannot be determined by direct measurement, but may be deduced with tolerable accuracy from their respective elasticities.

§ 222.

The undulations of incompressible fluids on our earth are of extreme importance, especially those of water, and, above all, the undulations of the sea. Did not water possess, as a consequence of the mobility of its particles among each other, the property of being thus set in motion, the ocean would soon be rendered putrid by the mass of organic matter it contains; its sweetness is preserved by its incessant undulations. The causes which contribute chiefly to maintain this state of constant movement are — the attractions of the sun and moon, which produce the ebb and flow of the tides; the rotation of the earth upon its axis, which originates the constant easterly current; and, lastly, the winds, which occasion the most violent disturbance of the waves, and stir up the ocean from its very depths. By means of the mechanical force associated with these violent movements in so large a body of water as the ocean, effects are produced which have brought about, and are still bringing about, important changes in the surface of our globe.

III. UNDULATIONS OF ELASTIC FLUIDS.

§ 223.

Elastic fluids are susceptible of undulatory movements, if the equilibrium of their particles be disturbed by any cause whatever. Their oscillations are in the main similar to those of non-elastic fluids, modified, however, by the extreme compressibility and expansibility of their particles.

As water was the liquid whose phenomena most demanded our notice, so in bodies of the class now under consideration the atmospheric air is that which may serve as a type of all other gases, and the results found to obtain from its movements may be held true of theirs.

§ 224.

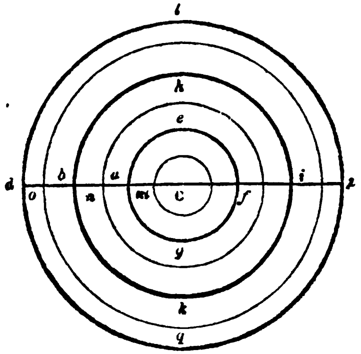
If a vibration be imparted to the air at any point whatever, it will be communicated to the surrounding particles, which will recede, in consequence of the facility with which they move among each other, with equal velocity in all directions, driving before them all those particles which lie in their path. Each of these, because of its inertia (§ 27.), opposes a certain resistance to this movement, and the air, by virtue of its compressibility, becomes condensed; consequently, the movement of the particle originally excited lasts only until overcome by this resistance. Hence there is formed about the point of impulse a *sphere of air*, which attains its maximum density at its boundary, at the very instant its further expansion ceases. The reaction of the condensed air then opposes the expansive tendency, the sphere contracts, and the particles resume their former positions. During this contraction of the particles originally set in motion, the undulation is imparted from one portion of the atmosphere to another at continually increasing distances from the original centre: thus, at the moment when the particles on which the impulse was first impressed, have recovered their former places, the undulation is transmitted to an equal distance in all directions. The radius of this hollow sphere of air, or the distance the undulation had traversed when the first particles resumed a position of rest, is called the *length of a wave*, and the entire sphere compressed within these limits constitutes *a wave*.

From the above explanation it follows, that the time of a particle's vibration, or the time which elapses whilst it attains its greatest density at its exterior, and thence returns to a state of rest, is exactly equal to the time in which the motion is propagated through the entire length of a wave, *i. e.* through the radius of the spherical cavity.

If the cause which excited the undulation continues to operate, there will arise a second, third, &c., wave within the first, and concentric with it. This radial propagation of the undulations can take place with equal velocity only when the atmosphere is equally dense and equally elastic in all directions. If this is not the case, the vibrations do not succeed each other with equal velocities in all directions, and consequently such a wave cannot have a spherical form.

Let *c*, *fig.* 157., be the point of impulse of the vibrations ; *cm* the distance at which the particles first excited began to return ; *ma* the space to which the undulation traversed before the particle *m* had arrived again at *c*, *ca* is therefore the length of the wave. The same holds good with regard to the points *n*, *o*, and *d*, and the distances *ab*, *bd*. Now conceive the sphere of air to be intersected by a plane passing through the point of impulse *c*, then the circles *aefg*, *bhik*, *dlpq* will be the sections of successive spheres.

Fig. 157.



§ 225.

If waves of air proceeding from two different points meet together, appearances present themselves exactly corresponding to those already described in the case of non-elastic fluids in

§§ 218. and 220.; the waves intersect and continue their progress freely. At those points of intersection where the velocities of both the particles are in the same direction, the effect is equal to the sum of the effects of the two particles; thus the effect produced will be doubled if the velocities be equal; but where one moves in an opposite direction to the other, their effects are neutralized, supposing their velocities to be equal, and in those places *interference* arises.

The following considerations may render this more intelligible.

Let a and b, *fig.* 158., be two points of impulse of waves of air; they are at a distance of an *entire wave's length* apart, and they are supposed to advance in the direction ac. Let the ordinates nm, op, &c. represent the displacement of the particles in the wave whose origin is a; then the line $amfpb$ will be the *curve of the intensity* of the first wave that proceeded from a; and if similar significations be attached to the ordinates qs, vu, drawn to the line bc, which marks the further progress of the wave, we shall have $bsguh$ as the curve of the intensity or displacement for the second wave. If now another undulation arise, in the same direction, at b, the beginning of the second wave, having a displacement qr at q, a displacement vw at v, then *the effect of both systems is increased*, and the particle at q acquires a displacement $qt = qs + qr$; that at v a displacement $vx = vu + vw$; and the curve of intensity generated by their com-

Fig. 158.

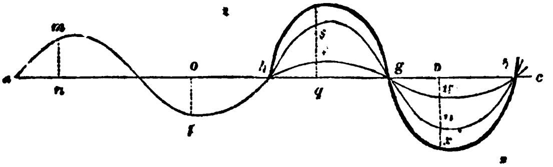
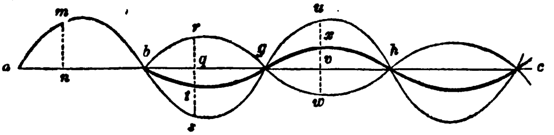
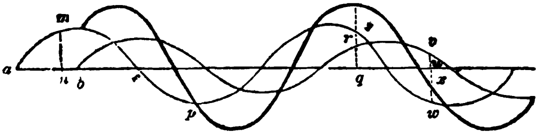


Fig. 159.



bined operation is the line $btgxh$. But if the points of impulse a and b be only the *length of half a wave* apart, as represented in *fig.* 159., then the particle at q acquires from the wave that starts from a the displacement qs; the particle at v the displacement vu; and from the wave at b the former particle receives the displacement qr, and the latter the displacement vw; but as these directions are exactly opposite the one to the other, the intensity of the vibration at q is $qs - qr = qt$; at v, $vu - vw = ux$; and $btgxhc$ is the curve of intensity formed in this case. Thus arranged, the systems of undulations will more or less *diminish* each other's force; and if equal, they will altogether *counteract* each other. In like manner, for any other two series of undulations, we may determine their curve of displacement by combining the displacements at each point of their path, as at q and u, *fig.* 160., the displacement at q being $qt = qr + qs$; that at u being $ux = uw - uv$, the strong line representing the curve of the wave's intensity throughout.

Fig. 160.



The effects described in the first case will necessarily obtain in similar systems of undulations, if their points of impulse, a and b, be 1, 2, 3, 4, &c., waves' lengths apart; those described in the second case, when their distance is ½, 1½, 2½, 3½, &c., waves' lengths from each other.

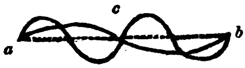
The observations made in §§ 216., 217. and 220., on the reflexion of the waves of non-elastic fluids from a solid body, and on the phenomena which arise when the fluid passes through an aperture in the resisting solid, are equally applicable in the case of the undulations of aeriform bodies: it must, however, be borne in mind, that these waves have a spherical form.

§ 226.

The extent of the vibrations, or the *intensity of the vibration*, *i. e.* the dimension of the spaces which the individual particles traverse while in this state of movement, depends on the force of the impulse which causes the undulation, and is generally proportional to it. The velocity, on the other hand, with which the waves are propagated, varies according to the specific elasticity of the medium in which they occur, and both large and small waves are transmitted at an equal rate so long as the elasticity continues the same.

If, *ex. gr.*, by a certain impulse a wave of air, *a b*, in length (*fig.* 161.) be formed, and the wave traverse this distance in one second of time, then, if a second wave be generated by another impulse, this second wave having only half the length of *a b*, two waves, *a c*, *c b*, will progress at the same rate, and both the undulations will arrive in equal times at *b*.

Fig. 161.



§ 227.

Waves of air which expand freely, form progressively increasing spheres (§ 224.), whose superficies are to each other as the squares of their radii, or distances from their respective points of impulse. The space which each particle of air in the wave traverses decreases in the same ratio. Hence, *the effects of waves of air expanding freely, diminish as the squares of their distances from their respective centres.* So soon, however, as the radial extension of the wave meets with any resistance which reflects the rays in a parallel or concentric direction, this rule ceases to be applicable.

§ 228.

Lastly the vibratory movements of one body are communicable to another; that they may be imparted it is requisite that the body to be set in motion be one susceptible of undulatory movements, that it be in a state of rest, and lastly, that the oscillations which act on it be violent enough to disturb the equilibrium of its particles.

Many distinguished philosophers have at different times devoted themselves to the theoretical investigation of this branch of natural science, which has now on many accounts become so important. As a whole, this was first satisfactorily accomplished by the brothers Weber, and published in their classical work "Die Wellenlehre, auf Experimente gegründet. Leipzig, 1825." The credit of having first given to the doctrine of undulations the form in which it now claims a separate section in a work on Physics is due to Professor Eisenlohr.

SECTION VII.

THE SONOROUS MOVEMENTS OF BODIES, OR THE DOCTRINE OF ACOUSTICS.

I. THE PRODUCTION OF SOUND.

A. *Sound in general and Musical Sounds.*

§ 229.

ALL those phenomena which affect our organs of hearing are called *Sounds;* sound is caused by the vibratory motions in bodies following each other with a certain velocity.—That this is really the immediate cause by which sound is produced may be proved by the following simple experiment.

> *Exp.* Secure by one end in a vice a piece of watch-spring about 1 foot in length and ¼ an inch in width, and communicate to it a vibratory motion; the vibrations may be clearly seen, though they are too rapid to be counted. Shorten the spring, it will move still more rapidly (§ 213.), so rapidly indeed that its vibrations cannot be recognised by the eye, but they may be heard even when the length of the wire has been greatly diminished.

Our organs of hearing are not immediately affected by these vibrations. To the production of sound, there is required, besides *the sonorous body,* one that shall be adapted to convey the sound, or rather the vibrations of the former body to our ear: bodies of this class are called *soniferous* bodies.

§ 230.

Any body may become sonorous if it makes the surrounding soniferous medium to vibrate with adequate force and rapidity. Consequently we hear with equal ease the flutter of an insect's wings, the report of a gun, or the discharge of fireworks. Those bodies are best suited to conduct sound which can transmit in all directions throughout their mass, and with equal velocity, the vibrations imparted to them by the sonorous body. A uniform density and elasticity are indispensable; consequently such bodies as wool, hair, feathers, and the like, do not possess the property of conducting sound.

§ 231.

If a sound be produced whose vibrations are irregular, the individual sounds composing it follow at unequal intervals; the human ear can not only recognise such irregular sounds, but distinguish one from another; most languages have appropriate words to mark these varieties of sound; it will suffice to mention rattle, buzz, roar, hiss, &c. If the vibrations succeed each other at equal intervals, they produce what is called a *musical sound.*

Those bodies only are capable of producing musical sounds which can be made to vibrate with considerable velocity. Each individual vibration or wave produces a sound, and if these follow in exactly equal times (§ 211.), we have that regular succession of sounds which has been just shown to constitute a musical sound.

A high degree of elasticity is indispensable to a body's yielding uniform and rapid undulations for any length of time; highly elastic bodies then can alone produce musical sounds. Elasticity is in fact the unceasing motive power which, like gravity in the case of the pendulum, sustains the permanence of the undulation.

Some solid bodies possess the requisite elasticity by virtue of the internal arrangement of their molecules; to this class belong several metals, steel especially; then glass, and even wood; to others we impart the necessary degree of elasticity by means of tension, as *ex. gr.*, to metallic wires, catgut, and animal membranes. To excite such undulations in aeriform bodies, we confine them in tubes. Recently even liquids have been made to emit musical sounds.

§ 232.

These vibrations can be excited only in those bodies whose dimensions of length or breadth exceed their thickness. For the production of musical notes we commonly use either rectilinear or curvilinear bodies, as tense cords, metal and glass rods, or columns of air inclosed in cylindrical or other tubes; or else bodies presenting a large surface, either plane or curved, such as membranes strained tight, plates of glass or metal, and bells.

Lineal bodies may consequently perform either *transverse, longitudinal*, or *torsion vibrations* (§ 210.), but bodies which have an extended superficies can vibrate only in the first of these modes. Again, a body may vibrate as a whole throughout its entire extent, or it may be divided into a number of aliquot parts, those which are conterminous moving in opposite directions; in other words, a stationary undulation may be induced; where these waves meet, the body will be at rest. Thus in cords, rods, and similar bodies, there will be *nodal points*, and in bodies with extended surfaces there will be *nodal lines.*

These musical undulations may be induced by various means in suitable bodies; for instance, by friction, or by a blow. The actual existence of such vibrations as have been described may be rendered visible by placing small light bodies in contact with the excited body, when they will be seen to acquire a similar movement. If you stop the vibrations of a body that is giving out musical sounds, the sound will cease at once, and so will the vibrations of the attached bodies.

First Exp. Attach a small ball by a string to a bell, ring the latter, and the ball will be seen to vibrate.

Second Exp. The string of any musical instrument, if it be made to give out musical sounds, will by its vibrations jerk off small pieces of paper that may be laid on it.

Third Exp. Strike a tuning-fork, and touch the surface of a mass of quicksilver with the tip of the fork, and the fluid will begin to undulate.

Fourth Exp. Touch the string of a monochord (§ 237.) with your finger or with a fine piece of cork at any aliquot part of its length, as at its third or fourth part; on the other nodal points lay small pieces of paper of this shape ∧; between these points also lay similar pieces; then touch that part of the string you had marked off with your fingers with a violin bow, so that it may give forth its note. When the string begins to vibrate, all the pieces of paper between the nodal points will be jerked off, whilst those at these points will continue in a state of rest.

Fifth Exp. Strew some fine sand on a disc of glass or metal. Hold this plate firmly between two fingers and draw a violin-bow down on its edge, and you will hear a musical note. At the same instant the sand will be in motion and gather itself to those parts which continue at rest, *i.e.* to the nodal lines.

Sixth Exp. Grasp any body that is giving out musical sounds with your hand, the vibrations and the sounds will cease simultaneously.

B. *The different Character of Musical Sounds, Notes, and Intervals.*

§ 233.

By the means already described we can render the vibrations of musical bodies visible; but we have not yet shown how these vibrations may be counted.

The velocity of the vibrations and their various modifications, but especially the *character* of the sound, are distinguished with remarkable accuracy by our organs of hearing; the mode, however, of the operation has not yet been satisfactorily explained. The character of musical sounds differs partly in regard to *loudness*, *pitch*, and *quality*.

The *loudness* depends on the extent of the vibration, that is the distance through which each particle moves to and fro. Thus, for instance, the further we draw a string from its natural position the louder will be its sound, and a bell will ring the more loudly the more violently it is shaken.

The sensation excited in our organs of hearing, according to the different velocities with which the vibrations succeed each other, is altogether distinct from that produced by the greater or less intensity of a sound. Musical sounds are either *high* or *low*, and when compared in this respect they are spoken of as *notes*. Of two notes heard at the same, or nearly at the same time, that which makes the greater number of vibrations in a given time has the higher *pitch*, and the other the lower.

> *Exp.* Let a cord vibrate throughout its entire length; then shorten it, its excursions will be more rapid (§ 211.) and accordingly the note produced will be higher.

It is still more difficult to explain how we are enabled to appreciate the *quality* (Fr. *timbre*) of a sound, though the faculty of distinguishing sounds is one in constant exercise; when notes equal in loudness and in pitch are produced by different bodies, we are able readily to distinguish one from another, and can assign them to the bodies from which they respectively proceed. No satisfactory explanation has yet been given of the emotion by which, for instance, we distinguish the human voice from that of a bird, or from the tones of an instrument, but it is probable that the difference arises from variety in the *form* of vibration.

§ 234.

The capacity of the human ear for perceiving the vibrations of bodies is confined within certain limits. It has been determined experimentally, that the lowest note we can appreciate is that produced by a body's performing 32 half-vibrations, or 16 impulses, in one second of time, and the highest when it performs about 16,000 in the same space of time. A fine ear is, however, able to recognise, as a distinct sound, a kind of hissing noise made by a body performing 48,000 half-vibrations, or 24,000 pulses, in a second.

> Chladni adopted the following simple method for determining the number of vibrations made by a body emitting musical sounds.

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§ 234. "The maximum and minimum of the intervals of successive pulses still appreciable by the ear as determinate sounds, have also been determined by M. Savart, more satisfactorily and more accurately than had previously been done. If their intensity is sufficiently great, sounds are still audible which result from the succession of 48,000 half-vibrations, or 24,000 impulses, in a second; and this, probably, is not the extreme limit in acuteness of sounds perceptible by the ear. Again, the sound resulting from 32 half-vibrations, or 16 impulses, in a second, is not, as has been supposed, the lowest appreciable note: on the contrary, M. Savart has succeeded in rendering sounds audible, which were produced by only 14 or 18 half-vibrations, or 7 or 8 impulses, in a second: and sounds still deeper might probably be heard if the individual pulses could be sufficiently prolonged." — *Müller's Physiology*, p. 1299.

Exp. A strong uniform strip of steel, several feet in length, about six lines in width, and one in thickness, is to be securely fixed at one extremity in a vice, in such a manner that when the free end is set in motion it shall make 120 half-vibrations, or 60 pulses in a minute; it will then make 2 of the former, or 1 of the latter, in a second. Fasten it a second time in the vice, so that the free end shall be only half as long as at first, then the number of half-vibrations in a second will be $4 \times 2 = 8$ (§ 211.). Halve this again, so that one-fourth of the original length shall vibrate, and we shall have for the number $4 \times 8 = 32$, and the half-vibrations will now constitute a note. Halve the length once more, and the note produced will be one that has $4 \times 32 = 128$ half-vibrations, or 64 pulses, in a second, and so on.

The *Siren*, invented by Caignard Latour, is an accurate but complicated instrument for determining the number of musical vibrations. The notes are produced in it by breaks in a constant stream of air, these breaks occurring at equal intervals of time, so that after each break there is a new wave of air. When the number of waves amounts to 32 in a second, the lowest note is emitted, and the number may be increased to about 16,000 per second, when the note degenerates into a mere whistle.

§ 235.

Besides the physical effect which musical notes produce on our sense of hearing, they also exert an influence of a purely psychical character; for we experience a sensation either of pleasure or pain on hearing several notes either simultaneously or in succession. The combination of harmonious sounds is a *chord*, an agreeable succession of notes is a *melody*, and a succession of chords constitutes *harmony*.

Music is the varied application and combination of harmonious sounds; it forms a language which addresses itself to the feelings of all mankind.

§ 236.

On closely investigating the reason why some combinations of sounds were consonant and others dissonant, it was found that this depended on the greater or less simplicity of the relation between the notes in regard to the number of their vibrations. Such a relation, expressed in the lowest numbers, is called an *interval.*

The simplest or most consonant interval is that of the *octave*, in which the higher note makes twice as many vibrations as the *fundamental note* from which it is derived; its ratio is therefore that of 1 : 2. This relation serves as the principal unit of measurement in music, its entire range embracing at most 9 entire octaves ($14-5=9$), rising from the deepest note which makes 32 vibrations in a second ($32=2^5$), to the highest which performs 16,384 vibrations ($16{,}384=2^{14}$) in the same time.

Between this fundamental interval, or each octave, there exist innumerable other notes, of which, however, there are but six that when heard in succession fall harmoniously on the ear. These eight notes of every octave constitute what in music is called a *scale;* the *diatonic scale* we have derived from the Greeks. In the following table, that which is most worthy of notice in it has been brought together at one view.

Notes - - - - - -	C	D	E	F	G	A	B	C′
Number of vibrations in upper note - - - -	1	9	5	4	3	5	15	2
Number of vibrations in fundamental note C - -	1	8	4	3	2	3	8	
Intervals from C - -	Fund. Note.	Second.	Third.	Fourth.	Fifth.	Sixth.	Seventh.	Octave.

There are three of these notes which together form the most agreeable combination to the ear, they are C, E, and G; this chord of three notes is called a *triad.* In this instance, again, it appears that the effectiveness of the combination is owing to the simplicity of the vibrations, for in the time that the fundamental note C performs 4 vibrations, its third E makes 5, and its fifth G 6. This combination of the fundamental note C with the major third E and the fifth G is called the *perfect major chord.*

Without entering fully into detail on this part of Acoustics, we may add, by way of supplement, a few remarks on the *chromatic scale,* and also explain what is meant by *even temperament.*

By glancing at the table of the *diatonic* scale, it will be seen that the ratio of the number of vibrations for any note to that for the note immediately preceding it, is not the same for all the notes—that is, the intervals which are determined by these ratios are unequal. The ratios are $\frac{9}{8}, \frac{10}{9}, \frac{16}{15}, \frac{9}{8}, \frac{10}{9}, \frac{9}{8}, \frac{16}{15}$, and it may be shown that the intervals are correctly represented by the numbers 51, 46, 28, 51, 46, 51, 28: the intervals E to F and B to C′ are therefore a little more than half the others. To enable us to select some other note than C and do away with the inequality between the intervals, *semitones* have been introduced between the whole tones, that is, in the intervals C to D, D to E, F to G, G to A, A to B. This scale, which contains 12 intervals is called the *chromatic* scale. But even in it the intervals are not exactly equalised. To make the 12 intervals exactly equal, the difference must be so distributed among the semitones of the octave, that, though they are not accurate with respect to any fundamental note whatever, the deviation is imperceptibly small. If the semitone were $25\frac{1}{12}$, the tone $50\frac{1}{6}$, it is easily seen that the notes would coincide almost exactly with those of the diatonic scale. An instrument tuned in this manner is said to have an *even temperament*. It can be played equally well in all keys.

The equal interval obtained between all the notes when an instrument has been thus tuned may be found mathematically, for it is represented by the common ratio of a geometrical series, the extremes being the ratio of the octave 1 and 2, and the number of terms 13, which is the number of notes in the chromatic scale. The common ratio of the series will therefore be $\sqrt[12]{2}$, or 1·05946.

We have now sufficiently shown that the science of music, apparently so independent of abstract mathematics, is notwithstanding based on certain fixed relations of numbers, and that the harmony of notes is intimately connected with the coincidence of simple numerical ratios.

C. *Musical Vibrations of Tense Cords and Elastic Rods.*

§ 237.

The phenomena presented by sonorous bodies and the ratios in their vibrations, which subjects have just passed under our notice, may be most easily illustrated by the transverse vibrations of tense cords. The apparatus employed for this purpose is called a *Monochord* or *Sonometer*.

In its most simple construction the monochord consists of a single string of wire or catgut strained by means of two pins across two wooden bridges, which are fastened in a pretty strong board; that part of the board below the string is graduated, so that the latter may be readily made to vibrate in any required number of parts.

As the number of vibrations made by a tense cord increases, *i. e.* as the note it gives forth becomes higher, in proportion as the cord is shortened, and as the height of the note is inversely as the length of the cord (§ 211.), it follows that all the notes in the scale may be obtained from a *single* string. Thus, *ex. gr.*, the half of any cord will give the octave, $\frac{2}{3}$rds will give the fifth, $\frac{3}{4}$ths the fourth, &c. of the fundamental note sounded by the entire cord.

First Exp. All the notes in both the diatonic and chromatic scales will be yielded by the string of a monochord, whose length is marked off on the register into 1000 equal parts by continually shortening it in the inverse ratio to the number of its vibrations.

The pitch may also be raised by increasing the force of tension that acts on the string, and the number of vibrations increases directly as the square roots of the force of tension (§ 211.), *i. e.* the number of vibrations will be doubled, or we shall get the octave, by straining a string with a power four times greater than the original power.

Second Exp. The truth of these remarks can be tested if the string be strained by means of weights, such as are used in Fischer's and Weber's monochords.

Lastly, the strength of the string and the material of which it is made materially affect the height of the notes, which will be higher in proportion to the strength of the string (§ 209.), and the elasticity and density of its material. Thus with equal tension, steel-wires yield a higher note than brass, and these again than catgut.

Third Exp. These facts may also be proved by means of a monochord furnished with weights to strain the string.

§ 238.

A string vibrating transversely may be divided into many vibrating portions; in other words, it may assume the form of a stationary undulation. This movement may be communicated to a string by touching it at any aliquot part of its length, and then drawing the bow of a violin across the part thus marked off. The whole cord will instantly divide itself into equal portions, and wherever two are conterminous there will be a nodal point. The greater the number of such parts into which the string is distributed, the higher the note, and the numbers of vibrations will be inversely as the length of the separate parts. (§ 211.)

Exp. This arrangement will present itself as you perform the fourth experiment described in § 232. — The height of the notes will be found to increase as the number of vibrating parts: half the string will yield the first octave, a quarter of it the second octave, an eighth the third, and so on, of the fundamental note — Notes of a flageolet; the Æolian harp.

§ 239.

Bars of a cylindrical or prismatic form are susceptible of transverse sonorous vibrations equally with cords. But as such bodies are elastic in all directions, and consequently their elastic force acts not lengthwise only, their vibrations do not obey the same laws, nor are they excited in the same manner as those of strings.

In similar bars the height of the note is directly as the thickness and inversely as the square of the length. — *Chladni's Sonometer*, § 234. The degree of the body's elasticity greatly influences the character of the note; thus steel gives a higher note than brass.

To produce these transverse vibrations, the bar may be either secured at both ends, or its extremities may be merely made to rest on some solid objects; or this arrangement may be made with one end only, the other being free; or lastly, both ends may be free, the rod being supported at two points.

The following instruments are some of those in which the sonorous vibrations of such bars are used: viz. the Jew's harp, clocks that play by means of steel springs, tuning forks, and the wood and straw instrument of the Russian *Gusikov.*

§ 240.

The second species of acoustic vibrations which linear-shaped bodies acquire are the longitudinal vibrations already spoken of. They may be excited in tense strings and elastic bars by drawing a violin-bow at an acute angle to them in the direction of their length, or even by friction lengthwise.

In consequence of this friction, the particles of the excited body alternately approach and recede. Compare with this the second experiment described in § 210. These longitudinal movements in this respect resemble the transverse vibrations, that they may either follow in one direction through the entire length of the body, or it may be divided into several aliquot parts. Where the vibrating portions impinge, nodal points exist, and conterminous portions move in opposite directions.

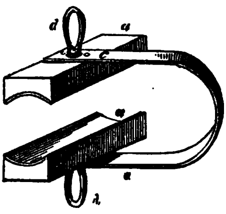
This oscillatory movement of the particles of bodies is owing to their elasticity; and, as it obtains in the interior of a body's mass, it must be regarded as a molecular movement.

The laws of longitudinal vibrations have nothing in common with the transverse oscillations, except that the height of the note depends on the length of the vibrating body, *i. e.* the note becomes deeper in proportion as the length increases, or the vibrations become slower in proportion to the length of the sonorous body. The notes produced by the longitudinal vibrations are, without exception, higher than those yielded by the transverse vibrations of the same body; they are but little, if at all, influenced by the thickness of the body; and in the case of metal wires of considerable thickness aud length, they are little affected by the degree of tension the body may suffer, they depend principally, as indeed the transverse vibrations do also, on the material of which the body is composed.

First Exp. If a violin-bow be drawn across the string of a monochord in the manner described above, it gives forth a sound, vibrating longitudinally; the note thus produced is considerably higher than that excited by the transverse vibration. If the instrument have two strings, it may readily be found that their thickness and force of tension exert no perceptible influence on the height of their tones.

Second Exp. Longitudinal vibrations may be generated in elastic bars, by holding them in the middle between two fingers, and rubbing repeatedly one of the ends with the fingers of the other hand. In experimenting on glass tubes, the friction apparatus represented in *fig.* 162. will be found convenient; *aa* are two pieces of wood hollowed out, having their cavity padded with cloth or leather; *cc* is a steel spring connecting them, and *dd* are two rings intended to receive the fingers with which the friction is excited. Moisten the padding with spirit of wine, and sprinkle on it a little finely pulverized pumice-stone. If metal or wooden bars are used, the readiest mode will be for the operator to put on a leather glove, on the thumb and index finger of which is some pounded resin, and with these to rub the rods.

Fig. 162.



Third Exp. The existence of nodal points may be verified by sliding small paper rings loosely on the rod, according to *Savart's* method.

These vibratory movements have been applied to musical purposes in some instruments.—*Kaufmann's Harmonichord,* and *Chladni's Euphon,* act on this principle.

§ 241.

Beside the two species of vibration described already, elastic rods admit of a third, viz. that by *rotation.* It is most easily generated in cylindrical bodies, by securing one end in a vice, and communicating to the other a rotatory motion by means of a bow, or by friction. An alternate expansion and contraction of the body's particles ensue in a direction perpendicular to its axis.—Compare with this the third experiment (§ 210.).—Different high and low notes succeed each other, of which, as yet, no use has been made in music.

D. *Of the acoustic Vibrations of elastic Planes and Bells.*

§ 242.

If elastic plates, of glass or metal in particular, be held tightly either by the fingers or by means of a clamp, at any one point, and the bow of a violin be drawn across the edge of the plate, sonorous undulations are immediately produced.

These oscillations resemble those of elastic rods, inasmuch as the surface is divided into a greater or less number of perfectly symmetrical parts, and such as are conterminous, vibrate in opposite directions. (§ 214.)

These boundary lines are all in a state of repose, and form the *nodal lines*, of which a full explanation has been given above; their position depends on the places at which the plane is held and excited, as one of these nodal lines invariably runs through the point at which the plane is held, whilst the plane itself receives the vibratory motion at the other point. These lines form certain peculiar figures, called, after their discoverer, *Chladni's Acoustic Figures.*

> To make these figures visible, and to render them permanent, adopt the plan described in the fifth experiment § 232.; they may also be seen if a small quantity of water be poured on the plate, nay, even by the rays of light falling on it. *Wheatstone* was the first to remark that, in using the last-named mode, still more delicate divisions in the figures were observable.

These acoustic figures are composed sometimes of right lines, sometimes of curves either parallel to or intersecting each other. The shape of the plane greatly affects them, as they are differently arranged, according as it may be a square, a rectangle, a triangle, a circle, an ellipse, or some other figure. A perfectly distinct and well-defined figure is produced only when the plate gives a very clear note.

By experiments made on such plates the following laws were detected by Chladni:—

1. Any particular note will always produce the same figure with the same plate; but a small change may often be produced in the figure by slightly changing the place at which the plate is held without causing any difference in the tone. If the tone be changed, the existing figure disappears at once, and a new one arranges itself.

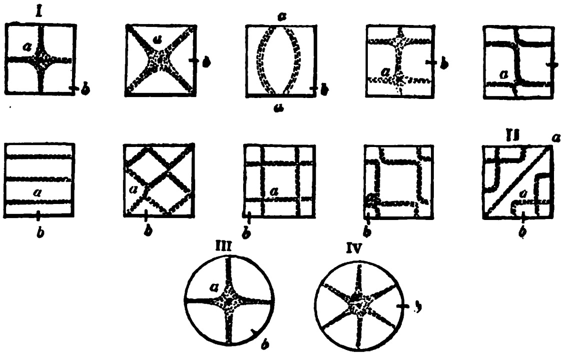
2. The deepest note any plate gives generates the simplest figure, and the higher the note the more complex the figure, *i.e.* the more nodal lines there will be.

3. If similar planes of various sizes be treated in the same manner, similar figures will be generated in each; by the same treatment, we mean that they shall be held at the same point, and that the bow shall pass over the same point in each. The notes will, however, differ, for the larger plates will give out the deeper notes; and if their dimensions be equal, the stronger will give the higher notes.

> *First Exp.* If the plates, *fig.* 163., be strewed with fine sand, and held at the point *a*, whilst the bow be made

to pass at *b*, the figures there depicted will in each case be produced.

Fig. 163.



A striking effect is obtained by making the same figure on several plates of equal size and similar form, and then so arranging them as to make one figure on a larger scale. The figure thus produced will be both a *compound* and *connected* one, and such as may not unfrequently be met with on a large plate.

Second Exp. If a large square be formed out of four

Fig. 164.

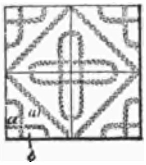
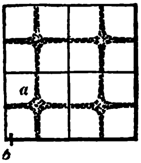


Fig. 165.



squares, bearing the figures (*fig.* 163.) I. and II., we shall have *figs.* 164. and 165.

If a large square plate, *figs.* 164, 165., be held at *a*, touched at *a'*, and a bow be drawn across at *b*, similar compound figures will be generated.

Cymbals, and the Chinese *Gong*, are practical applications of acoustic plates.

§ 243.

The oscillating motions of sonorous bells resemble those made by circular planes. In this case, too, the higher note produces the more complex figure.

> *Exp.* To render these undulations visible, fill a bell-shaped glass rather more than half full with water; draw a violin-bow across the rim, at the same time touching the glass at two opposite points of the rim with the fingers.
>
> The surface of the water will acquire an undulatory motion. To make the sonorous figures generated by these means permanently visible, powder the surface of the water with *semen lycopodii.*
>
> If the point excited by the bow be at an angle of 45° from that touched by the finger, a four-rayed star (*fig.* 163. III.) is produced; but if it be at an angle of 30°, 60°, or 90°, a six-rayed star appears (*fig.* 163. IV.).
>
> Such a cup gives musical sounds if its edge be rubbed with the moistened finger. These vibrations result from torsion, on which principle the well-known *musical glasses* act.

E. *Musical Notes generated by Vibration in the Air.*

§ 244.

A mere rapid concussion of the air does not produce a musical note, it causes only a noise or report. To obtain agreeable sounds from this fluid, it must, like the solid bodies, be made either to vibrate with quickness and regularity, or assume a stationary undulation.

> By the aid of the Siren (§ 234.) any one may convince himself that a series of exact and recurring pulsations in the air, or interruptions in a current of air, cause it to yield musical sounds, as was shown in the section just referred to; no note follows unless there be at least 32 pulsations of the air in one second.

These oscillating motions may be variously imparted to the air:—

1. If it be quickly forced through a narrow passage with elastic edges, as, *ex. gr.*, whistling through the mouth.

2. When a column of air is made to vibrate by the impulse of a current forcibly impinging on it, as happens with horns and trumpets.

3. When a current of air is so broken against a sharp edge, that a portion of it enters, causing a condensation which passes lengthwise over the column of air enclosed within the instrument, as in organ-pipes, flutes, and similar wind instruments.

4. When a current is driven between an elastic flat surface and

the edge of a channel or aperture, into which the flat piece fits, leaving but a small space between. On this principle the hautboy, clarionet, bassoon, and the well-known mouth-harmonicon act.

§ 245.

When a column of air confined in any tube, such as the wind-instruments referred to above, has a tremulous motion communicated to it in the manner described in the preceding section, the current of air blown into the instrument acts like a violin-bow, by which longitudinal vibrations are imparted to a chord. It condenses the particles of air within the tube; by virtue of their elasticity they again expand; as the current continues, they are once more condensed, and so on. By these successive condensations and expansions, the enclosed column of air is made to give musical sounds.

The flat elastic mouth-piece of such instruments as were mentioned (4. § 244.) receives an oscillating motion from the air blown into the instrument. This motion is the exciting medium of the musical notes, as the column of air contained in the instrument is by it put in a state of vibration.

§ 246.

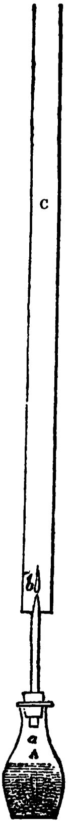
That the enclosed air is really the sonorous body, and not the substance of which the instrument is made, appears from the fact, that its tone is not at all affected by the instrument's being grasped in the hand; whereas, merely touching a solid sonorous body, at once either altered its tone or caused its vibrations to cease altogether. The human ear possesses a wonderful aptitude for appreciating the quality of the note, or the various modifications which the sonorous column of air receives from the inner surface of the instrument; modifications which vary according to the material composing it.

> The distinction felt by the ear when the same note is sounded, for instance, on a trumpet and a flute.

In wind-instruments the height of the note depends chiefly on the length of the enclosed column of air to which it is inversely proportional; thus, a column half the length of another, gives a note as high again as the latter. For this reason many wind-instruments have openings at the side, by which the length of the column of air producing the musical sounds may be increased or diminished at pleasure. The height of the note is further affected by the nature of the column's pulsation; *i. e.* whether it vibrate through its entire length as a whole, making but one wave, or be divided into several vibrating portions, in which case it is in a condition of stationary undulation. The notes increase in height just as those of sonorous chords and rods. In wind instruments the note is raised by blowing more quickly and forcibly,

§ 247.

Fig. 166.



The air may be rendered sonorous in a remarkable manner by means of the *Chemical Harmonicon*, as it is sometimes called. If a small flame of hydrogen gas be made to burn in a tube of glass, or any other material, from 1 to 3 inches wide, the column of air contained in the tube will in a short time be heard to give forth musical sounds. As was observed in wind instruments, so here too, long tubes give deeper tones than short ones, and if a pretty long tube be used by elevating the flame to different heights within it, various modulations of the original note can be obtained. It may be known when the air has acquired its sonorous property by observing the little puffs which rapidly follow each other; they are occasioned by the combustion of the hydrogen in the atmospheric air whch the tube contains, and thus an influx of the surounding atmosphere follows.

Exp. To perform this experiment, obtain some hydrogen in the usual way, by pouring dilute sulphuric acid on granulated zinc in a flask, A, *fig.* 166. The flask must remain uncorked till it is supposed the hydrogen has expelled all the common air. Then close the mouth with a cork, through which a glass tube, *ab*, about 6 inches long and from $\frac{1}{6}$th to $\frac{1}{4}$th of an inch wide, passes : this tube is tapered off at *b*, where it has a very fine opening; the gas is to be carefully lighted as it issues from *b*. Over the small lambent flame place the larger tube c, as represented in the *fig.*, and the expected phenomenon will before long begin to show itself. It is worthy of remark that, generally speaking, the musical sounds are not audible so long as the flame burns brightly; when it becomes small and steady, the notes begin to sound forth. If a copper tube be placed over an argand gas-burner, the diameter of the tube being about $\frac{3}{8}$ in. greater than that of the burner, a very loud note will be produced.

F. *The Organs of Voice.*

§ 248.

The *organs of voice* in man and in the inferior animals closely resemble those instruments in which the air is rendered sonorous by means of an elastic mouth-piece.

The human organs of voice are the *trachea*, or windpipe, the *larynx*, and the *cavity of the mouth*. The lungs and chest act like the bag of a bagpipe, the trachea answers to the pipe; at the head of the trachea is the larynx, which is the principal organ in producing the voice. The larynx is a widening of the upper part of the trachea, over the opening to which two semicircular membranes are stretched, so as to contract the trachea into a small cleft called the *glottis*, which is narrowed or widened at will, according as these membranes are contracted or relaxed. Besides these organs, especially formed for producing the voice, we have the roof of the mouth, the pliant lips, the ready tongue, and the nose, by the aid of which we variously modulate the sounds we utter.

§ 249.

The organs just named are thus used: in expiration the air passes from the lungs through the trachea and larynx into the mouth. If the glottis be open, the air passes without obstruction, and its motion is inaudible; if, on the contrary, the membranes of the larynx and glottis be contracted so as to narrow the aperture, the air is driven through forcibly, and these membranes impart their vibrations to the current of air. In this manner an *inarticulate sound* is produced; and it is plain that the mode in which it is generated is closely analogous to that in which musical notes are excited in clarionets and other similar instruments. (§ 244. 4.). Different impediments are offered, at the pleasure of the speaker, to these pulsations of the air, which he checks with the tongue, lips, and teeth, thus presenting very differently formed apertures for the escape of the air, by which means the sound is modified so as to make that variety of tones called *articulate sounds*.

The air which produces the voice has two passages in the mouth, one at the head of the trachea, which the tongue can close more or less exactly, and the outer one, which is opened and closed by the lips. By opening or closing one or both of these passages we form those sounds called by grammarians *vowels*; the *consonants* are made by impeding the escape of the breath by means of the tongue, lips, or teeth; hence are derived the terms labial, palatal, lingual, gutteral, &c., applied to the consonants.

§ 250.

The narrower the glottis is made, or the more the oscillating membranes at the head of the trachea are contracted, the higher is the musical note of the voice, for just so much more rapidly do these membranes vibrate. The possible contraction of the trachea itself, and perhaps also the degree of velocity with which the breath is forced through it may in some degree effect the tone of the voice.

The compass of the general human voice is about three octaves. The deepest notes in a man's voice make 192 pulsations, and the highest 633; the lowest in a woman's voice 576, and the highest 1720 oscillations. It rarely happens that the same voice has a compass of more than two octaves of complete notes.

§ 251.

The voices of the lower animals closely resemble that of man; all brutes, however, are not able to utter a vocal sound; those only which have lungs are endowed with this faculty; that is to say, mammalia, birds, and amphibious quadrupeds.

G. *Resonance of Bodies.*

§ 252.

The vibrations of sonorous bodies are imparted to others with which they come in contact. It is a property of such bodies that they readily acquire the same vibrations as the body which emits the sound possesses, and by thus vibrating with it, or by their *resonance* as it is called, they strengthen the original note. Elastic bodies are most susceptible of being thus acted on; loose and inelastic substances, on the contrary, enfeeble or altogether stifle sound by destroying the vibrations. Our ordinary musical instruments, especially such as are stringed, owe the strength and fulness of their tone to this property of bodies, which we have already said is called *Resonance.*

First Exp. Louder sound of a tuning-fork by placing its handle on a table.

Second Exp. The same fact may be shown by tuning the string of a monochord, holding it first in the air and afterwards resting it on a table.

Third Exp. Resonance of the second string of a monochord when the first string only is struck.—Nodal points generated in the string which is not struck.

II. TRANSMISSION OF SOUND.

A. *Transmission of Sound through Atmospheric Air, through Water, and through solid Bodies.*

§ 253.

It has been already shown in §§ 229. and 230. that for us to be able to perceive a sound a *conductor* is required that will connect our organs of hearing with the sonorous body; in the same part of this work the characteristics of such a conductor were mentioned.

The most general conductor of sound is our atmosphere. It is in contact with our organs of hearing, and, being co-extensive with our earth, it necessarily touches every sonorous body upon it. Being an elastic fluid, it is peculiarly adapted to transmit the pulsations of sonorous bodies by its own undulatory motion in all directions with equal velocity. Every vibration of the sonorous body produces a progressive wave in the air, which moves in a direction perpendicular to the surface of such body. These undulations are called *waves of sound*, and every right line, perpendicular to their surface or to that of the sonorous body, is called a *ray of sound*. It generally shows the direction whence the sound emanates; for which reason we always imagine the sonorous body to be in that direction in which our ear is affected by the rays of sound.

> From the explanation just given it follows, that the motion of the air in the transmission of sound is distinct from its sonorous motions. In the former case it merely extends the motions generated by some other body by its progressive waves, in the latter it assumes a stationary undulation.

§ 254.

When waves of sound are excited simultaneously in different places, each advances, according to the laws of undulatory motion (§ 218.), independently of the other. Where they meet, they cross each other's path, and after this intersection each pursues its course unaltered. From this circumstance we may comprehend how several sounds and notes may be heard at once. It is only at their points of intersection that they affect each other by adding to or diminishing their force, or even by neutralising each other, when *interference* arises.

> *Exp.* Hold a tuning-fork in a state of vibration, so near to the ear that its tone can be easily distinguished; if its handle be turned round with moderate quickness, it will be

observed that the sound seems almost to cease, when the aperture of the ear is in a line with the diagonal drawn through one of the prongs, and that it is most plainly heard when the flat face of the prong is towards the ear. This change in the increase and decrease of the strength with which the sound falls upon the ear takes place four times in each revolution.—To perform this experiment so that several persons may perceive it at the same time, turn the tuning-fork round over the mouth of a glass flask.

The waves of sound become *curved* (§ 220.) by passing through apertures. A difference is perceptible if a sound be listened to before and after its passage through any opening: in the latter case it is less forcible.

§ 255.

The velocity with which the waves of sound travel in a homogeneous medium, depends principally on its density and specific elasticity. Consequently, sound is transmitted more rapidly in liquids than through air, and in solids more quickly than in either of the other kinds of bodies. As the density and elasticity of the air, and indeed its general condition, is not always alike, so neither can the velocity with which sound travels be invariably the same.

It has been found, as the result of careful experiments (a), that when the atmosphere is dry, at the temperature 32° Fahr., and in a perfectly quiescent state, sound travels at the rate of 332·147 metres (or 1089·75 English feet) per second. The wind either accelerates or retards the progress of sound, according as it may be moving in the same or in an opposite direction. When the air is damp, sound travels more slowly than when it is free from moisture, as Goldingham of Madras has proved almost beyond a doubt. Heat exerts the greatest influence on its rate of motion, since the air's elasticity is principally modified by the presence of this powerful agent (b). The pitch, quality, and loudness of sound do not affect its rate of travelling. If the opening be large the sound is fully heard only immediately opposite the hole: for example, when a carriage turns the corner of a street. But when a sound passes through a small aperture, it is propagated almost equally in every direction.

a. The rate at which sound travels has been determined by firing cannon from two places at some known distance asunder, and measuring the time required for the report to travel from one point to the other. Moll and Van Beek performed such experiments in the neighbourhood of Utrecht in 1823.

The velocity of sound furnishes a ready mode of finding approximately the distance of places. To find how far a thunder-cloud or artillery is from the spectator, count the number of beats of the pulse between the flash and the report; multiply this number by 370, and the product will be the distance in yards nearly. It is supposed that the pulse beats 72 times in a minute.

b. The velocity increases with every degree of Fahrenheit's thermometer above freezing point about 1·1 foot. At a mean temperature of 60°, the air being in an average state of moisture, and perfectly at rest, we may reckon that sound moves 1120 feet in a second.

These results accord with what has been noticed respecting the vibrations made by columns of air in tubes, such as organ-pipes; a column of air intended to make the deepest note in an organ, *i. e.* to make 32 vibrations in 1 second, will therefore require a pipe rather more than 32 feet in length.

c. A band of music does not appear to keep bad time when at a distance, as it would if high notes travelled with greater or less speed than low ones.

§ 256.

The velocity with which a wave of sound advances in water, according to experiment (a), is about 4 times greater than its velocity in air. It amounts to 4708 feet in a second. This is also pretty nearly its rate on ice.

The velocity of sound in solid bodies has scarcely been investigated, as direct experiments can rarely be performed on a sufficiently extended scale. It is generally believed that solid bodies transmit sound most quickly, but with by no means equal velocity. (b)

a. These experiments were made by Colladon and Sturm in the Lake of Geneva. (Compare § 221.)

b. Biot found, by direct experiment, the velocity of sound in cast-iron, using pipes 3000 feet in length, made for the purpose of conveying water into Paris; he informs us that it is 10½ times greater than in air. Chladni has inferred that sound travels in tin 7½, in copper 12, in glass 17, in different kinds of wood from 11 to 17 times quicker than in air.

B. *Strength or Intensity of Sound.*

§ 257.

The force or intensity with which the emotion of sound is excited depends on the violence of the impulse which the conductor of the sound impresses on our organs of hearing. In this view of the matter, the greater or less susceptibility of the organ to appreciate sound forms no part of our inquiry. The causes which immediately affect its intensity are the nature and size of the undulations made by the sonorous body itself; next, the nature of the medium through which the sound is transmitted; and lastly, the distance of the sonorous body.

§ 258.

The greater the mass of the sonorous body, and the greater its vibrations, the denser will be every wave of sound that it produces, and consequently the greater the intensity with which it affects our organs of hearing.

For the reason just assigned, we feel the sound of a large church-bell to be more powerful than the same note when issuing from a small bell, or from any similar instrument; so also the same note struck on a plate of glass or metal is more forcible than when produced by a tuning-fork; the pressure of the bow on the strings of a violin increases the strength of the note; for a similar reason the report of a cannon is louder than that of a musket.

§ 259.

The denser the medium through which the sound is conveyed, and the less it disperses the oscillations of the sonorous body, the greater will be the density of each individual wave of sound, and the more intense the effect.

This explains why a sound conducted by wood, iron, &c., is heard more plainly than if it had passed through air; why, too, sound dies away in hydrogen and in highly rarefied air (Third Exp. § 188.); why the same sound, if heard when the air is cold or at night, is far louder than in the day-time. The difference is most perceptible in the polar regions and in the torrid zone.

The above remarks will account for the following phenomena:—

The ticking of a watch laid at one end of a long beam is distinctly heard, if the ear of an observer be applied to the other end. So also a sound is heard from a greater distance by listening with the ear close to the ground.—Application of this to military purposes by outposts and reconnoitering.

First Exp. Suspend a silver spoon by a piece of whipcord, hold the two ends of the string in your hands, and apply them to the orifice of the ears, and the ring of the metal will be heard with increased strength.

Second Exp. The sound excited by a bell in an exhausted receiver (Third Exp. § 188.) is greatly increased by connecting the bell with the metal rod which passes through the leathern stuffing-box at the top of the receiver.

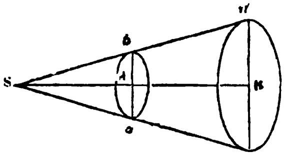
The report of fire-arms on a high mountain is much less loud than when the discharge takes place in the valleys, because of the greater rarity of the atmosphere in elevated regions.

The elasticity of the air is diminished by moisture, for which reason, the human voice, the ringing of a bell, the roar of artillery, the blast of a trumpet, and indeed all sounds whatever, are more dead in foggy weather than when the air is dry and pure.

§ 260.

When the waves of sound expand freely, the progressive undulations of the air become constantly larger in extent, and rarer as they recede from the sonorous body; consequently the density of their parts, which impinge on the ear, decreases in the same ratio, and these parts themselves constitute a smaller proportion of the entire moving mass of air according as the distance becomes greater. Hence the intensity of a sound decreases as the length of its rays increases; the proportion which has been found to obtain is, that the force of the sound decreases as the square of the distance from the sonorous body increases; *i. e.* if the distance be twice as great the sound will be one-fourth as loud, if the distance be three times as great the sound will be one-ninth as loud, &c.

Fig. 167.



This law of the decrease in the intensity of sounds may be deduced from § 227., it may also be inferred from the following considerations: — Let S, *fig.* 167., be the point at which a sound is excited, the rays of sound will consequently emanate from it (§ 253.), like S*d*, S*c*, *&c.* A disc A, at the distance S A = 1, would evidently be impinged upon by the same rays as the disc B, at the distance S B = 2. But

$$
\begin{array}{lcccccc}
 & SA & : & SB & = ab & : & cd \\
 & & & & = 1 & : & 2 \\
\text{and} & A & : & B & = ab^2 & : & cd^2 \\
 & & & & = 1^2 & : & 2^2 \\
 & & & & = 1 & : & 4
\end{array}
$$

i. e. the circular areas are to each other as the squares of the distances; whence at double the distance the same number of rays of sound is spread over four times as large a space, whence it is obvious that one-fourth of the effect will be produced.

§ 261.

The distance at which sounds are audible does not admit of precise measurement. It depends principally on the absolute intensity of the sound itself, next on the nature of the conducting medium; it is frequently affected by a number of accessory circumstances, some of which have been already glanced at; and, lastly, on the delicacy of hearing possessed by individuals. Generally speaking, a sound will be heard farther, the greater its original intensity, and the denser the medium in which it is propagated.

The greatest known distance which sound has been carried through the atmosphere, is 345 miles, as it is asserted that the very violent explosions of the volcano at St. Vincent's have been heard at Demerara. Sound travels farther and more loudly on the earth's surface than through the air. Thus, for instance, in 1806, the cannonading at the battle of Jena was heard in the open fields near Dresden, a distance of 92 miles, though but feebly, whilst in the casements of the fortifications it was heard with great distinctness. So also it is said that the cannonading of the citadel of Antwerp, in 1832, was heard in the mines of Saxony, which are about 370 miles distant.

When the air is calm and dry, the report of a musket is audible at 8000 paces; the marching of a company may be heard on a still night, at from 580—830 paces off; a squadron of cavalry at foot-pace, 750 paces; trotting or galloping at 1080 paces distant; heavy artillery, travelling at a foot-pace, is audible at a distance of 660 paces, if at a trot or gallop, at 1000 paces.—A powerful human voice in the open air, at an ordinary temperature, is audible at a distance of 230 paces, but in the polar regions it is said that a conversation may be easily carried on between two persons a mile apart.

§ 262.

The progress of sound is uniform and undisturbed only so long as it travels in a perfectly uniform medium; if the rays of sound be transmitted into another medium, whether of greater or less density, some of them become broken and reflected, by which the force of the advancing sound is necessarily weakened. As a general rule it may be assumed that the waves of sound mostly lose something of their intensity in passing out of a rare into a dense medium, and inversely, though in the latter case the loss is not so great as in the former. The more frequently they undergo this process, the less the distance at which sounds will be audible. Bodies of loose texture exert the greatest influence on the transmission of sound, as their non-elastic nature prevents their receiving a tremulous motion.

Blows struck on a diving-bell 30 feet below water may be distinctly heard at the surface, but a sound immmediately above the water will not be audible to persons within the bell. A noise in the open air is plainly heard in a room if the window be open. The striking of a clock is rendered less loud by covering the clock with a glass shade or any similar body.—To march unobserved by the enemy the horses' feet are sometimes wrapped in straw and rags, so as to check the propagation of sound.

C. *Phenomena arising from the Reflection of Sound.*

§ 263.

When the waves of air on which sound is being borne, impinge in the course of their expansion, on any objects by which their advance is stopped, the particles of air recoil from such objects, agreeably to the laws regulating the impact of elastic bodies (§ 211.); their return is made with equal velocity and under an equal but opposite angle, to that under which they advanced.

To this subject are applicable the laws developed in § 225., as regulating the undulations of elastic fluids.

§ 264.

If the wave of sound fall perpendicularly on the wall whence it is reflected, it returns in the same direction, and arrives again at the spot from which it originated.

If the place whence the sound arose is not far from that at which the wave is reflected, as, for example, in a room, hall, or church, it happens, in consequence of the velocity with which sound travels, that the returning wave is generated immediately after the original wave, and the ear becomes sensible of both simultaneously; by this means the sound is both prolonged and rendered more loud; it frequently also makes it difficult to hear words and notes distinctly.

An *Echo* is produced when the ear is able to distinguish the original sound from its reflexion: a good ear will perceive clearly about 9 sounds in a second; *i.e.* the sounds must succeed each other at intervals of $\frac{1}{9}$th of a second in order to be heard singly. The sound and the echo are therefore to be regarded as successive sounds, of which the latter will be plainly heard if it fall upon the ear after that organ has formed a distinct perception of the former. The least distance at which this is possible will be that which the wave of sound can traverse, so as to impinge on the resisting body, and return thence as a reflected wave in $\frac{1}{9}$th of a second of time. Now, as sound, at its mean rate of 1120 feet, will travel about 125 feet in $\frac{1}{9}$th of a second, half this distance, or 62—64 feet, must be the least distance at which the reflecting plane can be placed to produce a complete echo.

Echoes that repeat one or more syllables — those that repeat the sound more than once. At Adernach, in Bohemia, is an echo that repeats seven syllables thrice; at Lürley-Fels on the Rhine is one that repeats 17 times. Among the ancients the echo at Capo-di-Bove was famed, as was that in the tomb of the Metelli at Rome, which is said to have distinctly repeated the first verse of the Æneid, which contains fifteen syllables, 8 times distinctly. An echo on the Villa Simonetta, near Milan, which repeated a shout 30 times. An echo in a building at Pavia, which used to answer a question by repeating its last syllable 30 times.

In an experiment by Messrs. Herschel and Babbage at the Suspension Bridge over the Menai Straits, in which they were placed on opposite sides of one of the piers, each could hear distinctly a blow from the other by reflexion at the next pier, though he could scarcely hear the echo of one struck by himself. This is an illustration at once of the rectilinear propagation of sound, and its reflexion at an angle equal to that of incidence.

Exp. Tune two violin-strings nearly to unison. If they be struck together, alternate augmentation and cessation of sound may be observed. This arises from the interference of the waves from each. Thus if one string make 212 and the other 213 vibrations per second, at the half-seconds the vibrations are opposite and destroy each other. A similar effect occurs when two tuning-forks of nearly the same pitch are made to vibrate together over the end of a tube. By observing the time of these *beats*, as they are called, we may determine the difference of the notes, when they are far too near for direct observation even by the finest ears.

§ 265.

The waves of sound generated in every contracted space will be reflected agreeably to the laws already explained; the course of their undulation must therefore depend on the form of the boundaries enclosing such space. Accordingly, on these principles, several acoustic contrivances and buildings are constructed.

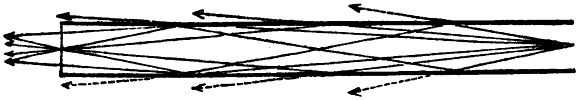
The *speaking tube*, which serves to transmit a whisper further than it would be carried by the air. A similar effect may be noticed in shafts, wells, chimneys; in pipes intended to convey hot air, and in water conduits.

Thus Biot, as has been already observed, heard distinctly at one end of a water-pipe every word whispered at the other end, which was 3000 feet off. On shipboard a speaking tube is not unfrequently carried from the captain's cabin to the top; so also in large establishments to save the attendants running from room to room.

This apparatus acts thus: the divergent rays of sound entering at one end of the tube strike against the column of air enclosed within the tube, whose inner surface again causes these waves to be reflected, so that at length they issue simultaneously from the other opening.

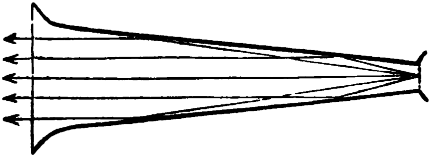
The diagram, *fig.* 168., will perhaps render this more easy to be comprehended. The dotted lines show the direction in which the rays of sound would have diverged in the open air, whilst the dark lines mark the path taken within the tube.

Fig 168.



The *Speaking Trumpet*, commonly funnel-shaped, but best when made of a parabolic form, serves to render the human voice audible at a considerable distance. This object is accomplished by the rays of sound being reflected from the interior of the trumpet, as the words are spoken at the smaller end, in such a manner that they issue from the expanded mouth parallel to each other, and consequently agitate the air in the direction in which the axis of the trumpet is held, more forcibly than they would had they proceeded immediately from the mouth of the speaker. It has been proved experimentally that the efficiency of this instrument increases with its length.

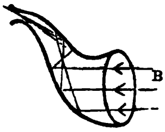
Fig. 169.



The diagram above (*fig.* 169.) will render this intelligible without further remark. As a mean of many experiments it appears that a strong man's voice, sent through a trumpet from 18 to 24 feet in length, may be heard at a distance of 3 miles. — Utility of this instrument at sea.

The *Hearing Trumpet*, which is intended to assist persons who are hard of hearing, and enable them to hear the human voice distinctly, is similar to the speaking trumpet. In fact it inverts the operation, the rays of sound being received at the large opening are so reflected as to become united at the small aperture which is inserted into the ear.

Fig. 170.



The mode in which this instrument acts is shown in *fig.* 170., where the convergence of the rays, and the means by which it is effected, may be at once perceived.

Whispering Galleries are domes or rooms so constructed that words uttered in a low tone at one place are audible at some other, whilst persons standing between these two points cannot hear what is said. To produce this effect the room should have an elliptical form, so that words spoken in one focus (§ 217.) may be heard in the other.

The observatory at Paris contains an apartment of this kind. The ear of Dionysius is celebrated in ancient history; it was a grotto cut out of the solid rock at Syracuse, in which a person placed at one point, could hear the faintest whisper spoken in the grotto. Probably it was a parabola in shape, as may be inferred from § 217.

Music Halls, Theatres, and similar buildings, intended for purposes of hearing, must be constructed in such a manner as to diffuse the sounds that are uttered throughout the part occupied by the audience unimpaired by any echo or resound. It is, therefore, of the utmost importance to give to the walls of these edifices such a form as will cause the waves of sound to fulfil these conditions. On theoretical grounds (§ 217.) the parabolic form would best answer the purpose. Further, every thing must be avoided that would interfere with a uniform diffusion of the sound; all needless hollow and projecting spaces, as generating echoes, are to be avoided.

The following experiment will illustrate, in a very simple manner, the consequences arising from the reflection of the rays of sound from the interior of a parabola.

Exp. Place a watch in the focus *a* of the mirror *m d n* (*fig.* 152.), the best form is that of the parabola (see § 217.), and all the rays of sound that fall on the concave surface will be reflected in the direction indicated by the arrows. The ticking of the watch will be plainly heard within the space in which the rays fall, but it will not be audible at a small distance on either side.

Now place a second reflector, *o r p*, opposite to the former, *m d n*, and at some distance from it, the rays of sound will be received by it and thrown into the focus *b*. If the ear, or, better still, the mouth of a hearing-trumpet, be applied to this point, the ticking of the watch will be heard as plainly as at *a*.

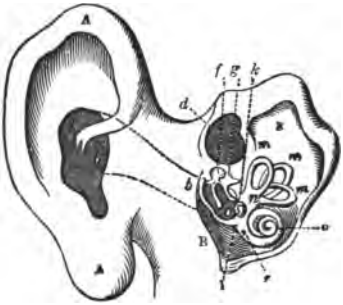
D. *Perception of Sound by the Organs of Hearing.*

§ 266.

The *ear* is that organ by means of which we are made sensible of the various undulations of sonorous bodies, the usual conductor of which is the atmosphere.

This ingeniously constructed organ is generally divided into the *external*, the *middle*, and the *internal ear*. The external ear consists of a cartilaginous funnel, A A, *fig.* 171., called the *ala* or wing, the pendent part is named the *lobe*, and the *meatus auditorius*, *b b*,

Fig. 171.



which is a canal leading to the interior parts of this organ; its inner extremity is closed by an elastic membrane, the *membrana tympani*, or drum, *c*. Immediately beyond this membrane, is the middle ear, or *cavity of the tympanum*, B B, a small opening in the hardest bone of the skull. It communicates with the mouth by means of the *Eustachian tube*, *d*, a canal through which it is filled with the air we breathe; no air is admitted to it from the *meatus auditorius*, the aperture being closed by the *membrana tympani*. In the cavity of the tympanum are three small bones, the *malleus*, *f*, the *incus*, *g*, with the *os orbiculare*, on which is articulated the *stapes*, *k*, so called from its supposed resemblance to a stirrup. These bones form a connected chain, the first link, viz. the malleus, *f*, being attached to the membrana tympani, and the last, the stapes, *k*, being fixed with its basis in the *fenestra ovalis*, *l*, which is a small aperture closed by a fine membrane, and leading into the inner ear or *labyrinth*. The labyrinth is likewise a cavity lying behind the cavity of the tympanum, with which it communicates by the small aperture of the fenestra ovalis. The labyrinth is composed of three parts: the *vestibulum*, *n*, the three *semicircular canals*, *m*, placed at right angles to each other, and a spiral course *cochlea*, *o*, not unlike the shell of a snail. Its interior is divided by a thin lamina or septum into an upper canal, which opens behind the fenestra ovalis into the vestibulum, and a lower one that communicates through the *fenestra rotunda*, *r*, with the cavity of the tympanum. The interior of the labyrinth is almost filled with a certain fluid. The *auditory nerve* divides itself into two bundles of fine threads, floating in this fluid, one bundle terminating in the vestibulum, and the other in the skull.

Each of the senses by whose aid we obtain a knowledge of what is transpiring in the external world around us, has its own appropriate nerve, which, on being affected, conveys to our sensorium that peculiar information of which it is the channel. The auditory nerve is therefore so formed, and its subordinate organs are so arranged, that it receives a tremulous motion from the waves of sound propagated in the air, which tremor it conveys immediately to the sensorium. This nerve runs to both sides of the brain: it alone produces the sensation of hearing, whilst the bundles of nerves distributed in the interior of this organ excite the motion necessary to hearing.

§ 267.

Thus much is clear with regard to the office of the different parts and the functions they discharge in the sense of hearing, that the external ear receives the waves of sound in its funnel-shaped cavity, and transmits them along the auditory passage to the membrana tympani. This, like any other membrane similarly circumstanced, is set vibrating, and these pulsations, by the three small bones are carried to the labyrinth and the nerves there distributed.

The Eustachian tube serves to keep the cavity of the tympanum always filled with air of uniform temperature and density, which appears to be indispensably necessary to the parts performing their different functions with accuracy.

The vertical, oblique, and horizontal semicircular canals appear to constitute an organ for distinguishing the direction of sound; the skull itself seems admirably adapted for the musical relations of sounds.

APPENDIX.

TABLE I. § 21. p. 14.

For comparing different miles.

(From Gehler's phys. Wörterb. *n.* A. Bd. 6. 3 Abth. S. 1778.)

Name of the State.	Name of the Mile.	Subdivisions in the respective Countries.	Paris Feet.	Number of Miles to 1°.
Austria -	mile	24000 Vienna feet -	23355·0	14·67
Baden -	mile	29630 Baden feet -	27366·6	12·50
Bavaria -	great mile	- - -	39425·8	8·69
	less mile	- - -	24212·7	14·15
	chaussée mile	- - -	22826·6	14·50
Brunswick -	mile	2811·2 Rheinland roods	32569·4	10·52
Denmark -	mül	2400 roods=4000 faden	23165·0	14·79
England -	statute mile	8 furlongs=1760 yards	4956·6	69·12
	geog. mile	- - -	5710·1	60·00
	league	1 league=3 miles -	17130·5	20·00
France -	myriametre	1000 metres - -	30784·4	11·11
	poste	400 toises - -	24000·0	14·27
	lieue	- - -	13704·4	25·00
	sea mile	- - -	17130·5	20·00
Hanover -	mile	32000 Calenberg feet -	28800·0	11·89
Holland -	myl	- - -	18032·1	19·00
Hungary -	mile	- - -	25760·2	13·30
Italy - -	miglio	1000 geographical paces	5710·1	60·00
Netherlands	stunde	- - -	17117·8	19·67
	sea mile	- - -	17130·5	20·00
Poland -	mile	- - -	17130·5	20·00
Portugal -	legua	- - -	19034·0	18·00
Prussia -	mile	2000 roods=24000 feet	23113·0	14·78
Russia -	verst	1500 arschines -	3284·8	104·30
Saxony, k. of	Landesvermessungs meile	12000 Dresden ells -	20907·8	16·21
	post mile	13100 Dresden ells -	22803·3	15·00
	old police mile	16000 Dresden ells -	27877·1	12·29
Sweden -	mile	18000 ells - -	32911·6	10·41
Switzerland	mile	- - -	25760·2	13·30
Spain -	- -	5000 veras - -	12882·0	26·63
Turkey -	Berri	- - -	5138·9	66·67
	sea mile	- - -	3965·4	86·47

TABLE II. § 61. p. 57.

Table of Simple Substances, their Symbols and Equivalents.

Names.	Symbols.	Equivalents.		Symbols of important Compounds.
		Hydrogen = 1	Oxygen = 100	
Aluminium	Al.	13·69	171·17	$Al_2\ O_3$, Alumina
Antimony Stibium	Sb.	129·03	1612·90	Sb. O_3, Oxide of Antimony
Arsenic	As.	75·	937·50	As. O_3, Arsenious Acid
Barium	Ba.	68·64	858·	Ba. O, Baryta
Bismuth	Bi.	70·95	886·92	Bi. O, Oxide of Bismuth
Boron	B.	10·90	136·20	B. O_3, Boracic Acid
Bromine	Br.	78·26	978·30	Br. H, Hydrobromic Acid
Cadmium	Cd.	55·74	696·77	Cd. O, Oxide of Cadmium
Calcium	Ca.	20·	250·	Ca. O, Lime
Carbon	C.	6·	75·	C. O_2, Carbonic Acid
Cerium	Ce.	46·	575·	Ce. O, Oxide of Cerium
Chlorine	Cl.	35·50	443·75	Cl. H, Hydrochloric Acid
Chromium	Cr.	28·15	351·82	Cr. O_3, Chromic Acid
Cobalt	Co.	29·52	368·99	Co. O, Oxide of Cobalt
Copper Cuprum	Cu.	31·66	395·70	Cu. O, Oxide of Copper
Didymium		not ascertained		
Fluorine	F.	18·70	233·80	H. F, Hydrofluoric Acid
Glucinum	Gl.	26·50	331·26	$Gl_2\ O_3$, Glucina
Gold Aurum	Au.	93·33	1229·16	$Au_2\ O_3$, Sesquioxide of Gold
Hydrogen	H.	1.	12·5	H. O, Water
Iodine	I.	126·36	1579·50	H. I. Hydriodic Acid
Iridium	Ir.	98·68	1233·50	Ir. O, Protoxide of Iridium
Iron Ferrum	Fe.	28·	350·	$Fe_2\ O_3$. Sesquioxide of Iron
Lanthanum	Ln.	48·	600·	Ln. O, Oxide of Lanthanum
Lead Plumbum	Pb.	103·56	1295·50	Pb. O, Oxide of Lead
Magnesium	Mg.	12·67	158·35	Mg. O, Magnesia
Manganese	Mn.	27·67	345·90	Mn. O, Protoxide of Manganese
Mercury Hydragyrum	Hg.	100·07	1250·9	Hg. O, Oxide (Red Oxide)
Molybdenum	Mo.	47·88	598·52	Mo. O_3, Molybdic Acid
Nickel	Ni.	29·57	369·68	Ni. O, Oxide of Nickel
Niobium				
Nitrogen	N.	14·	175·	NO_5. Nitric Ac. NH_3, Ammonia
Osmium	Os-	99·56	1244·49	Os. O_4, Osmic Acid
Oxygen	O.	8·	100·	
Palladium	Pd.	53·27	665·90	Pd. O, Protoxide of Palladium
Pelopium				
Phosphorus	Ph.	32·02	400·3	Ph. O_5, Phosphoric Acid
Platinum	Pt.	98·68	1233·50	Pt. O_2, Binoxide of Platinum
Potassium Kalium	K.	39·	487·50	K. O, Potash
Rhodium	R.	52·11	651·39	R. O, Protoxide of Rhodium
Rhutenium	Ru.	52·11	651·39	Ru_2. O_3, Sesquioxide of Rhutenium
Selenium	Se.	39·57	494·58	Se. O_3, Selenic Acid
Silicon	Si.	21·35	266·82	Si. O_3, Silicic Acid, or Silica
Silver Argentum	Ag.	108·	1350·	Ag. O, Oxide of Silver
Sodium Natronium	Na.	22·97	287·17	Na. O, Soda

TABLE II.—*continued.*

Names.	Symbols.	Equivalents.		Symbols of important Compounds.
		Hydrogen = 1	Oxygen = 100.	
Strontium	Sr.	43·84	548·02	Sr. O, Strontia
Sulphur	S.	16·	200·	S. O_3, Sulphuric Acid
Tantalum	Ta.	92·30	1153·72	Ta. O_3, Tantalic Acid
Tellurium	Te.	66·14	801·76	Te. O_3, Telluric Acid
Thorium	Th.	59·59	744·90	Th. O, Oxide of Thorium
Tin (Stannum)	Sn.	58·82	735·29	Sn. O, Protoxide of Tin
Titanium	Ti.	24·29	303·66	Ti. O_2, Titanic Acid
Tungsten (Wolfram)	W.	94·64	1183·	W. O_3, Tungstic Acid
Uranium	U.	60·	750·	U. O, Oxide of Uranium
Vanadium	V.	68·55	856·89	V. O_3, Vanadic Acid
Yttrium	Y.	32·20	402·51	Y. O, Yttria
Zinc	Zn.	32·52	406·59	Zn. O, Oxide of Zinc
Zirconium	Zr.	33·62	420·2	Zr_2. O_3, Zirconia
				This Table, which contains the Equivalents on both the Hydrogen and the Oxygen scales, corrected up to the period of the latest investigations, has been copied from Professor Graham's Elements of Chemistry.

TABLE III. § 88. p. 76.

Of the absolute Strength of some of the most important Substances.

(Eytelwein's Handbuch der Statik fester Körper.)

Bodies in the form of Prisms, the area of the end being 1 sq. inch.		
Name.	Lb. avoir.	Observations.
1. *Metals.*		If any of these measures of the absolute strength of materials be applied practically, it will be advisable, on account of the various results obtained, to reckon the strength of metals at only one half, and that of woods and cords at one third of their estimated value. Tredgold gives the following rule for finding the weight in lbs. which any hempen rope will be capable of supporting: Multiply the square of the cir-
Steel, untempered -	110,690 — 127,094	
———, tempered -	114,794 — 153,741	
Iron, bar - -	53,182 — 84,611	
—— plate, rolled -	53,920	
—— wire - -	58,730 — 112,905	
Cast-iron - -	16,243 — 19,464	
Platinum-wire - -	56,392	
Silver, molten - -	40,997	
Copper, molten - -	20,320 — 37,380	
———, hammered -	37,770 — 39,968	
Brass, molten - -	17,947 — 19,472	
——— wire - -	47,114 — 58,931	
——— plate - -	52,240	
Gold - - -	20,490 — 65,237	
Tin - - -	3,228 — 6,666	
Bismuth - -	3,137	
Zinc - - -	2,820	
Antimony - -	1,062	
Lead, molten - -	887 — 1,065	
——— wire - -	2,543 — 3,823	

Table III.—*continued.*

Name.	Lb. avoir.	Observations.
2. *Woods.*		cumference in inches by 200, and the product will be the quantity sought.
Oak - - -	10,367 — 25,851	
Alder - - -	11.453 — 21,730	
Lime - - -	6,991 — 20.796	
Box - - -	14,210 — 24,043	
Pinus sylv. - -	17,056 — 20,395	
Ash - - -	13,480 — 23,455	
Elm - - -	13,530 — 15,040	
Pine - - -	10,038 — 14,965	
Fir - - -	6,991 — 12,876	
3. *Cords.*		
Of Hemp twisted, ½ to 1 in. thick - -	8,746	
——— 1 to 3 in. thick -	6,800	
——— 3 — 5 — -	5,345	
——— 5 — 7 — -	4,860	

Table IV. § 88. p. 78.

Name.	*f.*	M.
1. *Metals.*		
Malleable iron - -	17,800 lb.	475 lb.
Hammered iron - -	- -	487 —
Cast-iron - - -	15,300 —	450 —
Brass - - - -	6,700 —	506·25 —
Zinc - - - -	5,700 —	439·25 —
Tin - - - -	2,800 —	455·7 —
Lead - - - -	1,500 —	709·5 —
2. *Woods.*		
Oak - - -	3,960 —	52 —
Fir (red or yellow) - -	4,290 —	34·8 —
Pine (American yellow) -	3.900 —	26·75 —
Fir (white) - - -	3,630 —	29·3 —
Ash - - - -	3,540 —	47·5 —
Elm - - - -	3 240 —	34 —

Table V. § 88. p. 82.

Of the Weights required to crush some of the most important Materials.

(Eytelwein's Handbuch der Statik fester Körper 2r. Bd.)

Name.	On the Square Inch.
1. *Metals.*	
Cast-iron - - -	115813—177776 lb.
Brass, fine - - -	164864
Copper, molten - -	117088
———hammered - -	103040
Tin, molten - - -	15456
Lead, molten - -	7728

TABLE V. — *continued.*

Name.	On the Square Inch.
2. *Woods.*	
Oak	3860—5147
Pine	1928
Pinus sylv.	1606
Elm	1294
3. *Stones.*	
Gneiss	4970
Sandstone, Rothenburg	2556
Brick, well baked	1092

§ 88. Tredgold, in his "Practical Essay on the Strength of Cast Iron," mentions an experiment by Muschenbröek on a piece of bar iron, which gives 63,286 lb. for the weight that would tear asunder a square inch of the specimen he examined. He refers also to a specimen tested by Captain S. Brown, which tore asunder with a weight of 16,265 lb.; and two experiments made by Mr. G. Rennie, where the bars respectively required a force of 18,656 lb. and 19,488 lb. per square inch, to produce rupture.

"The resisting body is supposed in the theory to be perfectly homogeneous, or composed of parallel fibres, equally distributed around the axis, and presenting uniform resistance to rupture. But this is not the case in a beam of timber; for by tracing the process of vegetation, it has been found that the ligneous coats of a tree, formed by its annual growth, are almost concentric; and that they are like so many hollow cylinders thrust into each other, and united by a kind of medullary substance which offers but little resistance: these hollow cylinders, therefore, furnish the chief resistance to the force which tends to break them. Now, when the trunk of a tree is squared, in order that it may be converted into a beam, it is evident that all the ligneous cylinders greater than the circle inscribed in the square or rectangle, which is the section of the beam, are cut off at the sides; and therefore, as Montucla remarks, almost the whole resistance arises from the cylindric trunk inscribed in the solid part of the beam. The portions of the cylindric coats which are towards the angles, add a little, it is true, to the strength of that cylinder, as they cannot fail to oppose *some* resistance to the straining force; but it is far less than though the ligneous cylinder were entire. Hence we cannot, by legitimate comparison, accurately deduce the strength of a joist cut from a small tree, by experiments on another which has been sawn from a much larger tree or block; the latter is generally weak, and very liable to break. As to the concentric cylinders we have been speaking of, they are evidently not all of equal strength. Those nearest the centre being the oldest, are likewise the hardest; which again is contrary to the theory, in which they are supposed to be uniform throughout. After all, however, it is still found that in some of the most important problems the results of the theory and well conducted experiments coincide, even with regard to timber; thus, for example, the experiments of Duhamel on rectangular beams, afford results deviating but in a slight degree from the theorem of Galileo, that the strength is proportional to the product of the breadth into the square of the depth." — *Gregory's Mechanics*, vol. i. p. 121.

Rennie says, that cast-iron may be crushed with a weight of 93,000 lb., and brick with one of 562 lb.

TABLE VI. § 174. p. 173.

Of the Densities and Volumes of Water at different Degrees of Heat (according to Stampfer), for every 2¼ Degrees of Fahrenheit's Scale.

(Jahrbuch des Polytechnischen Institutes in Wien, Bd. 16. S. 70.)

t Temperature.	*d* Density.	Diff.	*v* Volume.	Diff.
32	·999887		1·000113	
34¼	·999950	63	1·000050	63
36½	·999988	38	1·000012	38
38¾	1·000000	12	1·000000	12
41	·999988	12	1·000012	12
43¼	·999952	35	1·000047	35
45½	·999894	58	1·000106	59
47¾	·999813	81	1 000187	81
50	·999711	102	1·000289	102
52¼	·999587	124	1·000413	124
54½	·999442	145	1·000558	145
56¾	·999278	164	1·000723	165
59	·999095	183	1 000906	183
61¼	·998893	202	1·001108	202
63½	·998673	220	1·001329	221
65¾	·998435	238	1·001567	238
68	·998180	255	1·001822	255
70¼	·997909	271	1·002095	273
72½	·997622	287	1·002384	289
74¾	·997320	302	1·002687	303
77	·997003	317	1·003005	318
79¼	·996673	330	1·003338	333
81½	·996329	344	1·003685	347
83¾	·995971	358	1·004045	360
86	·995601	370	1·004418	373
88¼	·995219	382	1·004804	386
90½	·994825	394	1·005202	398
92¾	·994420	405	1·005612	410
95	·994004	416	1·006032	420
97¼	·993579	425	1·006462	430
99½	·993145	434	1·006902	440

To correct the specific gravity of a body found by weighing it in water at any other temperature than that of its greatest density, viz. 38¾°, multiply the specific gravity by the number which expresses the density of water at the particular temperature at which the body was weighed; thus, for instance, if a body be weighed in water at 59°, the density calculated for the temperature *t* of the water should be multiplied by ·999095.

In like manner use the factor *v* to determine how much the volume of any mass of water, at a temperature *t*, exceeds that of the same mass at its maximum density, multiplying it by the number expressing the *v* at the particular temperature.

TABLE VII. § 174. p. 173.

Of the Specific Gravities of some of the most important Bodies.

(The density of distilled water is reckoned in this table at its maximum 38¾° F. = 1·000.)

Name of the Body.	Specific Gravity.	
I. SOLID BODIES.		
A. METALS.		
Antimony (of the laboratory)	4·2 — 4·7	
Brass	7·6 — 8·8	
Bronze for cannon, according to Lieut. Matzka	8·414 — 8·974	
Do. mean	8·758	
Copper, molten	7·788 — 8·726	
Do. hammered	8·878 — 8·9	
Do. wire-drawn	8·78	
Gold, molten	19·238 — 19·253	
Do. hammered	19·361 — 19·6	
Iron, wrought	7·207 — 7·788	
Do. cast, a mean	7·251	
Do. grey	7·2	
Do. white	7·5	
Do. for cannon, a mean	7·21	
Lead, pure molten	11·3303	
Do. flattened	11·388	
Platinum, native	16·0 — 18·94	
Do. molten	20·855	
Do. hammered and wire-drawn	21·25	
Quicksilver, at 32° F.	13·568 — 13·598	
Silver, pure molten	10·474	
Do. hammered	10·51 — 10·622	
Steel, cast	7·919	
Do. wrought	7·840	
Do. much hardened	7·818	
Do. slightly	7·833	
Tin, chemically pure	7·291	
Do. hammered	7·299 — 7·475	
Do. Bohemian and Saxon	7·312	
Do. English	7·291	
Zinc, molten	6·861 — 7·215	
Do. rolled	7·191	
B. BUILDING STONES.		
Alabaster	2·7 — 3·0	
Basalt	2·8 — 3·1	
Dolerite	2·72 — 2·93	
Gneiss	2·5 — 2·9	
Granite	2·5 — 2·66	
Hornblende	2·9 — 3·1	
Limestone, various kinds	2·64 — 2·72	
Phonolite	2·51 — 2·69	
Porphyry	2·4 — 2·6	
Quartz	2·56 — 2·75	
Sand stone, various kinds, a mean	2·2 — 2·5	
Stones for building	1·66 — 2·62	
Syenite	2·5 — 3·	
Trachyte	2·4 — 2·6	
Brick	1·4 — 1·86	
C. WOODS.	Fresh-felled.	Dry.
Alder	0·8571	0·5001
Ash	0·9036	0·6440
Aspen	0·7654	0·4302
Birch	0·9012	0·6274
Box	0·9822	0·5907
Elm	0·9476	0·5474
Fir	0·8941	0·5550
Hornbeam	0·9452	0·7695
Horse-chesnut	0·8614	0·5749
Larch	0·9206	0·4735
Lime	0·8170	0·4390
Maple	0 9036	0·6592
Oak	1·0494	0·6777
Do. another specimen	1·0754	0·7075
Pine, *Pinus Abies Picea*	0·8699	0·4716
Do. *Pinus Sylvestris*	0.9121	0·5502
Poplar (Italian)	0·7634	0·3931
Willow	0·7155	0·5289
Do. White	0·9859	0·4873
D. VARIOUS SOLID BODIES.		
Charcoal, of cork	0·1	
Do. soft wood	0·28 — 0·44	
Do. oak	1·573	
Coal	1·232 — 1·510	
Coke	1·865	
Earth, common	1·48	
rough sand	1·92	
rough earth with gravel	2·02	
moist sand	2·05	
gravelly soil	2·07	
clay	2·15	

Table VIII.—*continued.*

Name of the Body.	Specific Gravity.	Name of the Body.	Specific Gravity.
Earth, clay or loam, with gravel	2·48	Lime, unslacked	1·842
Flint, dark	2·542	Resin, common	1·089
Do. white	2·741	Rock salt	2·257
Gunpowder, loosely filled in—		Saltpetre, melted	2·745
coarse powder	0·886	Do. crystallised	1·900
musket do.	0·992	Slate pencil	1·8 — 2·24
Do. slightly shaken down—		Sulphur	1·92 — 1·99
musket-powder	1·069	Tallow	0·942
Do. solid	2·248 — 2·563	Turpentine	0·991
Ice	0·916 — 0·9268	Wax, white	0·969
		Do. yellow	0·965
		Do. shoemaker's	0·897

II. LIQUIDS.

Name of the Body.	Specific Gravity.	Name of the Body.	Specific Gravity.
Acid, acetic	1·063	Ether, acetic	0·866
Do. muriatic	1·211	Do. muriatic	0·845 — 0·874
Do. nitric, concentrated	1·521 — 1·522	Do. nitric	0·886
Do. sulphuric, Eng.	1·845	Do. sulphuric	0·715
Do. concentrated (Nordh.)	1·860	Oil, linseed	0·928 — 0·953
Alcohol, free from water	0·792	Do. olive	0·915
Do. common	0·824 — 0·79	Do. turpentine	0·792 — 0·891
Ammoniac, liquid	0·875	Do. whale	0·923
Aquafortis, double	1·300	Quicksilver	13·568 — 13·598
Do. single	1·200	Water, distilled	1·000
Beer	1·023 — 1·034	Do. rain	1·0013
		Do. sea	1·0265 — 1·028
		Wine	0·992 — 1·038

III. GASES.

Name of the Body.	Water = 1. Temp. 38¾° F.	Barometer 30 In. Temp. = 34°.	Name of the Body.	Water = 1. Temp. 38¾° F.	Barometer 30 In. Temp. = 34°.
Atmospheric air = $\frac{1}{770}$ =	0·00130	1·0000	Hydrosulphuric acid gas	0·00155	1·1912
Carbonic acid gas	0·00198	1·5240	Muriatic acid gas	0·00162	1·2474
Carbonic oxide gas	0·00126	0·9569	Nitrogen	0·00127	0·9760
Carbureted hydrogen, a maximum	0·00127	0·9784	Oxygen	0·00143	1·1026
Do. from coals	0·00039 0·00085	0·3000 0·5596	Phosphureted hydrogen gas	0·00113	0·8700
Chlorine	0·00321	2·4700	Steam at 212° F.	0·00082	0·6235
Hydriodic gas	0·00577	4·4430	Sulphurous acid gas	0·00292	0·2470
Hydrogen	0·0000895	688			

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